Vortex Reconnection and the Role of Topology

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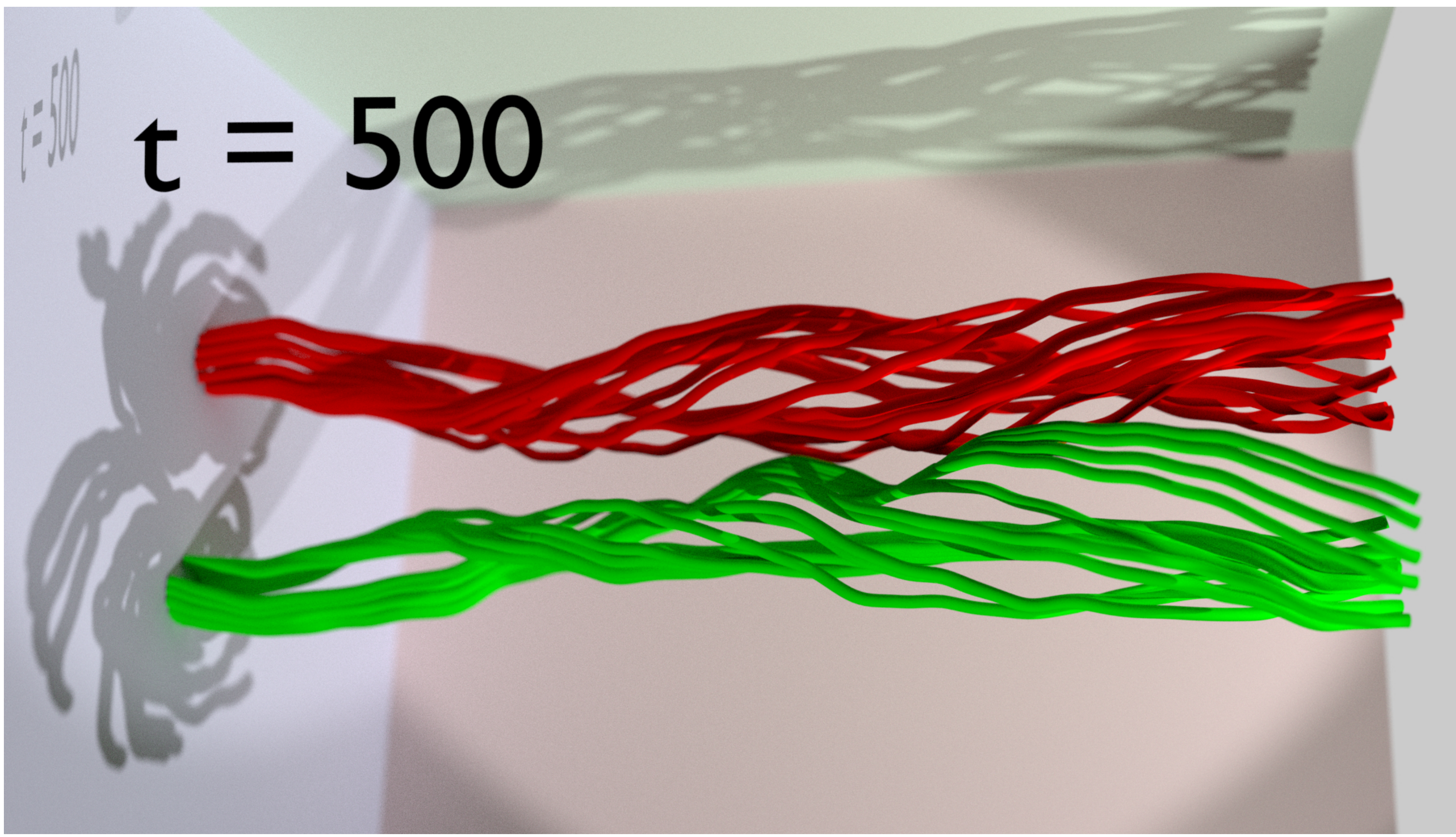


Magnetic Case

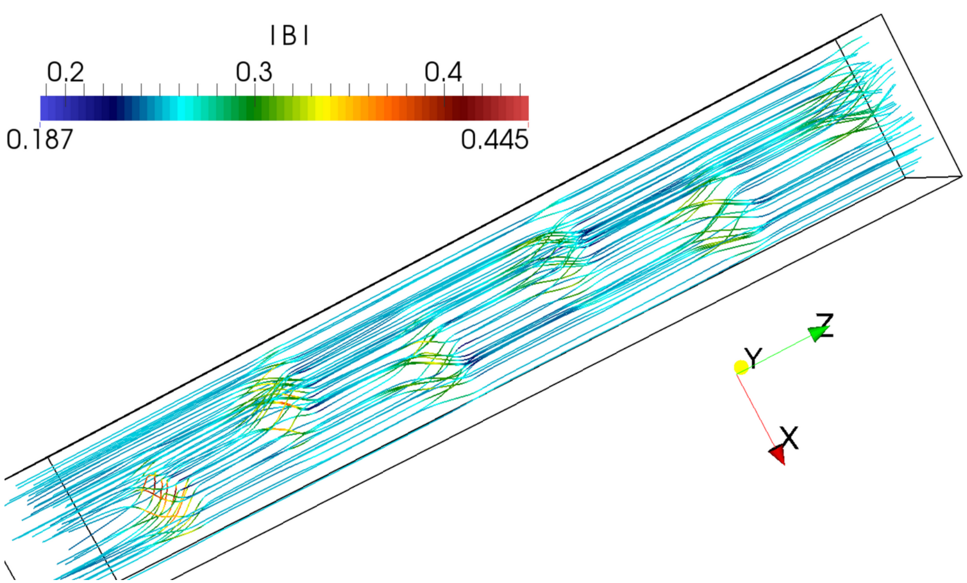
Conservation of magnetic helicity:

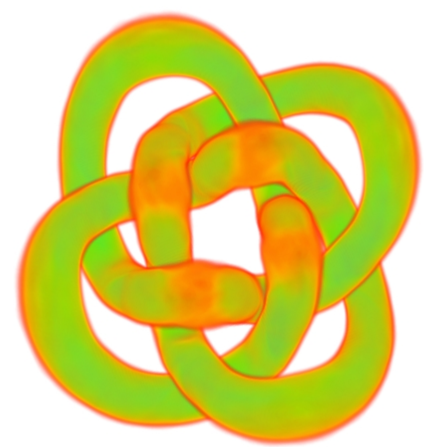
$$\lim_{\eta \to 0} \frac{\partial}{\partial t} \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0 \qquad \eta = \text{magnetic resistivity}$$

Realizability condition:

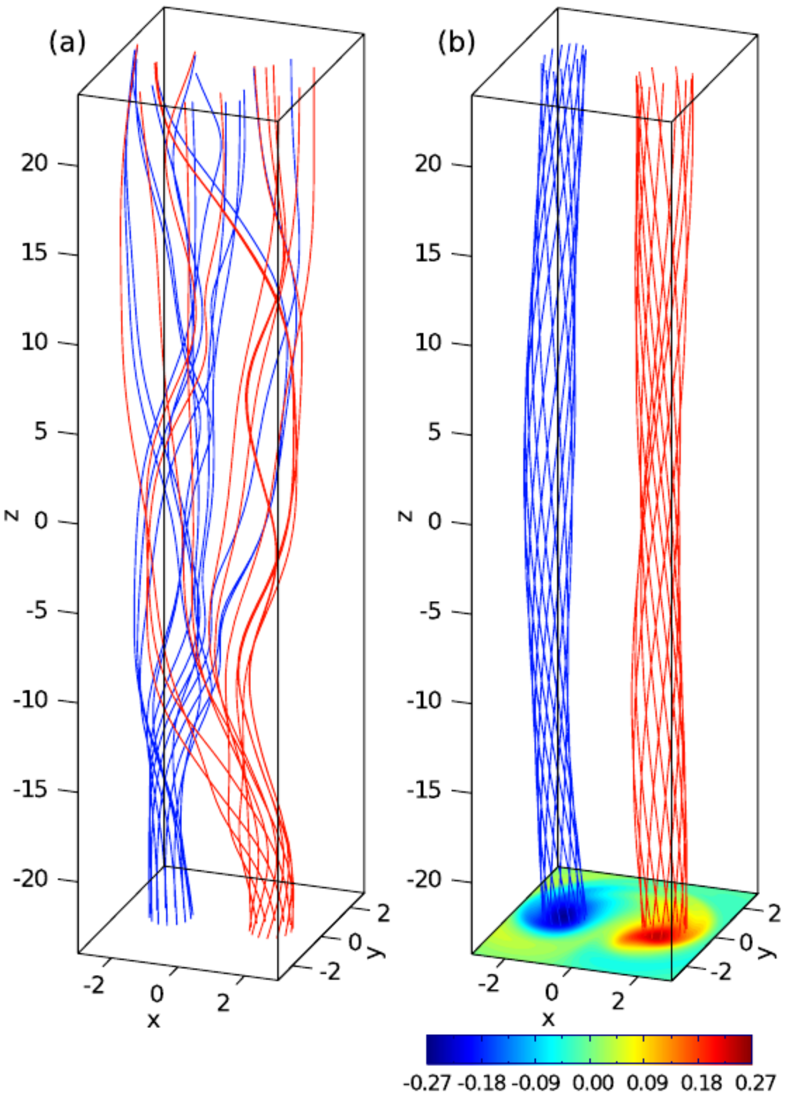
$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.

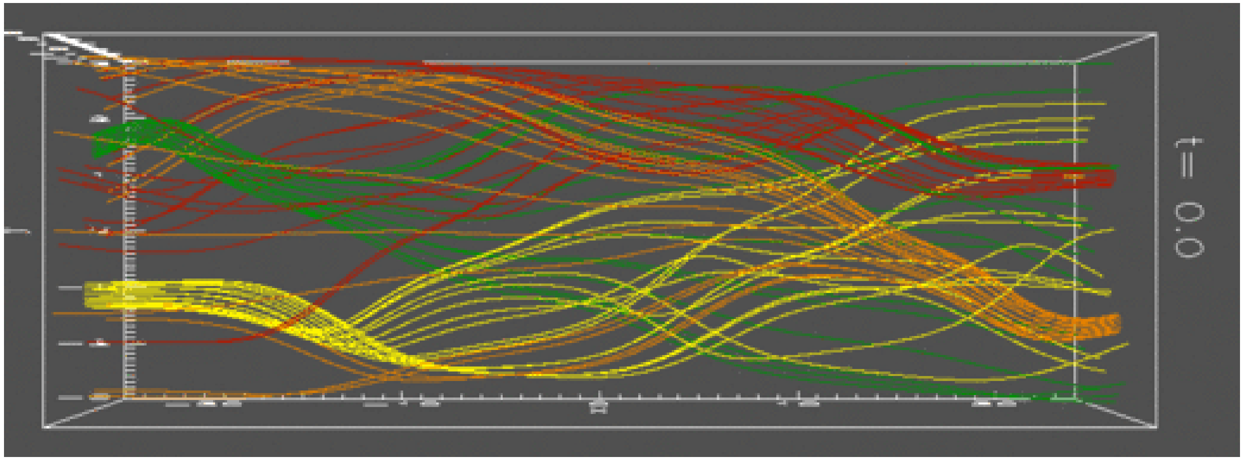




Magnetic Braid



(Yeates 2011)



(Wilmot-Smith 2010)

- Separation into two twisted field regions.
- Conserved invariants like fixed point index and field line helicity.

Navier-Stokes Case

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B} \qquad \boldsymbol{B} \to \boldsymbol{\omega} \qquad H_{\mathrm{m}} = \int \boldsymbol{A} \cdot \boldsymbol{B}\, \mathrm{d}V$$

$$\boldsymbol{\nabla} \times \boldsymbol{u} = \boldsymbol{\omega} \qquad \boldsymbol{A} \to \boldsymbol{u} \qquad H_{\mathrm{k}} = \int \boldsymbol{u} \cdot \boldsymbol{\omega}\, \mathrm{d}V$$

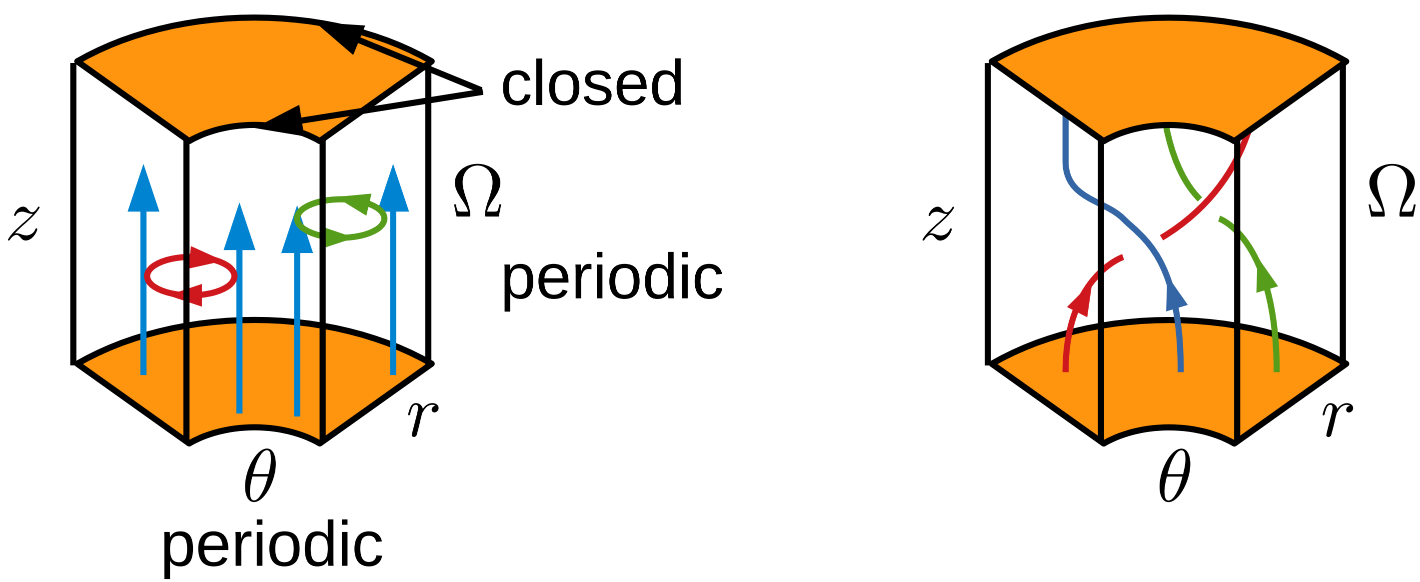
- No realizability condition for the hydro case.
- How does the field line topology affect the dynamics?

Vortex Braid Experiments

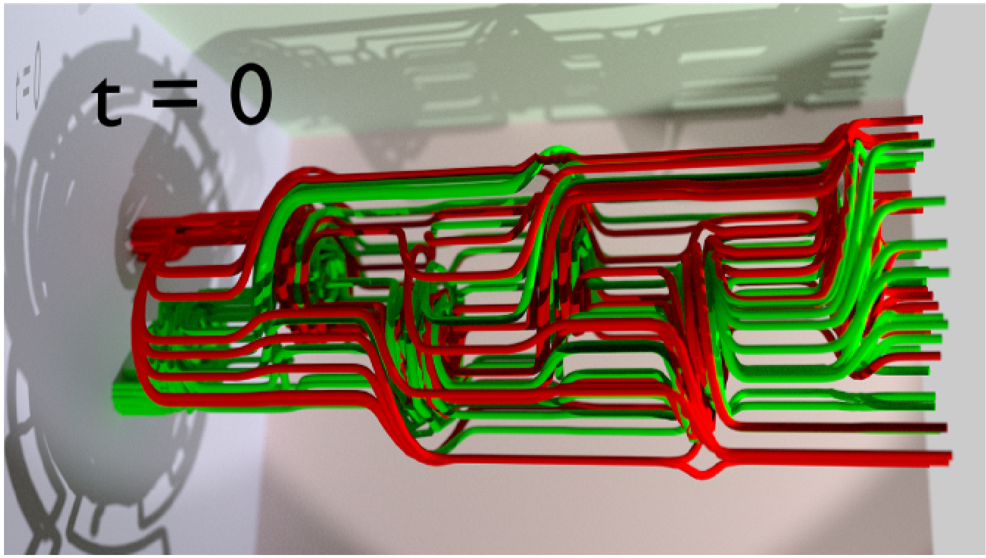
Full viscous simulations with the PencilCode.

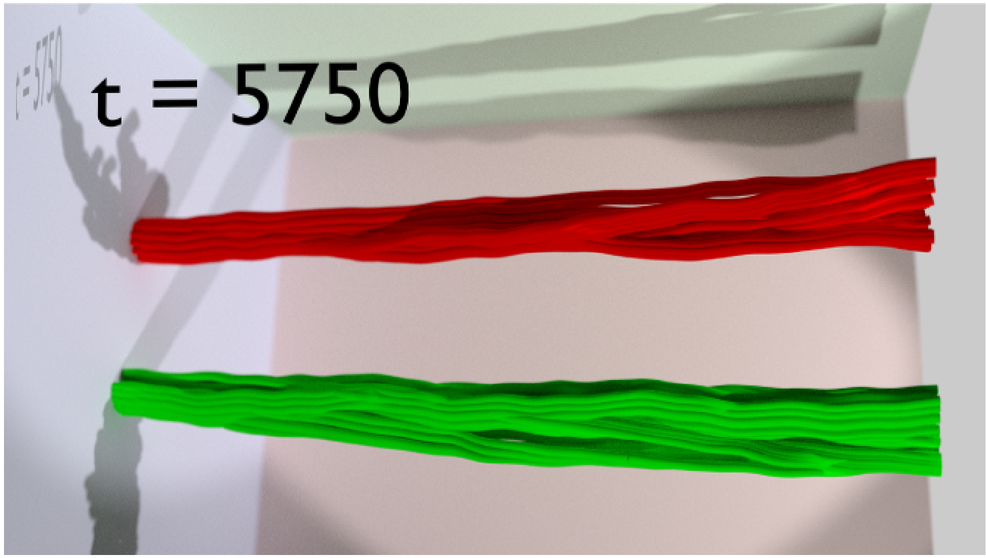
$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -c_\mathrm{s}^2 \boldsymbol{\nabla} \ln(\rho) + 2\boldsymbol{u} \times \tilde{\boldsymbol{\Omega}} + \boldsymbol{F}_\mathrm{visc}$$

$$\frac{\mathrm{D} \ln(\rho)}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{u}$$



Vortex Reconnection





Field lines untangle into two twisted vortex tubes.

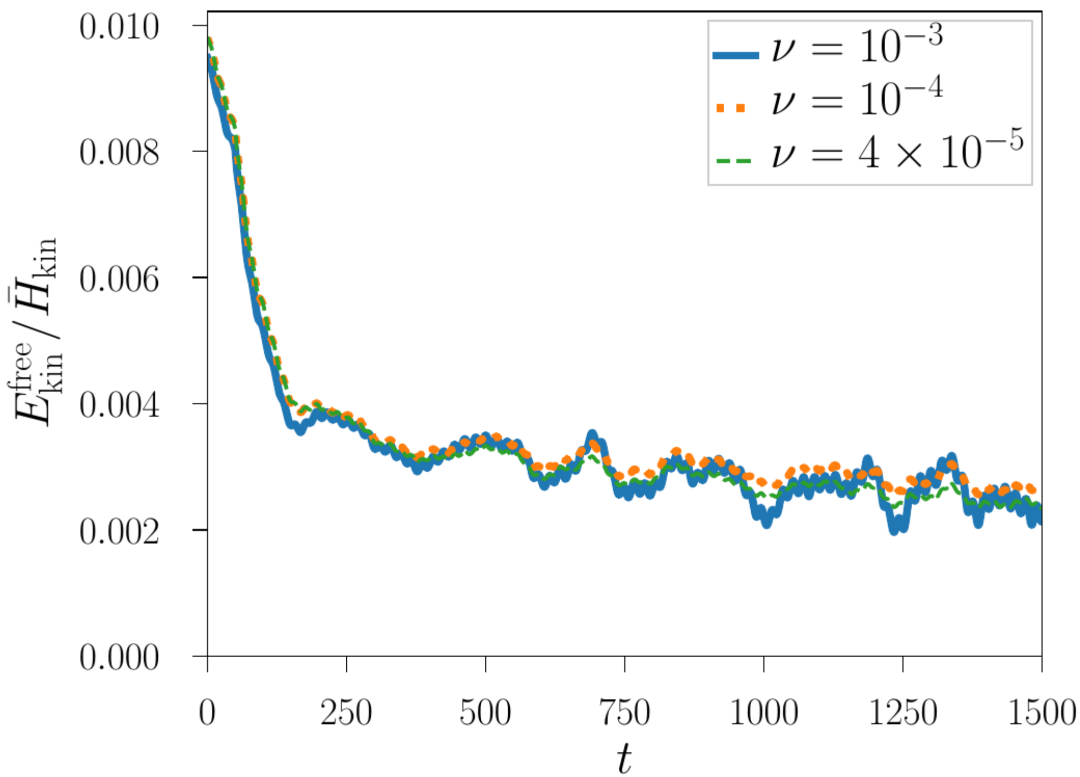
Kinetic Energy Bound

free kinetic energy: $E_{\mathrm{kin}}^{\mathrm{free}} = \int \rho \left(\frac{1}{2}\boldsymbol{u}^2 + \boldsymbol{u} \cdot \boldsymbol{U} \right) \, \mathrm{d}V$

$$\boldsymbol{\nabla} \times \boldsymbol{U} = \boldsymbol{\Omega}$$

unsigned kinetic helicity: $\bar{H}_{\mathrm{kin}} = \int \left| (\boldsymbol{\omega} + \boldsymbol{\Omega}) \cdot (\boldsymbol{u} + \boldsymbol{U}) \right| \, \mathrm{d}V$

Field topology affects relaxation.

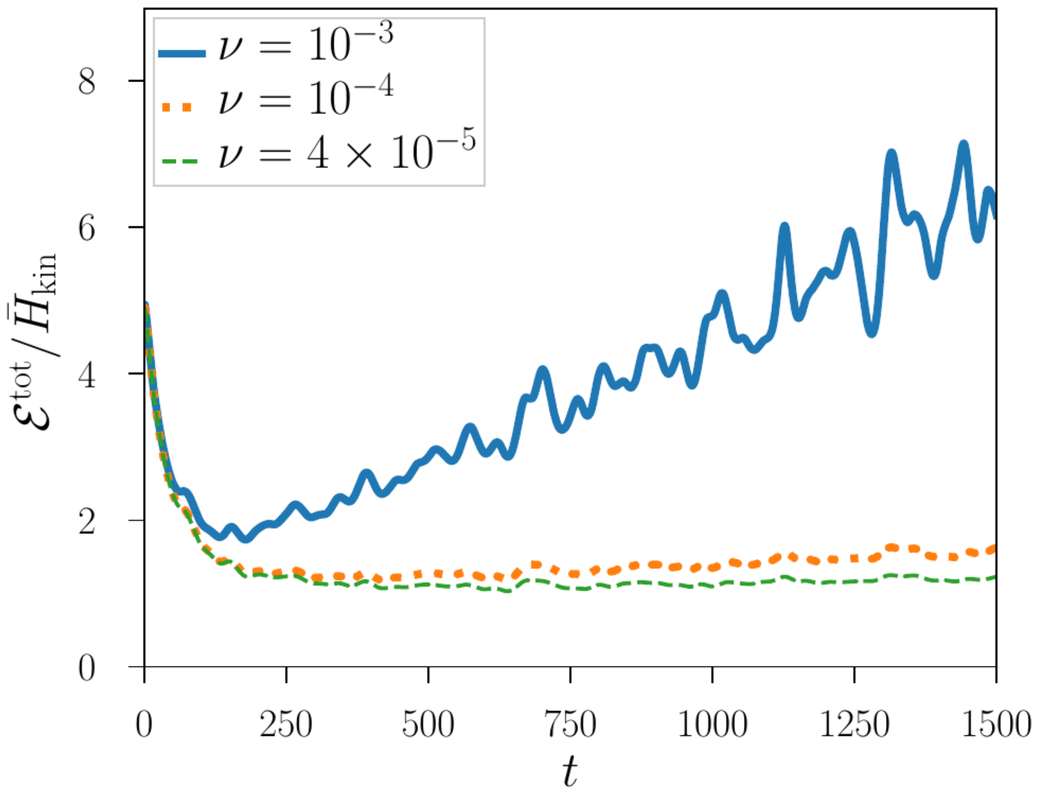


Vortex Braid Enstrophy

enstrophy: $\mathcal{E}^{\mathrm{tot}} = \int (\boldsymbol{\omega} + \boldsymbol{\Omega})^2 \; \mathrm{d}V$

$$\boxed{\bar{H} \leq \frac{1}{\lambda} \mathcal{E}}$$

λ depends on the geometry of the domain.



Hydrodynamical realizability condition.

Conclusions

- Topology preserving relaxation of vortex fields.
- Unbraiding into two twisted vortex flux tubes.
- Unsigned helicity limits energy decay.
- Realizability condition with enstrophy.

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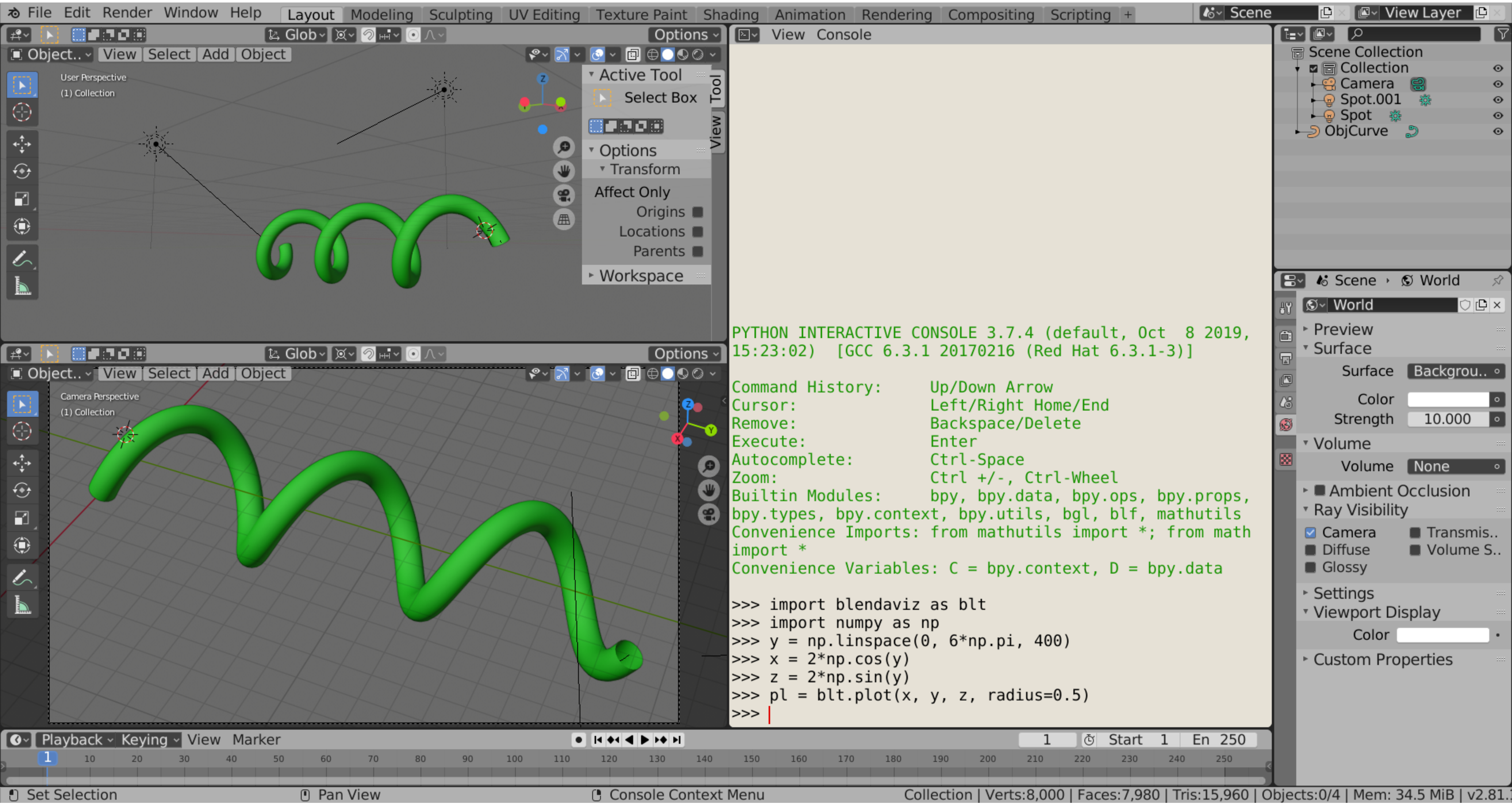
BlenDaViz



github.com/SimonCan/BlenDaViz