Magnetic Helicity Fluxes and their Effects in Dynamo Theory

Licentiate Thesis

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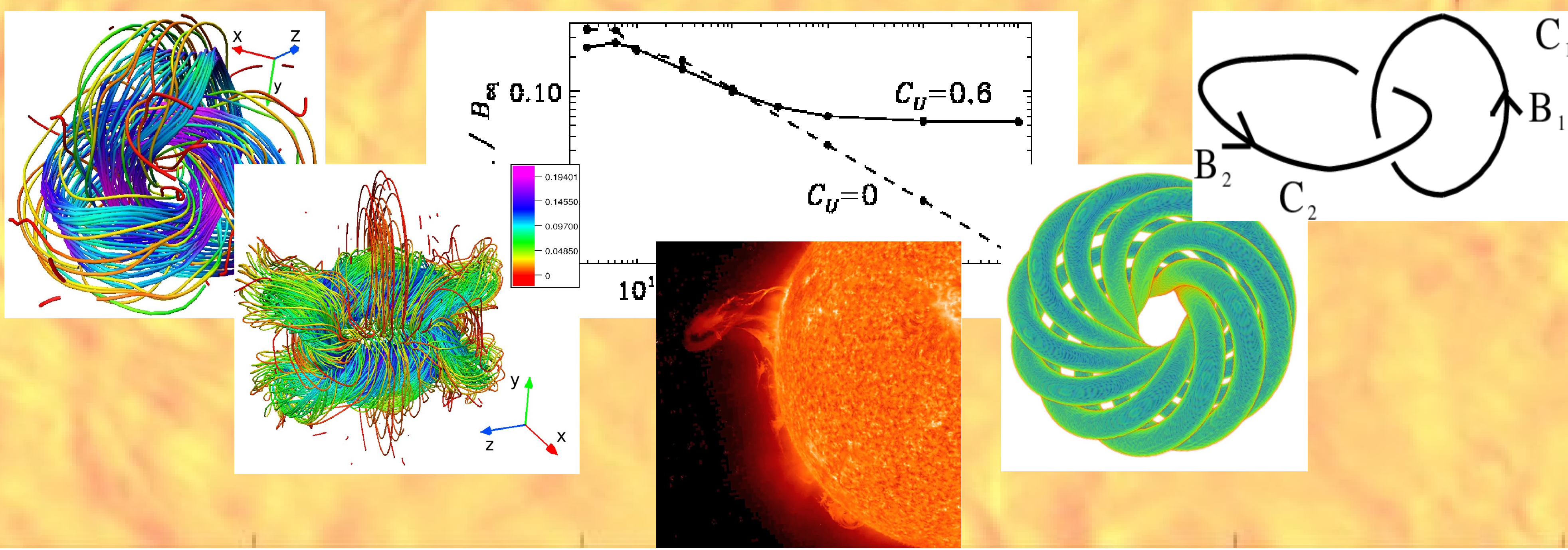
2011-02-11

Stockholm University



Papers included in the thesis

I. **Magnetic-field decay of three interlocked flux rings with zero linking number.**
Del Sordo F., Candelaresi S. and Brandenburg A.
Phys. Rev. E, 81:036401, Mar 2010

II. **Decay of trefoil and other magnetic knots.**
Candelaresi S., Del Sordo F. and Brandenburg A.
arXiv:1011.0417

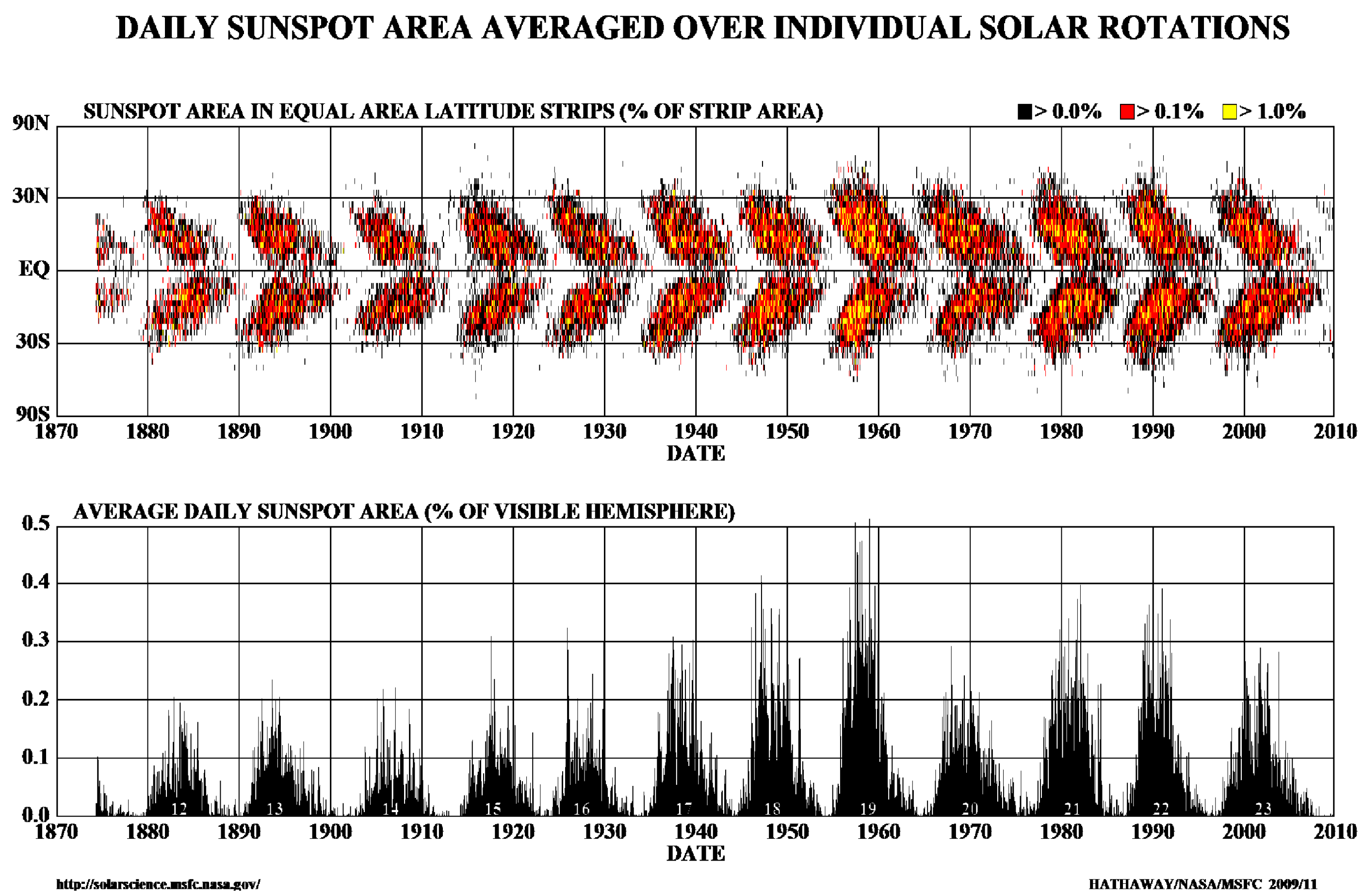
III. **Small-scale magnetic helicity losses from a mean-field dyamo**
Brandenburg A., Candelaresi S. and Chatterjee P.
Mon. Not. Roy. Astron. Soc., 398:1414-1422, September 2009

IV. **Equatorial magnetic helicity flux in simulations with different gauges.**
Mitra D., Candelaresi S., Chatterjee P., Tavakol R. and Brandenburg A.
Astronomical Notes, 331:130-135, January 2010

V. **Magnetic helicity transport in the advective gauge family.**
Candelaresi S., Hubbard A., Brandenburg A. and Mitra D.
Physics of Plasmas, 18:012903, January 2011

Introduction

11 year cycle



→ dynamo working

Introduction

plasma → currents → induction → dynamo effect

magnetohydrodynamics MHD

induction equation: $\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J})$

mean and fluctuating fields: $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$

$$\partial_t \overline{\mathbf{B}} = \eta \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}})$$

mean electromotive force (emf): $\overline{\boldsymbol{\mathcal{E}}} = \overline{\mathbf{u} \times \mathbf{b}}$

coupling with the mean field: $\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\mathbf{B}} - \eta_{\mathrm{t}} \boldsymbol{\nabla} \times \overline{\mathbf{B}}$

Introduction

$$\alpha = \alpha_{\rm K} + \alpha_{\rm M}$$

$$\alpha_{\rm K} = -\tau\overline{\boldsymbol{\omega}\cdot\mathbf{u}}/3 \qquad \alpha_{\rm M} = \tau\overline{\mathbf{j}\cdot\mathbf{b}}/(3\overline{\rho})$$

magnetic helicity density: $h_{\rm M} = \mathbf{A}\cdot\mathbf{B}$

helically driven dynamo $\overline{h}_{\rm K,f} = \overline{\boldsymbol{\omega}\cdot\mathbf{u}}$

↓

current helicity $\overline{h}_{\rm C,f} = \overline{\mathbf{j}\cdot\mathbf{b}}$

↓

production of magnetic helicity $\overline{h}_{\rm M,f} = \overline{\mathbf{a}\cdot\mathbf{b}}$ → $-\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}$

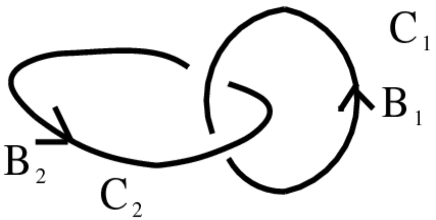
$\overline{\mathbf{a}\cdot\mathbf{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^9$ galaxies: ${\rm Re}_{\rm M} = 10^{29}$

Topological Interpretation

$$H_\mathrm{M} = \int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V = 2n_{ij}\phi_i\phi_j$$

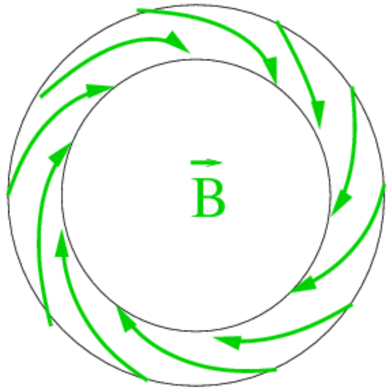
$$\phi_i = \int_{S_i} \mathbf{B}\cdot\mathrm{d}\mathbf{S}$$



Realizability condition:

$$E_\mathrm{m}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.



twisted field

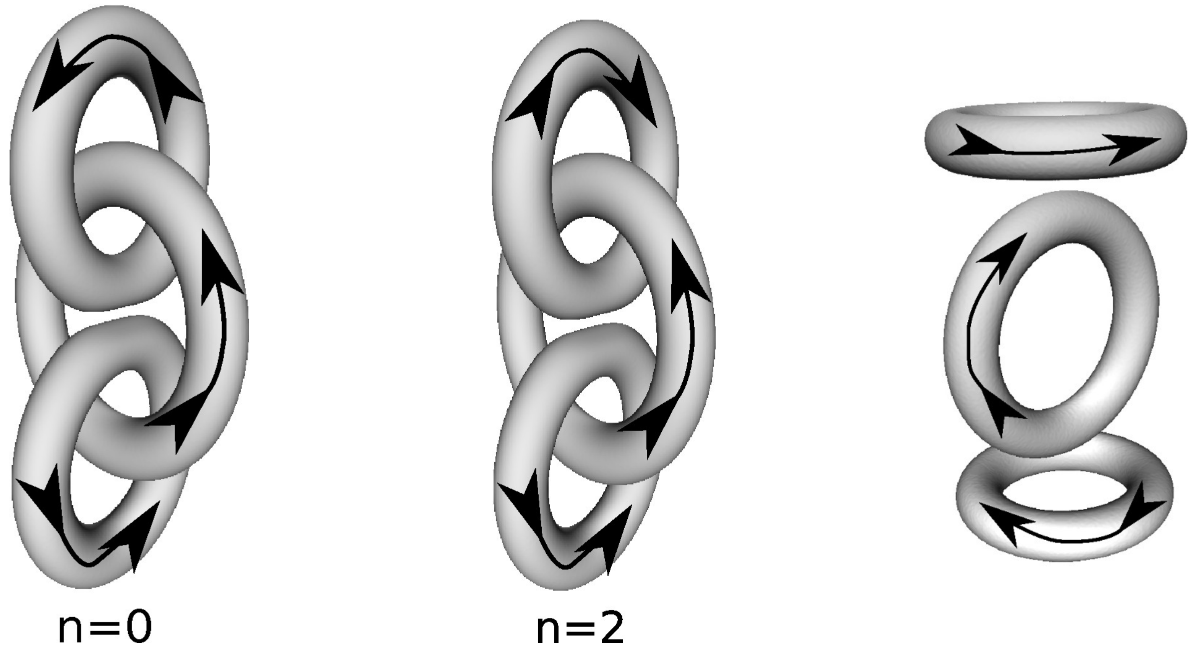
magnetic helicity conservation

$$\mathrm{Re}_\mathrm{M} \to \infty$$

$$\frac{dH_\mathrm{M}}{dt} = 0$$

trefoil knot

Topological Interpretation



compressible isothermal fluid

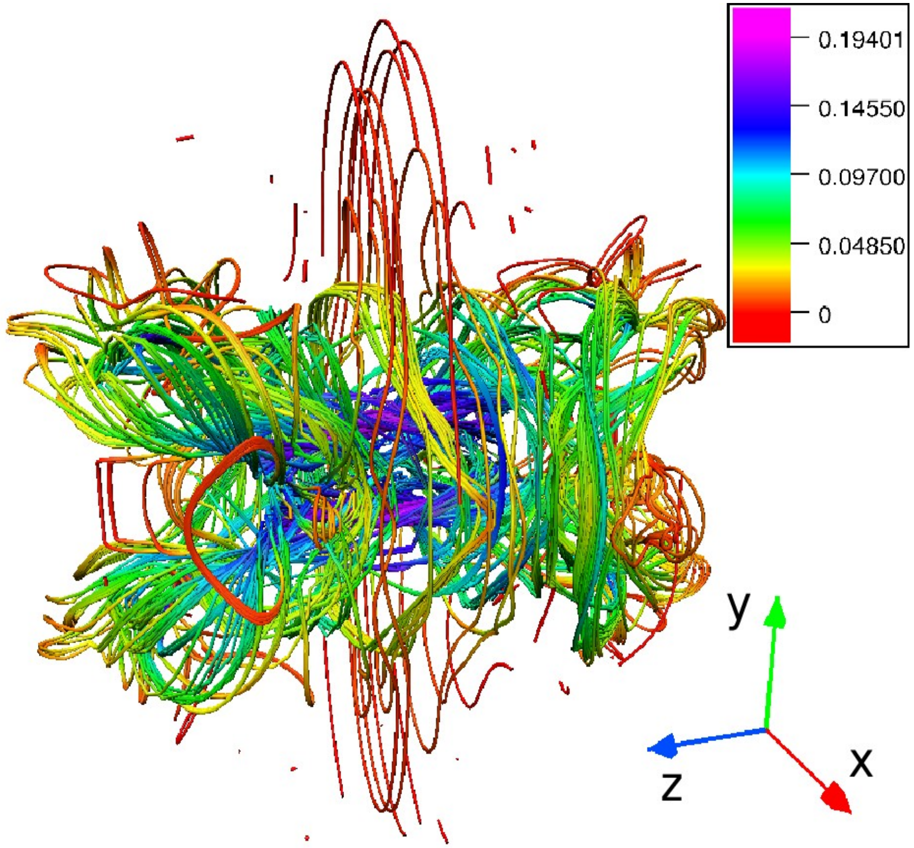
periodic boundary conditions

$$\mathrm{Re}_{\mathrm{M}} = \frac{UL}{\eta} = 10^3$$

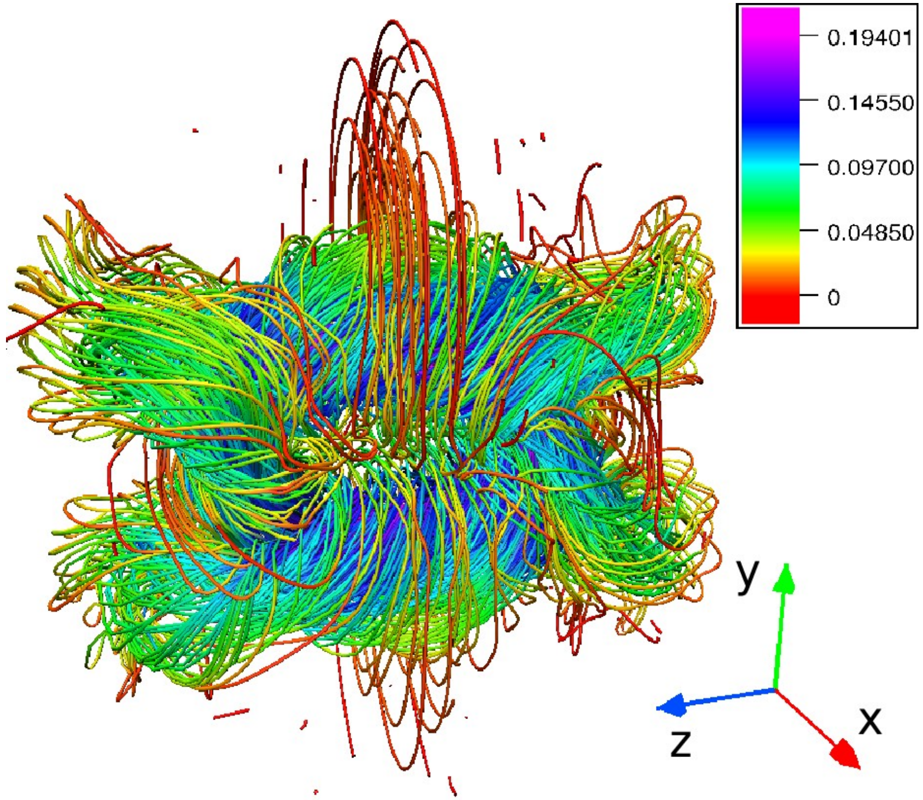
$$\Rightarrow \quad \frac{dH_{\mathrm{M}}}{dt} \approx 0$$

Topological Interpretation

$t = 4T_{\mathrm{A}}$ $\quad\quad$ $T_{\mathrm{A}} = \sqrt{\mu_0 \rho_0} R_0^3 / \phi$



$H = 0$



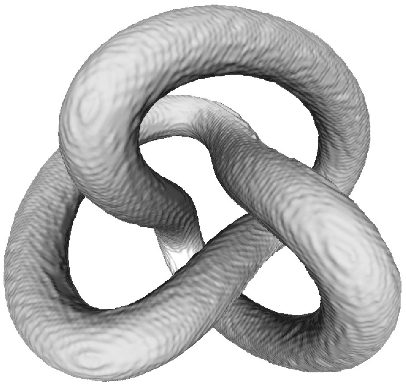
$H \neq 0$

Topological Interpretation

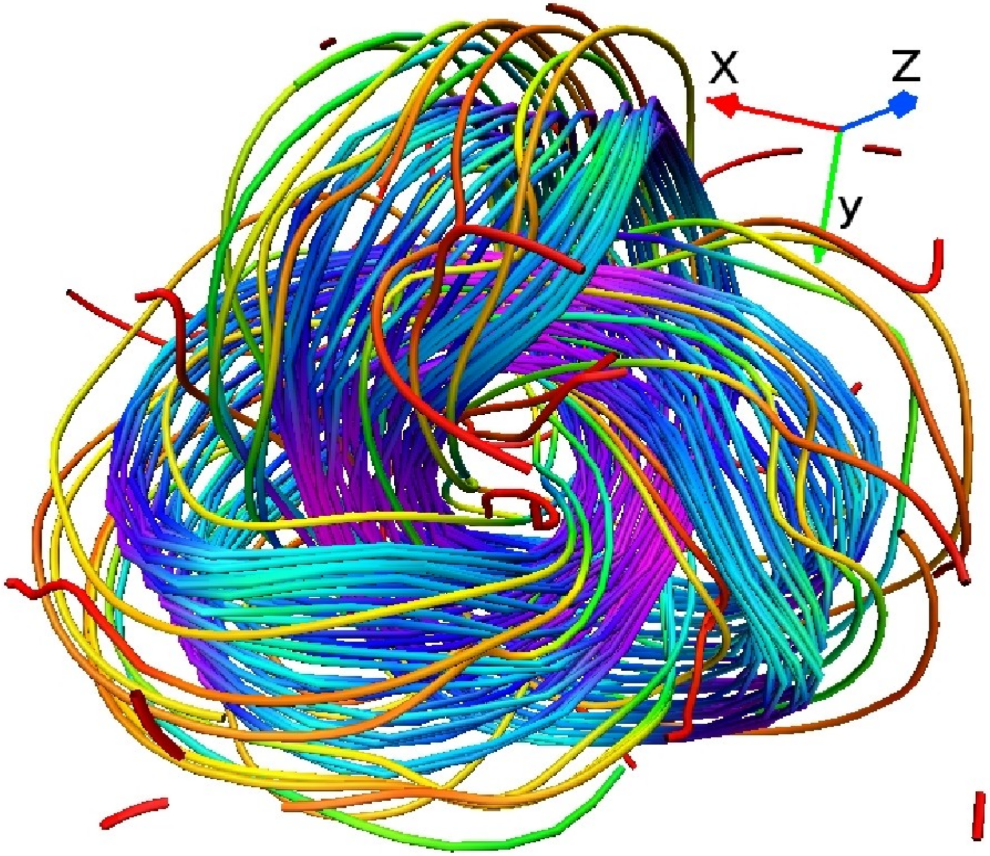
$$H_\mathrm{M} = \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V = 2 n_{ij} \phi_i \phi_j$$

$t = 5T_\mathrm{A}$

$t = 0$

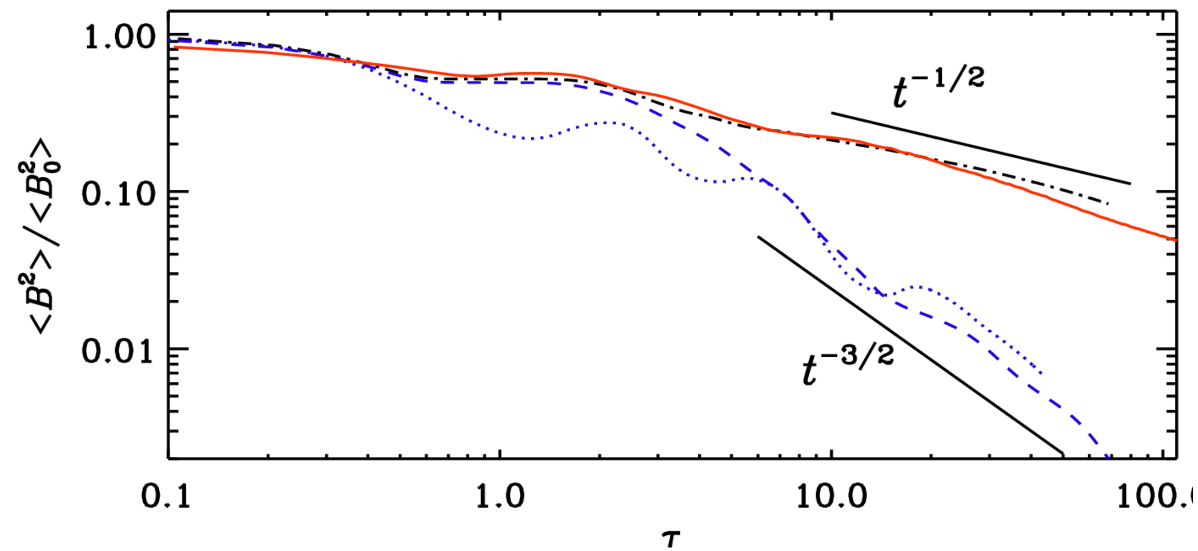


$\mathrm{Re}_\mathrm{M} = 100$

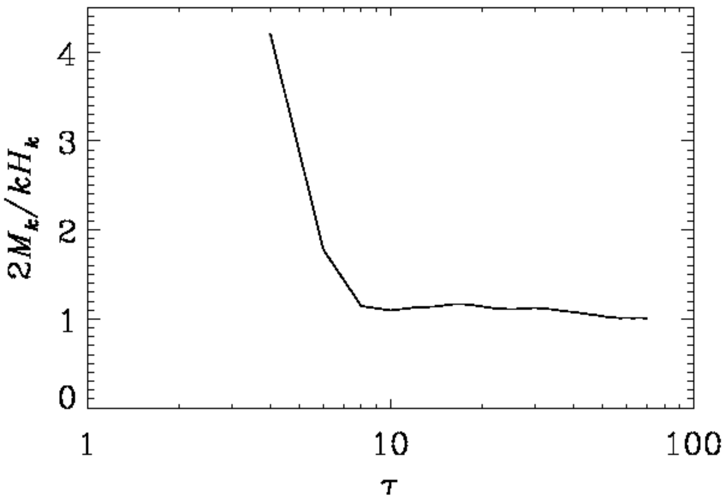


Topological Interpretation

Magnetic energy decay



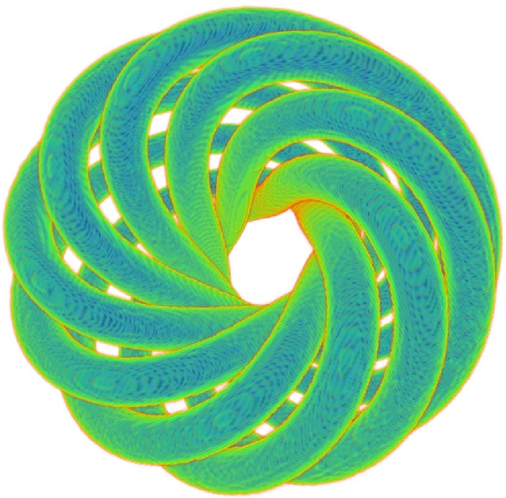
$k = 4$



- Magnetic helicity alone determines the field decay, not the actual linking.
- Entangled fields are indistinguishable from non-entangled if the magnetic helicity is zero.

Topology Outlook

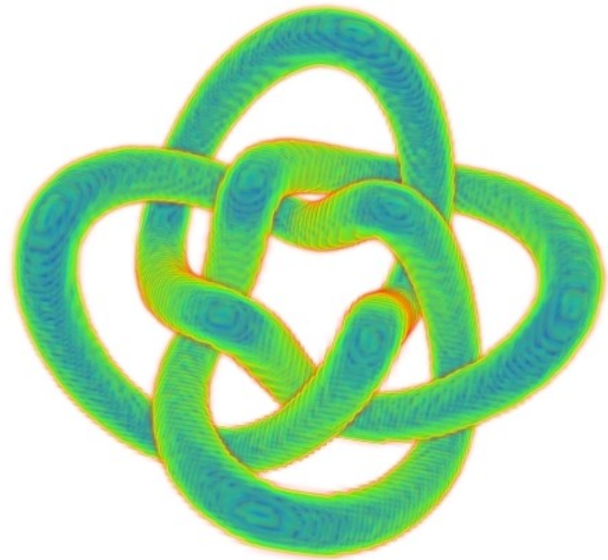
9-foil knot



$$\int_V \mathbf{A}\cdot\mathbf{B}\ \mathrm{d}V = 2\alpha\phi\phi$$

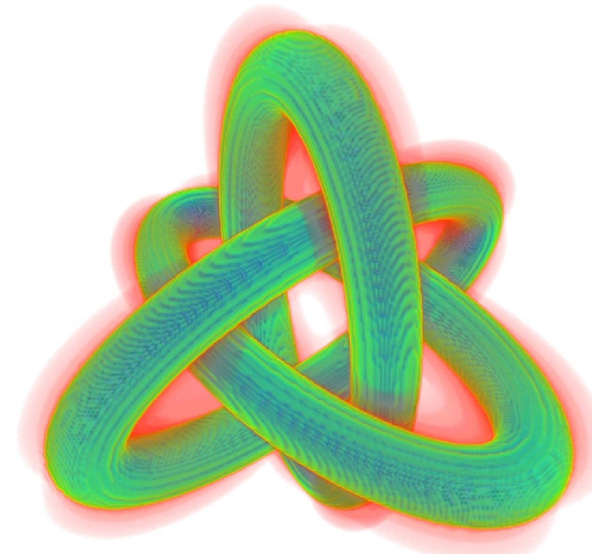
valid?

IUCAA* logo



$$H = 0$$

Borromean rings



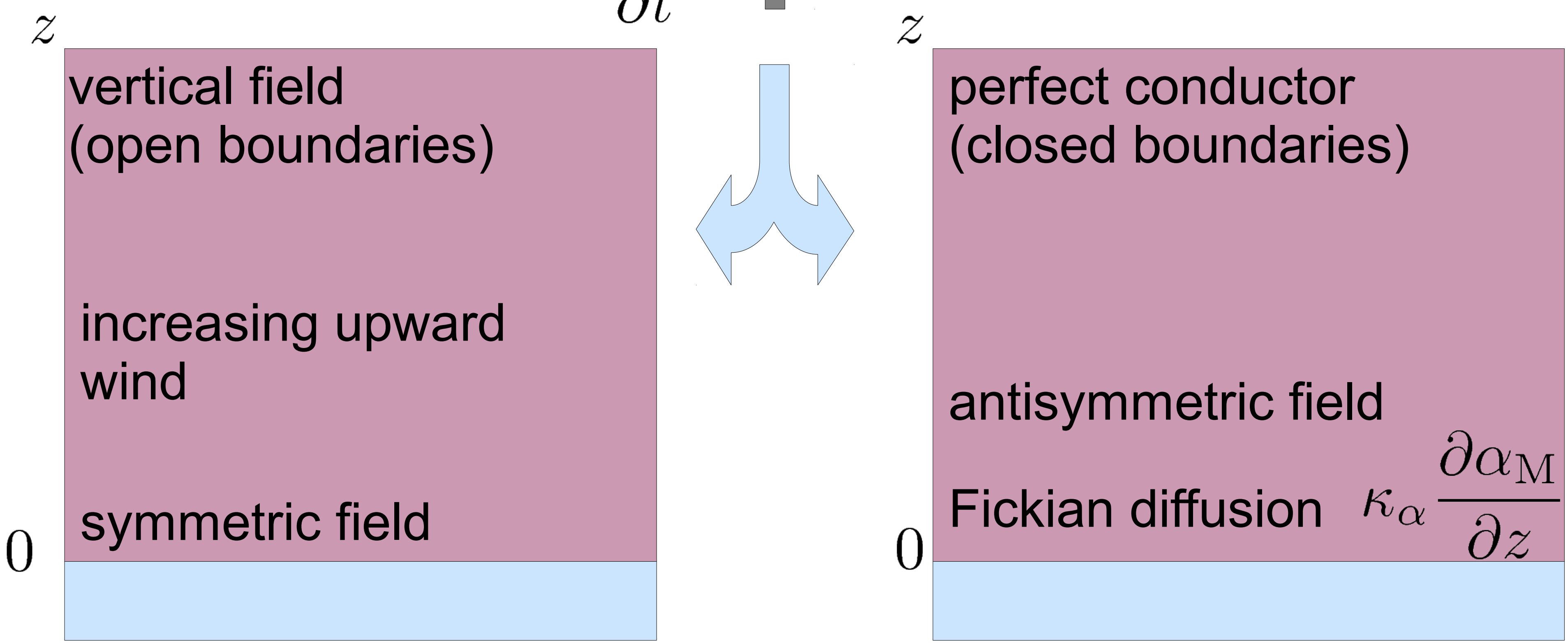
$$H = 0$$

*Inter-University Centre for Astronomy and Astrophysics, Pune, India

Dynamical alpha quenching

$$\frac{\partial \overline{h}_{\rm f}}{\partial t} = -2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} - 2\eta\mu_0\overline{\mathbf{j}\cdot\mathbf{b}} - \boldsymbol{\nabla}\cdot\overline{\mathbf{F}}_{\rm f}$$

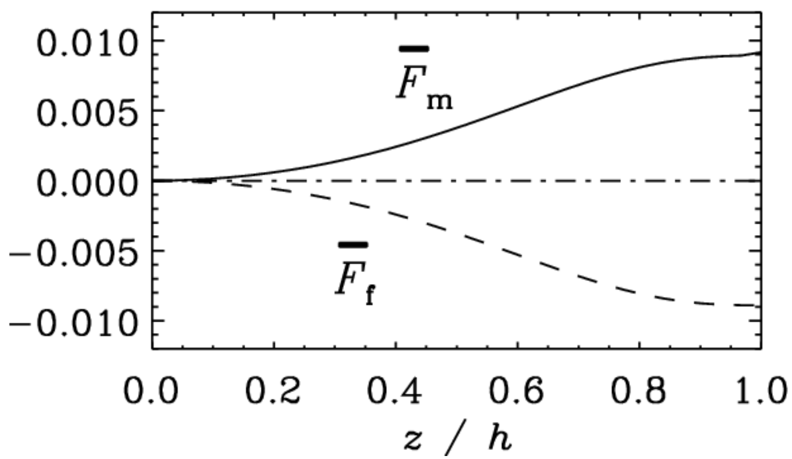
$$\frac{\partial \alpha_{\rm M}}{\partial t}$$ + 1d mean-field

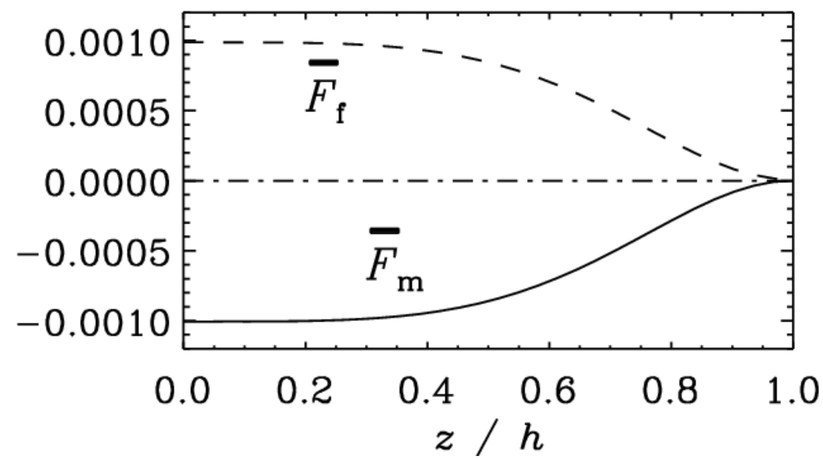


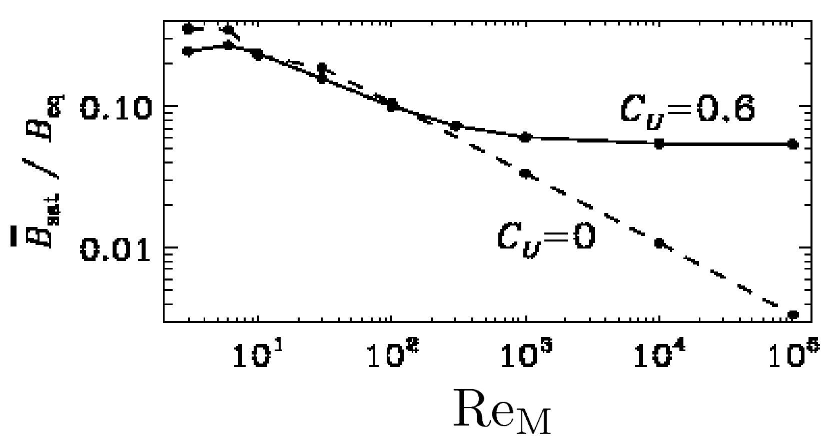
→ helical driving mechanism by $\alpha_{\rm K}$

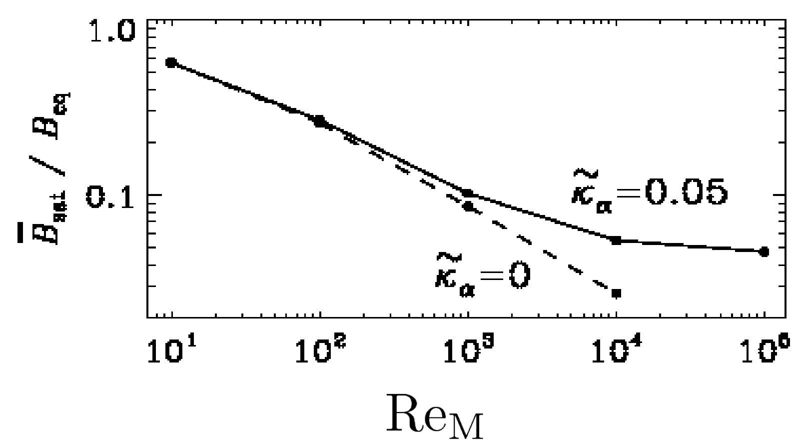
Dynamical alpha quenching

open boundary symmetric wind	vs.	closed boundary antisymmetric κ_α







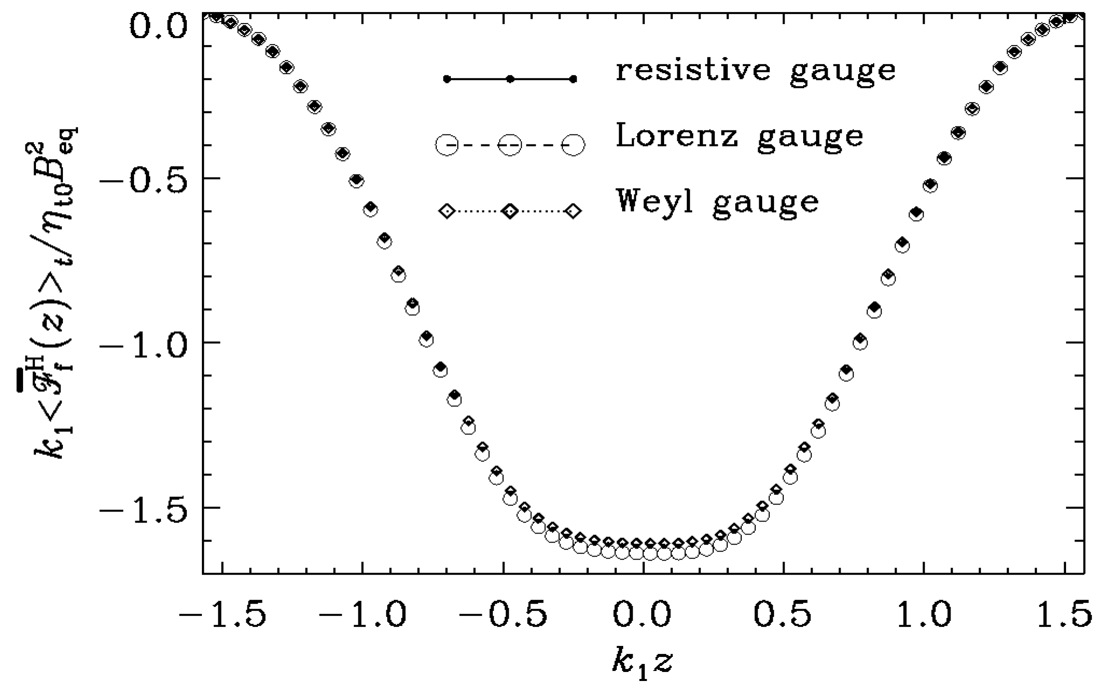


Gauge Issues

Gauge transformation: $\mathbf{A} \to \mathbf{A} + \boldsymbol{\nabla}\Lambda$

$$H_\mathrm{m} \to H_\mathrm{m} + \int_S \Lambda \mathbf{B} \cdot \mathrm{d}\mathbf{S}$$

- resistive gauge
- pseudo-Lorenz gauge
- Weyl gauge



- helical forcing analog. MF
- periodic boundaries
- 128X128x256 box

Time averaged magnetic helicity fluxes do not depend on the gauge.

Its importance for dynamos is saved.

Advective gauge

induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J})$$

uncurl

resistive gauge

$$\frac{\partial \mathbf{A}^{\mathrm{r}}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}^{\mathrm{r}}$$

advecto-resistive gauge

$$\frac{\partial \mathbf{A}^{\mathrm{ar}}}{\partial t} = \mathbf{U} \times \mathbf{B} - \eta \mathbf{J} - \boldsymbol{\nabla}(\mathbf{U} \cdot \mathbf{A}^{\mathrm{ar}} - \eta \boldsymbol{\nabla} \cdot \mathbf{A}^{\mathrm{ar}})$$

measure helicity transport

spatial distribution of the magnetic helicity

Advective gauge

But: Simulations are unstable.

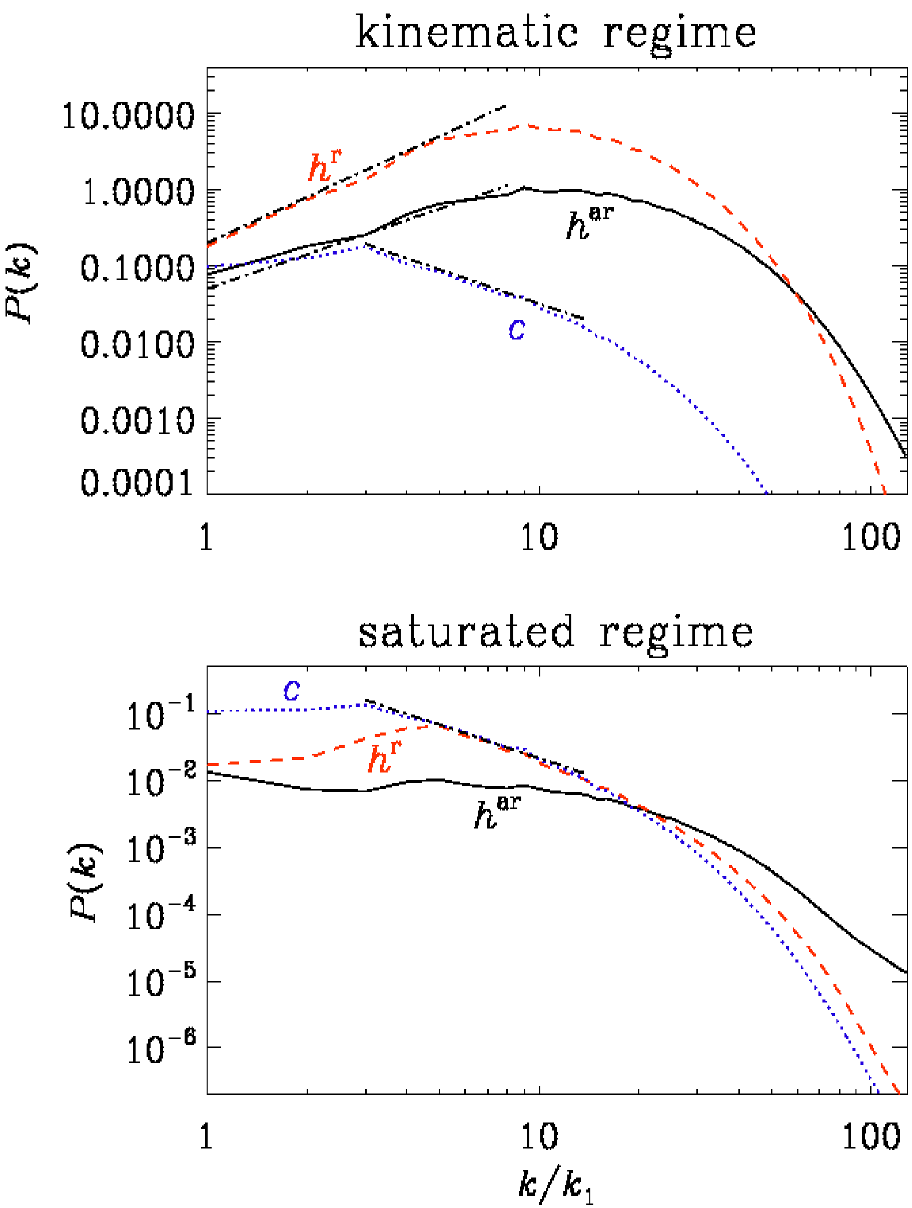
resistive gauge $$\frac{\partial \mathbf{A}^{\mathrm{r}}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}^{\mathrm{r}}$$

gauge transformation $$\mathbf{A}^{\mathrm{ar}} = \mathbf{A}^{\mathrm{r}} + \boldsymbol{\nabla} \Lambda$$

evolve Λ $$\frac{\mathrm{D}\Lambda}{\mathrm{D}t} = -\mathbf{U} \cdot \mathbf{A}^{\mathrm{r}} + \eta \nabla^2 \Lambda$$

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{U} \cdot \boldsymbol{\nabla}$$

Advective gauge



passive scalar: $\frac{\mathrm{D}C}{\mathrm{D}t} = -\kappa \nabla^2 C$

In the kinematic regime h^{ar} behaves like a passive scalar.

h^{ar} has strong high-k tail

↓

efficient turbulent cascade in the advecto-resistive gauge

Conclusions

- Magnetic helicity is the dominant quantity.
- No need (yet) for higher topological invariants.

- Magnetic helicity fluxes can alleviate catastrophic alpha quenching.
- Diffusive fluxes within the domain can also alleviate catastrophic quenching.

- Time averaged magnetic helicity fluxes are independent of the gauge.
- The advecto-resistive gauge efficiently makes magnetic helicity cascade to higher wave numbers.
- In the advecto-resistive gauge magnetic helicity behaves like a passive scalar in the kinematic regime and for high $\mathrm{Re_M}$.

RC