Magnetic Helicity and its Topological Interpretation

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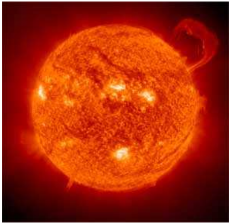
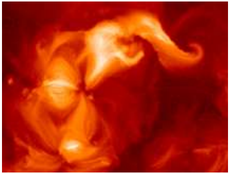
Overview

1. Motivation
2. Magnetic Helicity
3. Linking Number
4. Simulations
5. Crossing Number

Motivation

- Differential rotation and turbulent convection may give rise to flux rings (alpha and omega effect)[Moffatt, 1983].
- Reconnection of magnetic fields in the corona → heating of the corona.





Magnetic Helicity

magnetic helicity:

$$H_B = \int \mathbf{AB} dV \tag{1}$$

realizability condition:

$$M(k) \geq k|H(k)|/2\mu_0 \tag{2}$$

The magnetic energy at each scale is bound from below.

[Brandenburg and Subramanian, 2005]

Linking Number

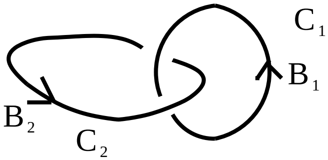


Figure 1: interlocked flux rings

$$
\begin{aligned}
F_1 &= \oint_{C_1} \mathbf{A} d\mathbf{l} && (3)\\
&= \int_{S_1} \nabla x \mathbf{A} d\mathbf{S} && (4)\\
&= \int_{S_1} \mathbf{B}_2 d\mathbf{S} && (5)\\
&= \phi_2 && (6)
\end{aligned}
$$

Gauss' law:

$$\alpha_{ij} = \frac{1}{2\pi} \oint_{C_i} \oint_{C_j} \frac{\mathbf{R}(d\mathbf{l}_i x d\mathbf{l}_j)}{R^3}$$

ϕ_i is the magnetic flux through the tube.

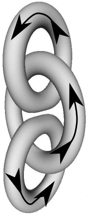
Linking Number ↔ Magnetic Helicity

Connection to linking number[Priest and Terry, 2000, p.266]:
mutual magnetic helicity:

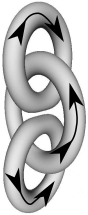
$$
\begin{aligned}
H_{Bmij} &= \int \mathbf{A}_i \mathbf{B}_j && (7) \\
&= 2\alpha_{ij}\phi_i\phi_j && (8)
\end{aligned}
$$

other derivations on [Arnold, 1974].
⇒ Change of mutual helicity changes topology of the flux distribution.

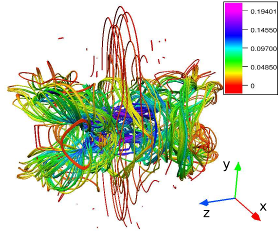
Simulation Results

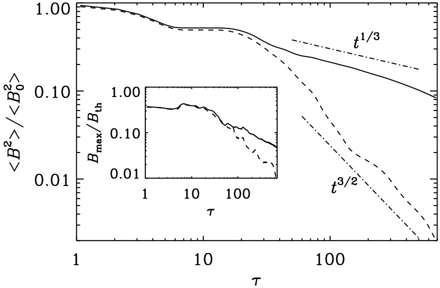


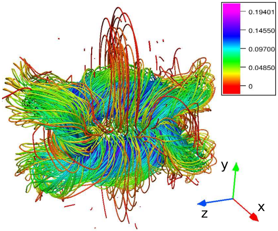
n=0



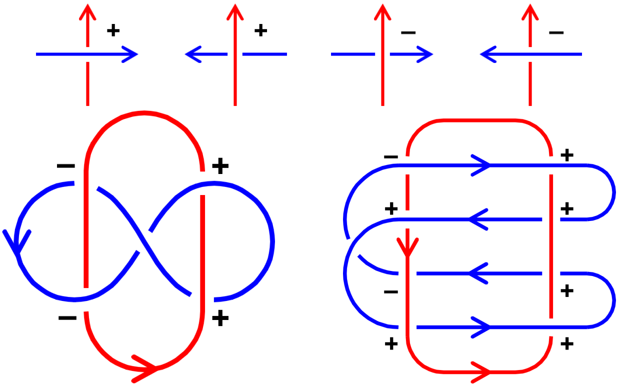
n=2







Crossing number



$$\text{linking number} = \frac{n_+ - n_-}{2} \quad (9)$$

References

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Moffatt, H. (1983).

Priest, E. R. and Terry, F. (2000).