Decay of helical and non-helical magnetic links and knots

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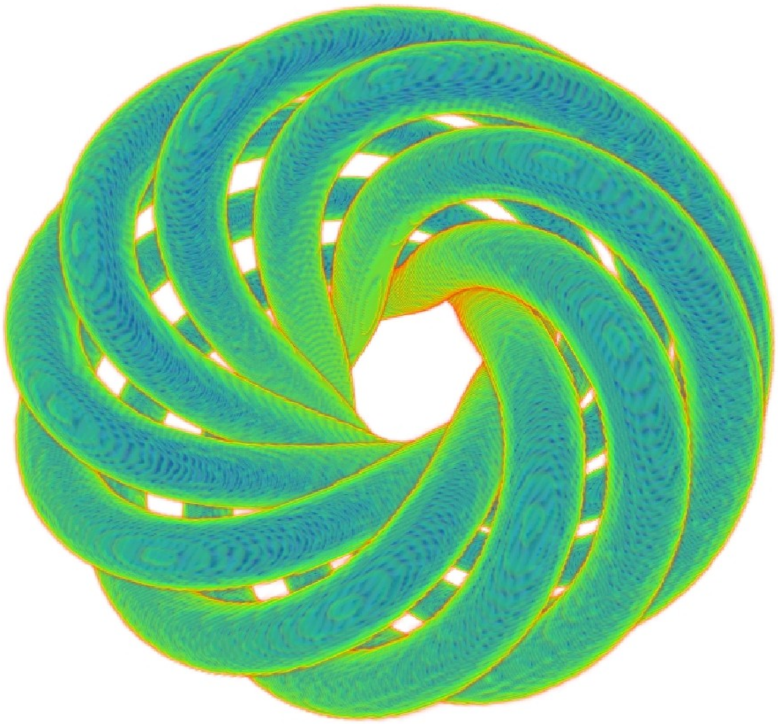
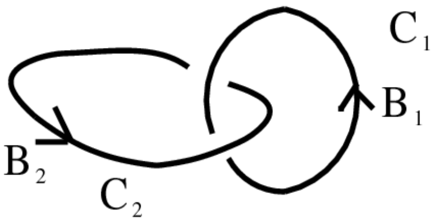


Magnetic helicity

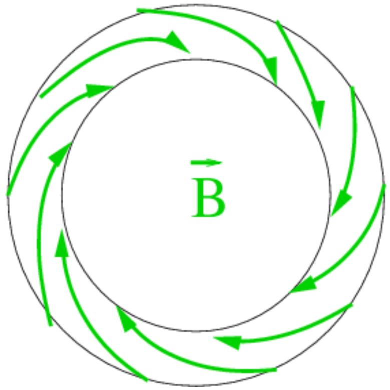
$$H_{\mathrm{M}} = \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V = 2n\phi_1\phi_2$$

$$\phi_i = \int_{S_i} \mathbf{B} \cdot \mathrm{d}\mathbf{S}$$



Realizability condition:

$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$



twisted field

Magnetic energy is bound from below by magnetic helicity.

magnetic helicity conservation

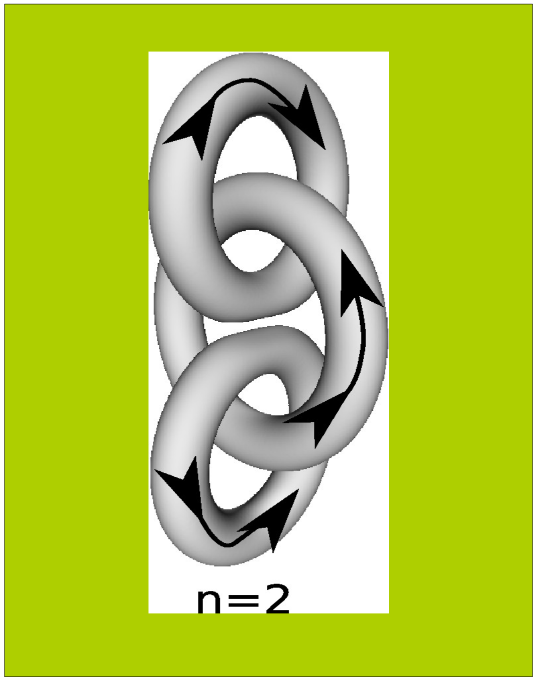
$$\mathrm{Re}_{\mathrm{M}} \to \infty$$

$$\frac{\mathrm{d}H_{\mathrm{M}}}{\mathrm{d}t} = 0$$

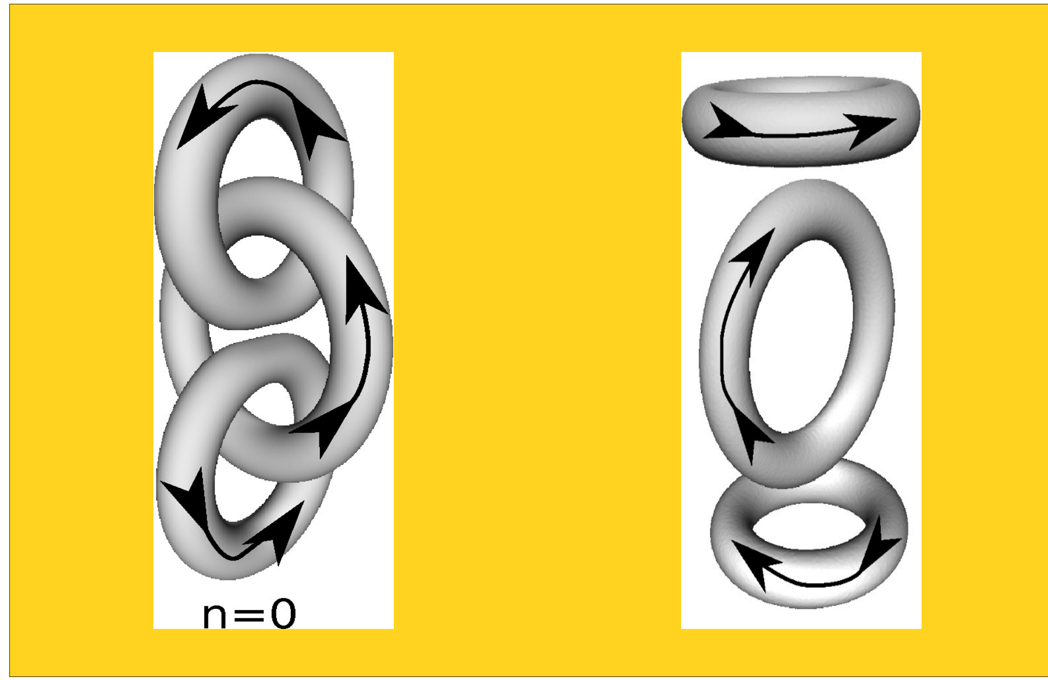
trefoil knot

Interlocked flux rings

$H_{\mathrm{M}} \neq 0$



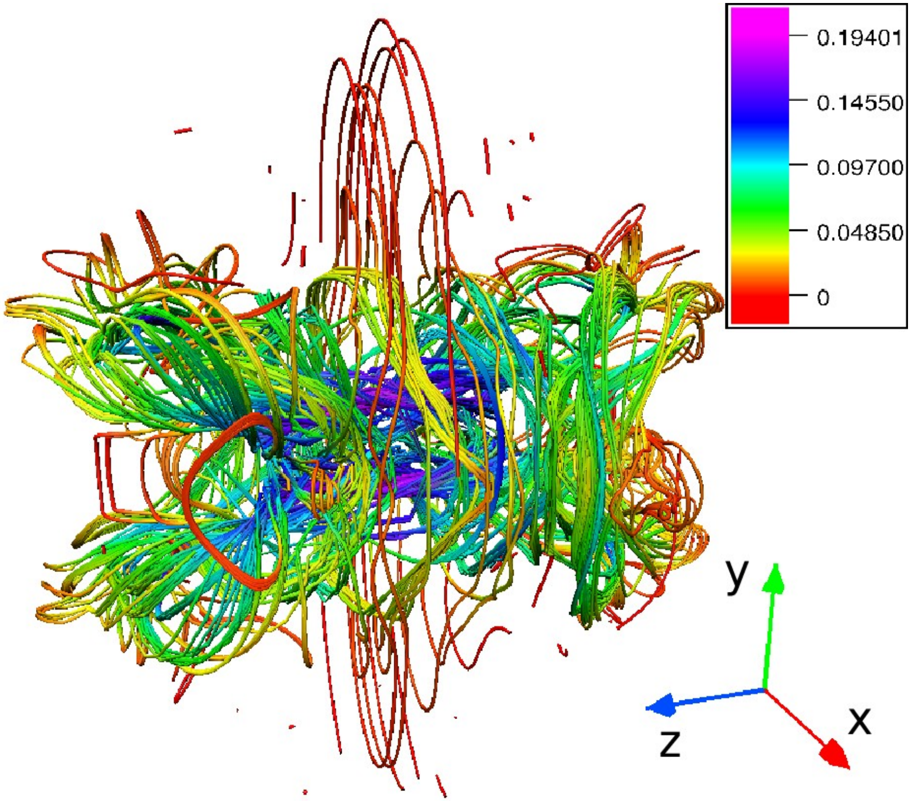
$H_{\mathrm{M}} = 0$



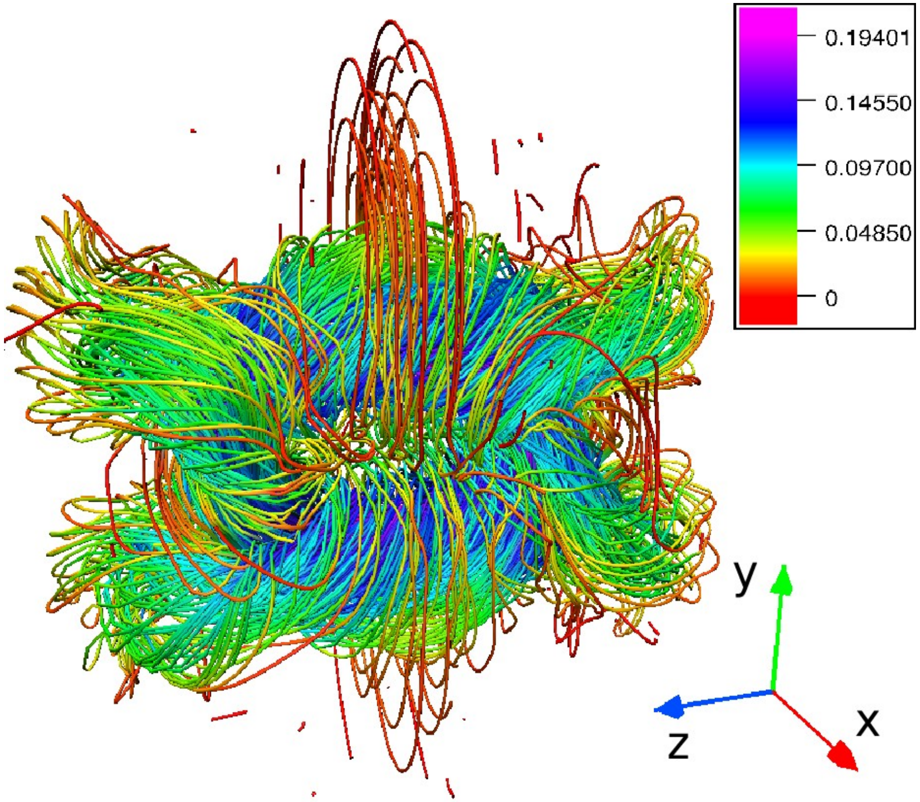
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

Interlocked flux rings

$\tau = 4$

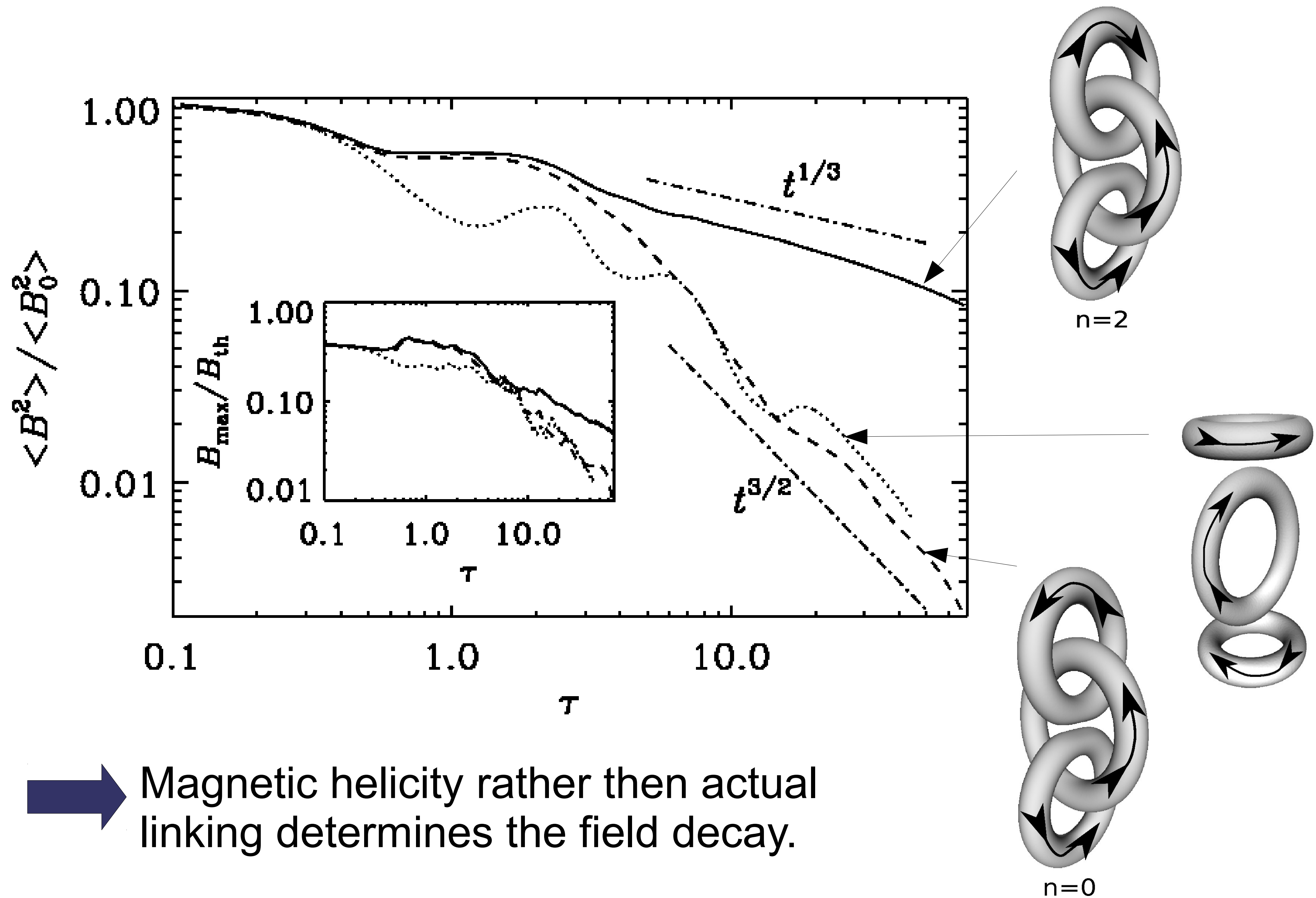


$H_{\mathrm{M}} = 0$



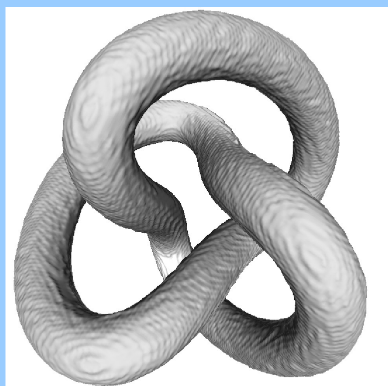
$H_{\mathrm{M}} \neq 0$

Interlocked flux rings

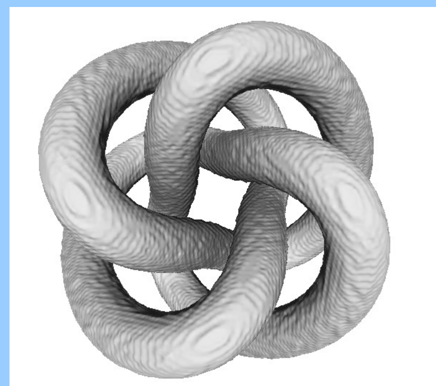


Magnetic helicity rather then actual linking determines the field decay.

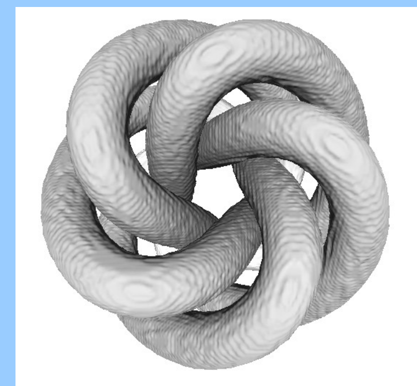
N-foil knots



3-foil

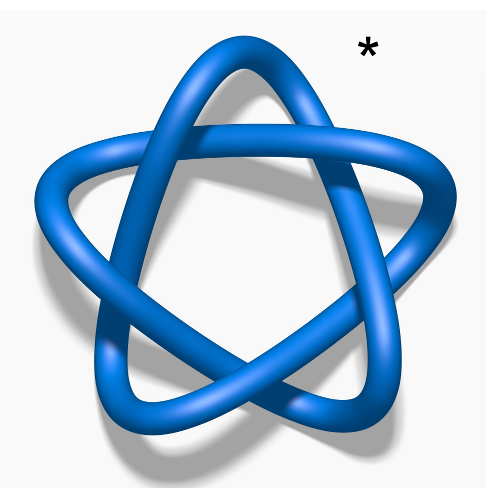


4-foil



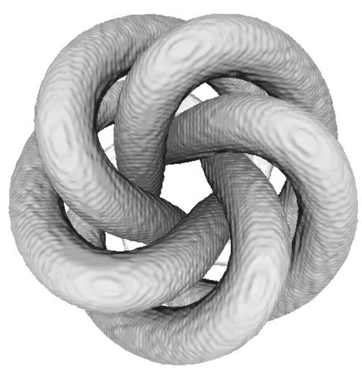
5-foil

6-foil 7-foil



cinquefoil knot

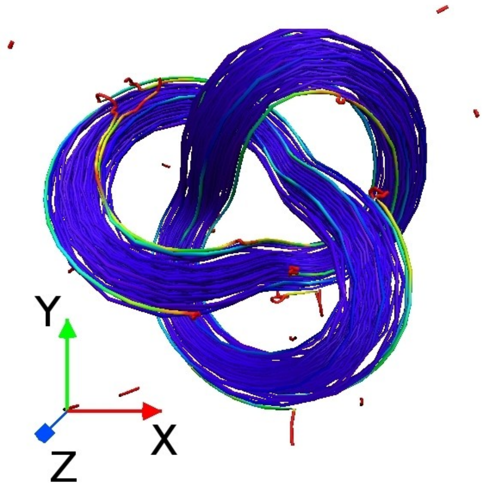
$\neq$



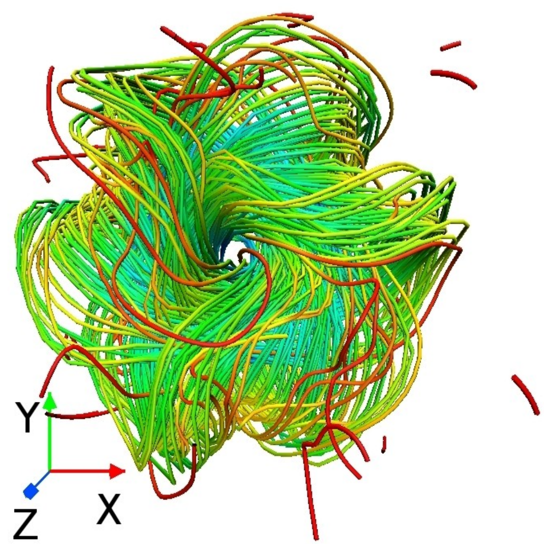
$$x(s) = \begin{pmatrix} (C + \sin s n_{\mathrm{f}}) \sin[s(n_{\mathrm{f}} - 1)] \\ (C + \sin s n_{\mathrm{f}}) \cos[s(n_{\mathrm{f}} - 1)] \\ D \cos s n_{\mathrm{f}} \end{pmatrix}$$

* from Wikipedia, author: Jim.belk

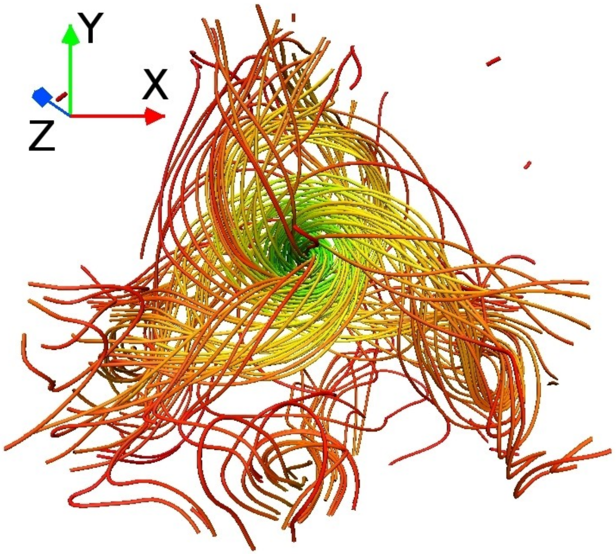
N-foil knots



$t = 0$



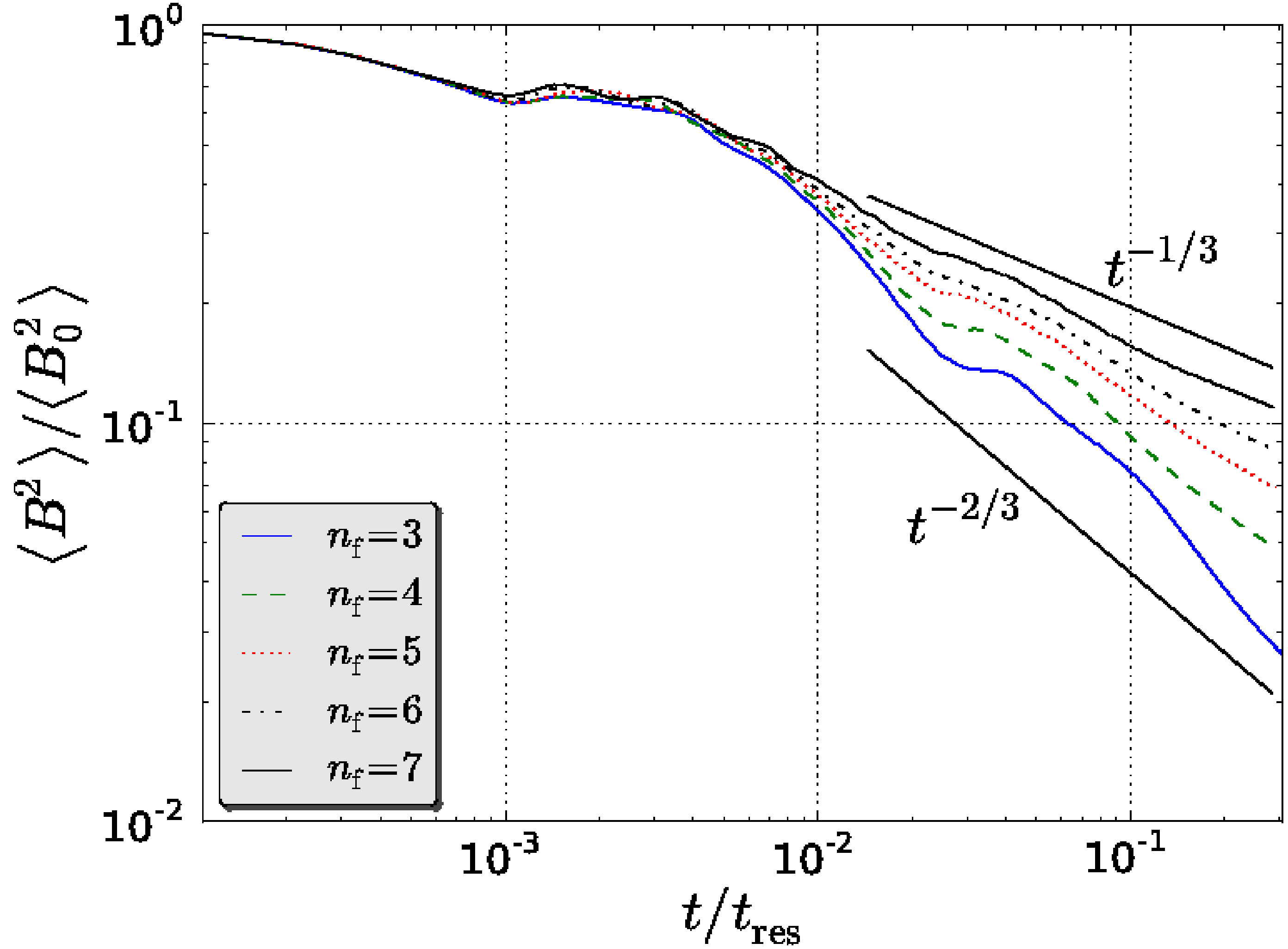
$t = 6$



$t = 39$

- Magnetic helicity is approximately conserved.
- Self-linking is transformed into twisting after reconnection.

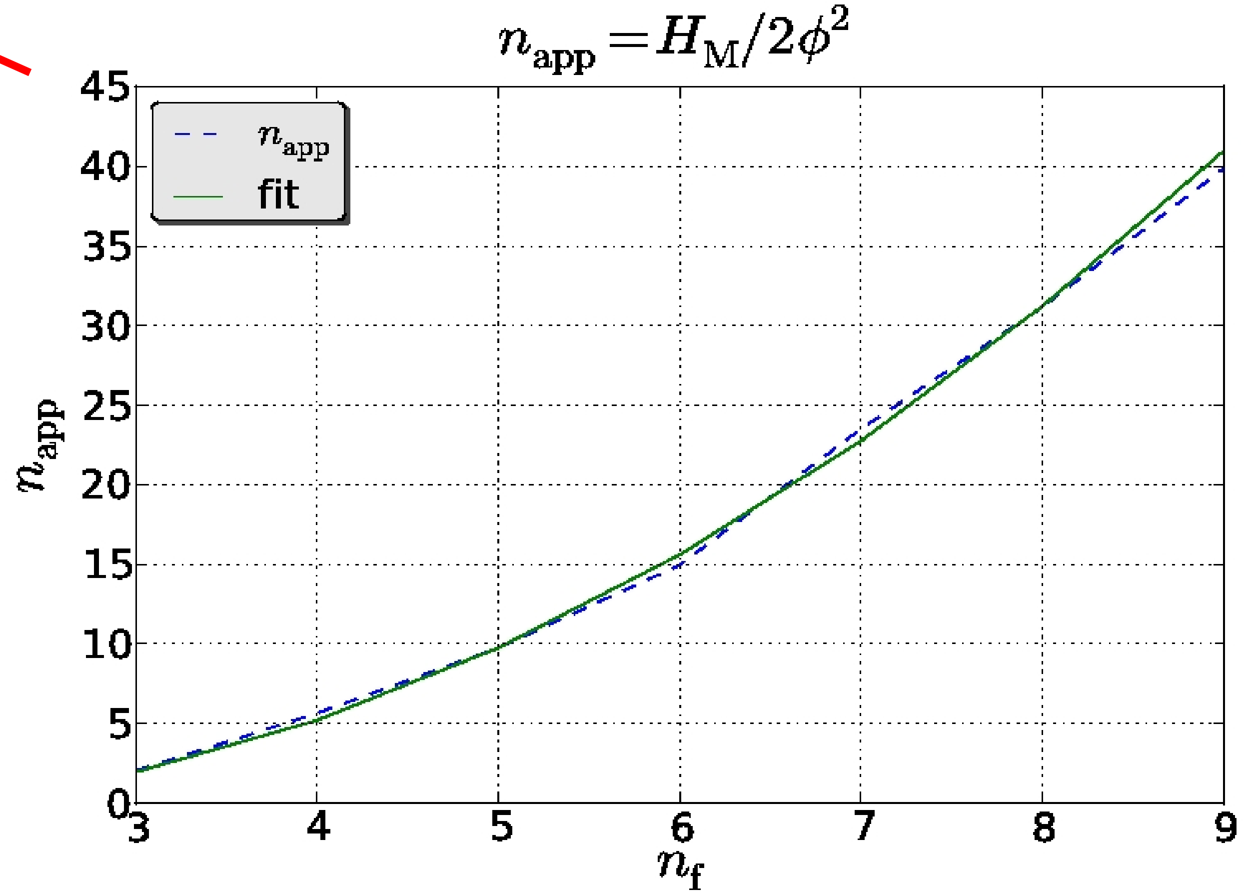
N-foil knots



Slower decay for higher $n_{\rm f}$.

N-foil knots

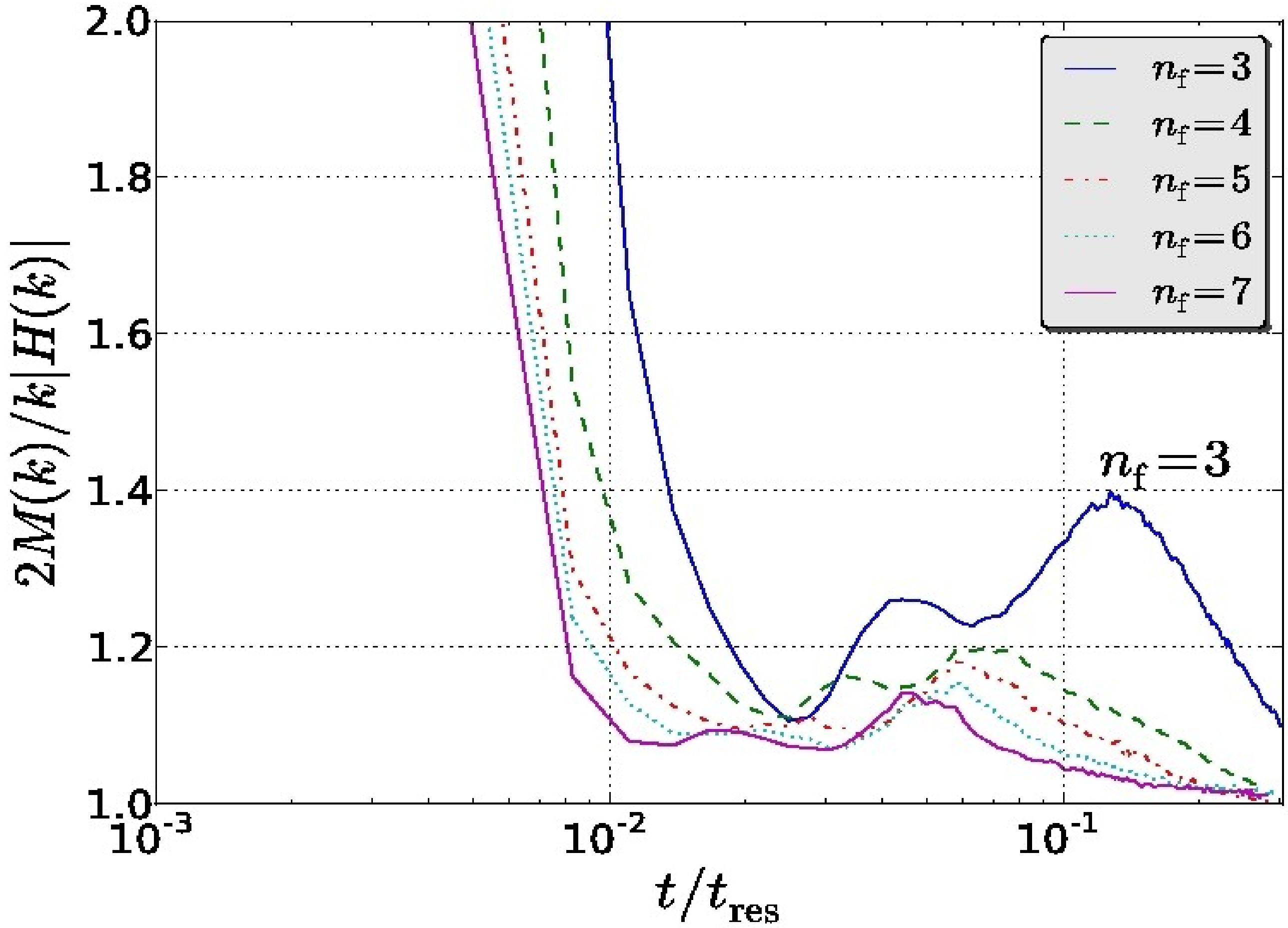
~~$H_\mathrm{M} = 2n\phi_1\phi_2$~~



$$H_\mathrm{M} = (n_\mathrm{f} - 2)n_\mathrm{f}\phi^2/2$$

N-foil knots

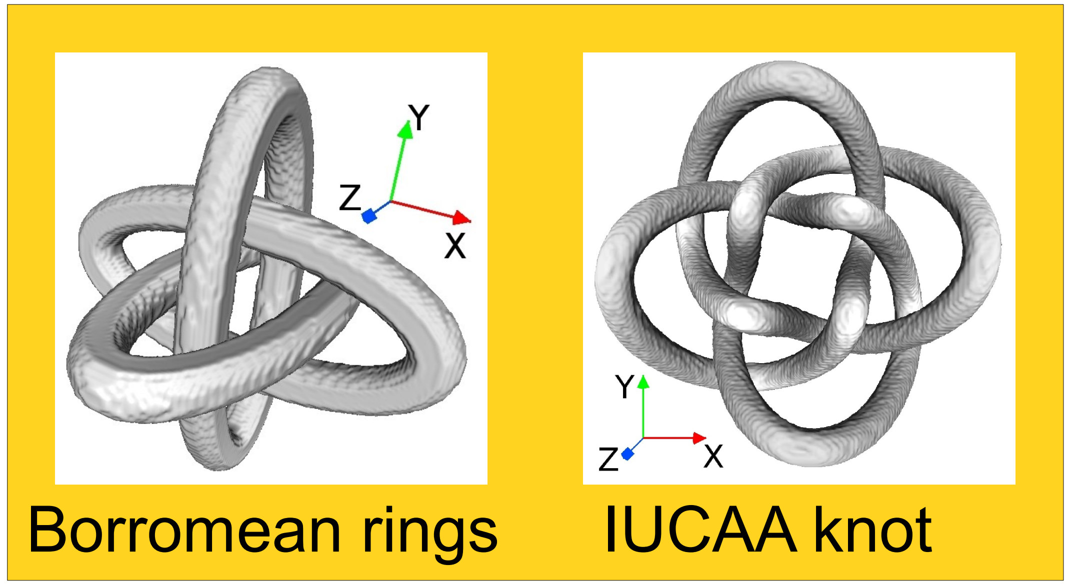
$2M(k)/|H(k)|k$



$k = 2$

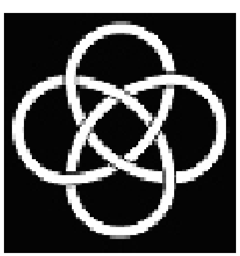
Realizability condition more important for high $n_{\rm f}$.

IUCAA knot and Borromean rings



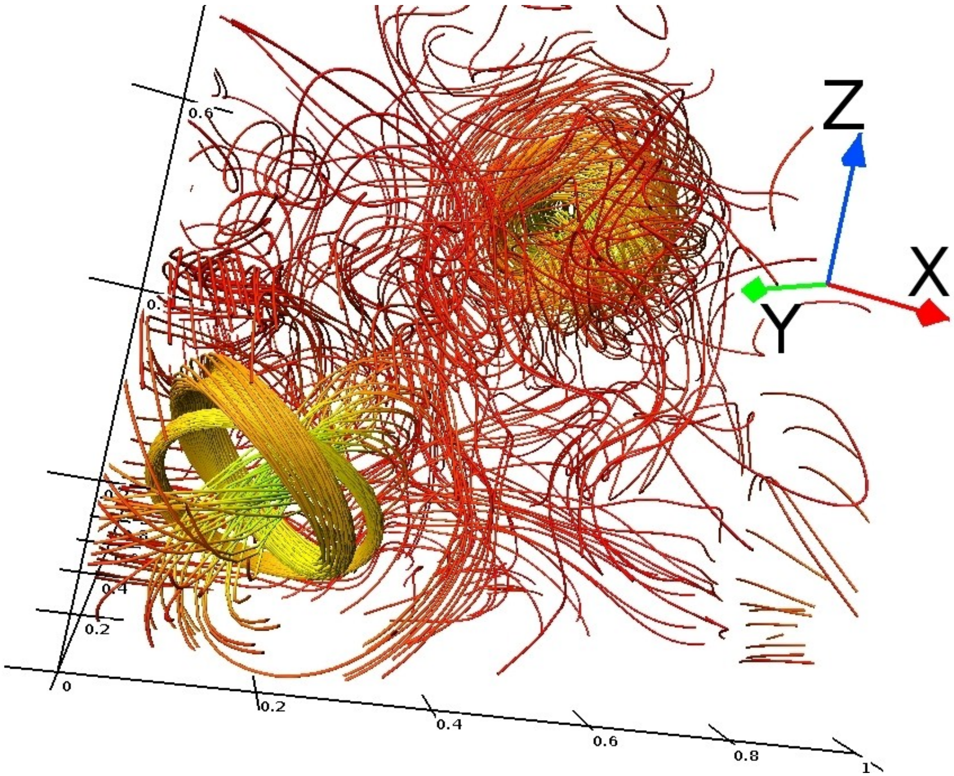
$$H_{\rm M} = 0$$

- Is magnetic helicity sufficient?
- Higher order invariants?

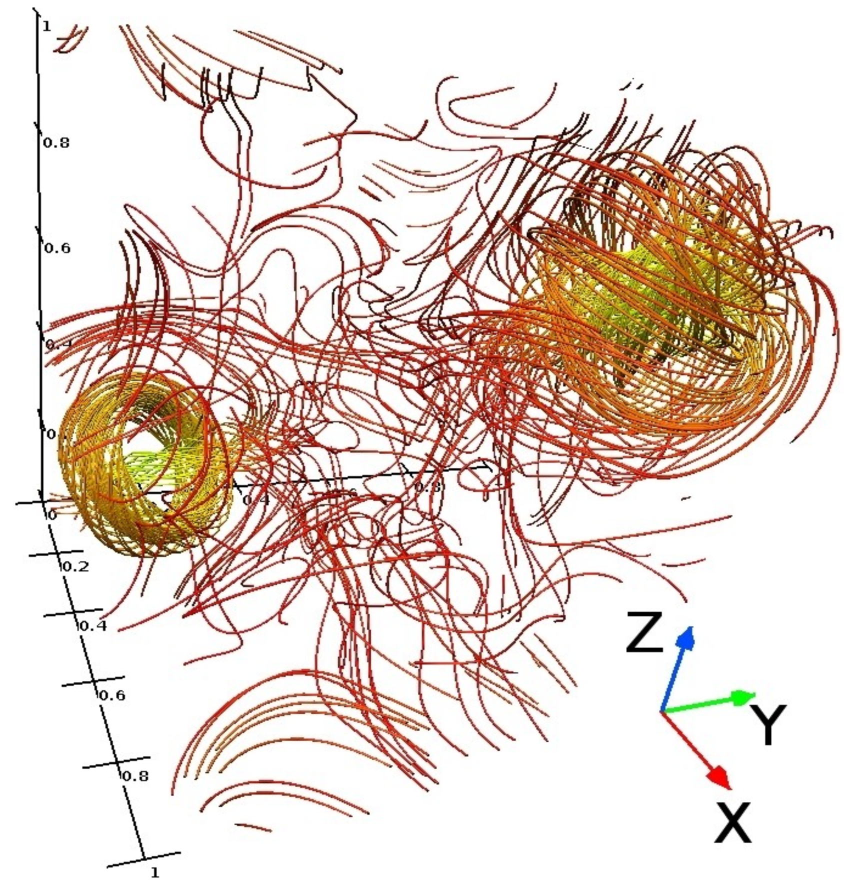


IUCAA = The Inter-University Centre for Astronomy and Astrophysics, Pune, India

Reconnection characteristics



$t = 70$

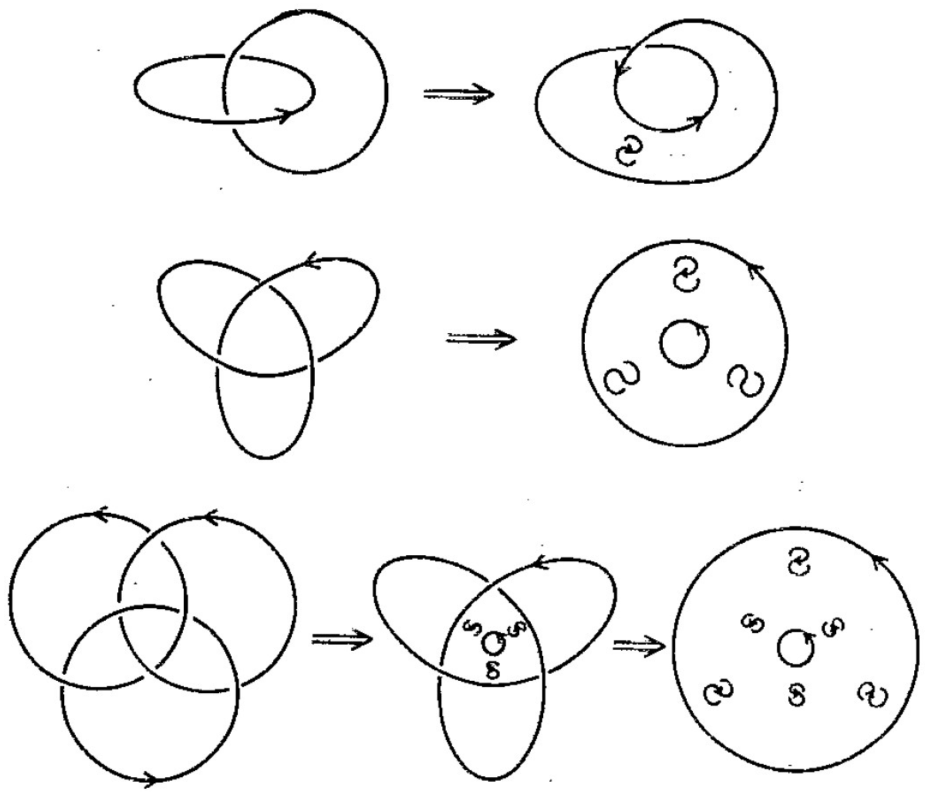


$t = 78$

3 rings → Twisted ring + interlocked rings → 2 twisted rings

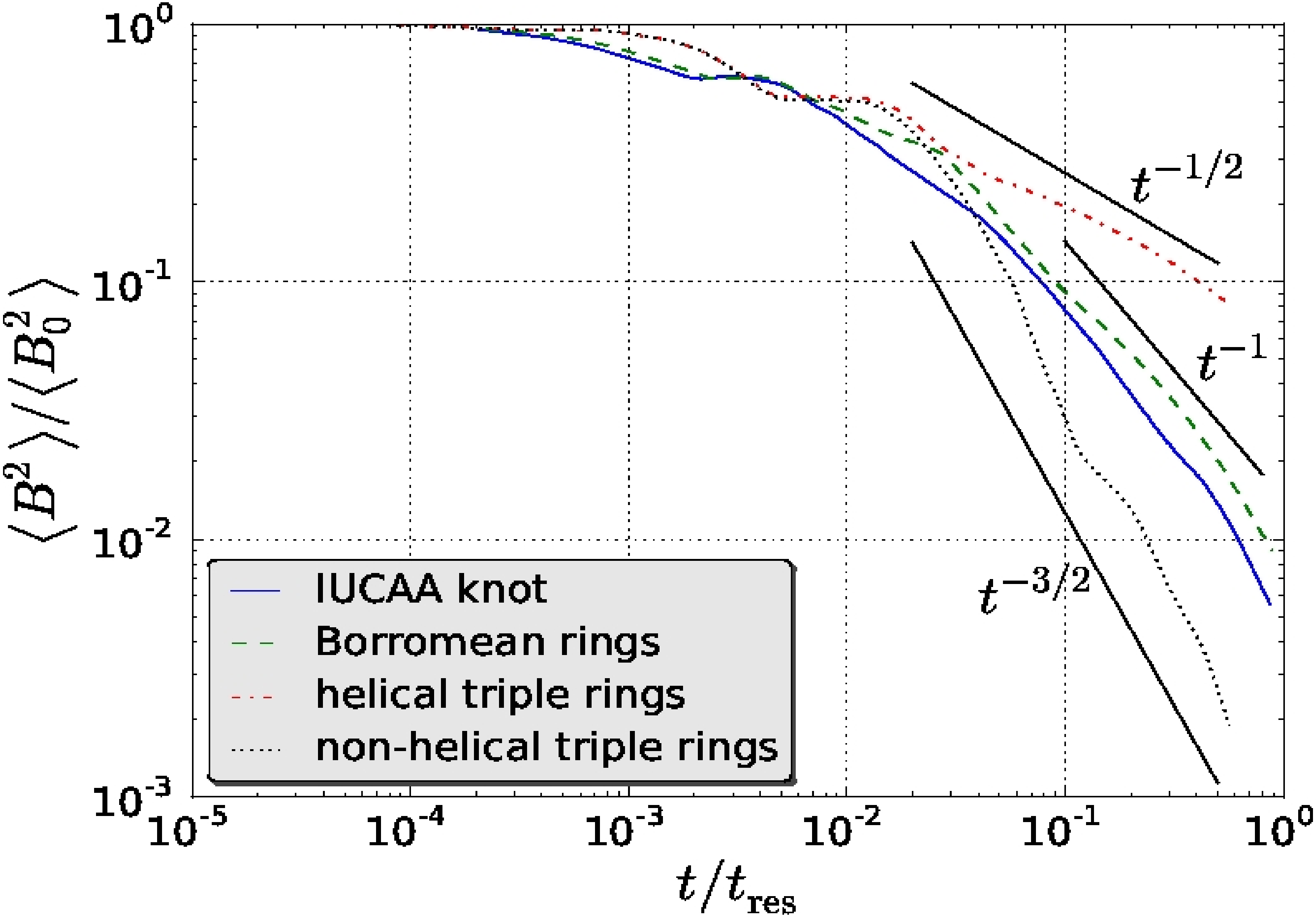
Reconnection characteristics

Conversion of linking into twisting



Ruzmaikin and Akhmetiev (1994)

Magnetic energy decay



Conclusions

- Topology *can* constrain field decay.
- Stronger packing for high n_{f} leads to different decay slopes.
- Higher order invariants?

- Non-forced ejection of magnetic field
- Isolated helical structures inhibit energy decay
- Reconsider realizability condition

References

Candelaresi and Brandenburg 2011

Simon Candelaresi, and Axel Brandenburg.
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Del Sordo et al. 2010

Fabio Del Sordo, Simon Candelaresi, and Axel Brandenburg.
Magnetic-field decay of three interlocked flux rings with zero linking number.
Phys. Rev. E, 81:036401, Mar 2010.

Ruzmaikin and Akhmetiev 1994

A. Ruzmaikin and P. Akhmetiev.
Topological invariants of magnetic fields, and the effect of reconnections.
Phys. Plasmas, vol. 1, pp. 331–336, 1994.

Simulations

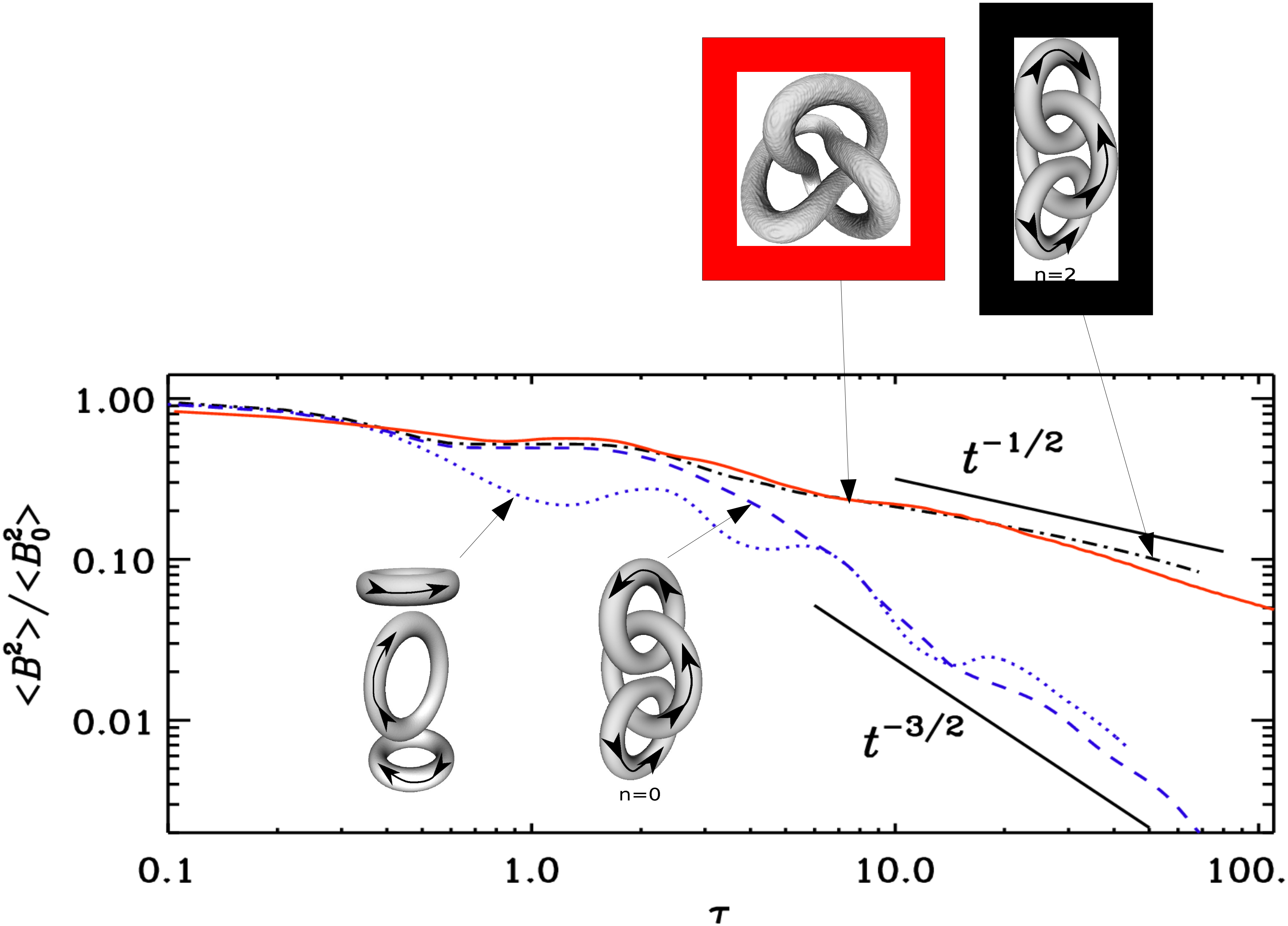
- 256^3 mesh point
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}$$

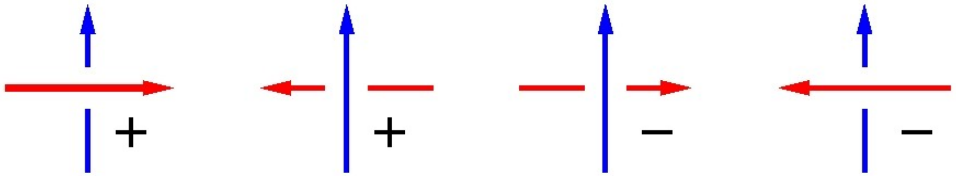
$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \mathbf{J} \times \mathbf{B}/\rho + \mathbf{F}_{\mathrm{visc}}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \mathbf{U}$$

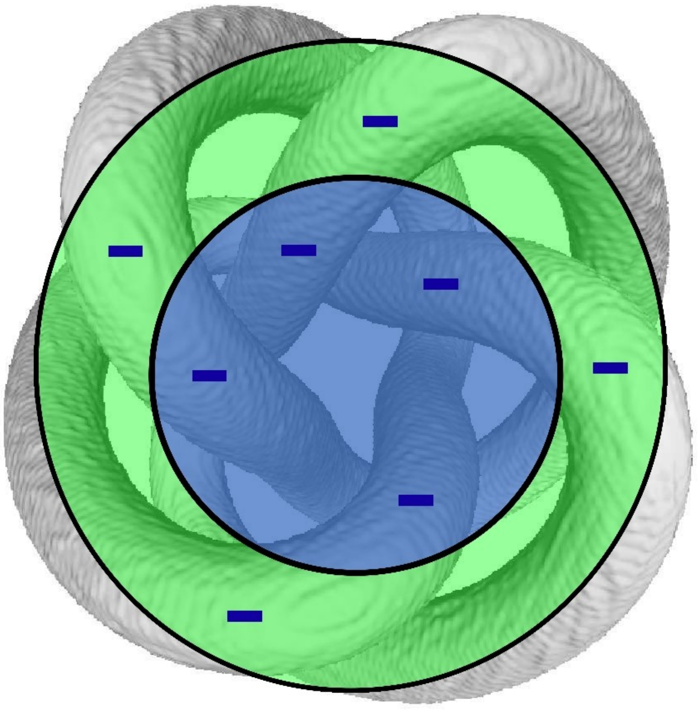
Magnetic energy decay



Linking number



$$n_{\mathrm{linking}} = (n_+ - n_-)/2$$



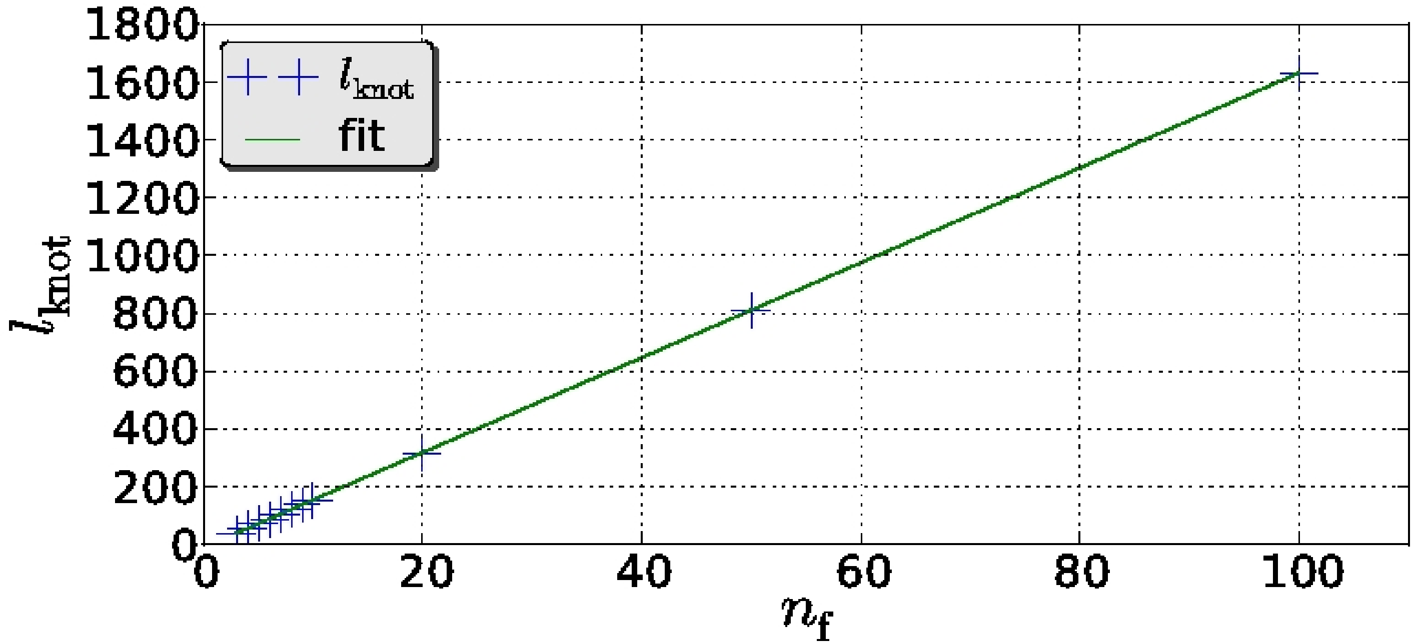
Sign of the crossings
for the 4-foil knot

Number of crossings increases like n_{f}^2

$$H_{\mathrm{M}} \propto n_{\mathrm{linking}}$$

$$H_{\mathrm{M}} \propto n_{\mathrm{f}}^2$$

Helicity vs. energy



$$E_M \propto l_{knot} \propto n_f$$

$$H_M \propto n_f^2$$

Knot is more strongly packed with increasing n_f.

Magnetic energy is closer to its lower limit for high n_f.