Numerical Wave Damping in the Pencil Code

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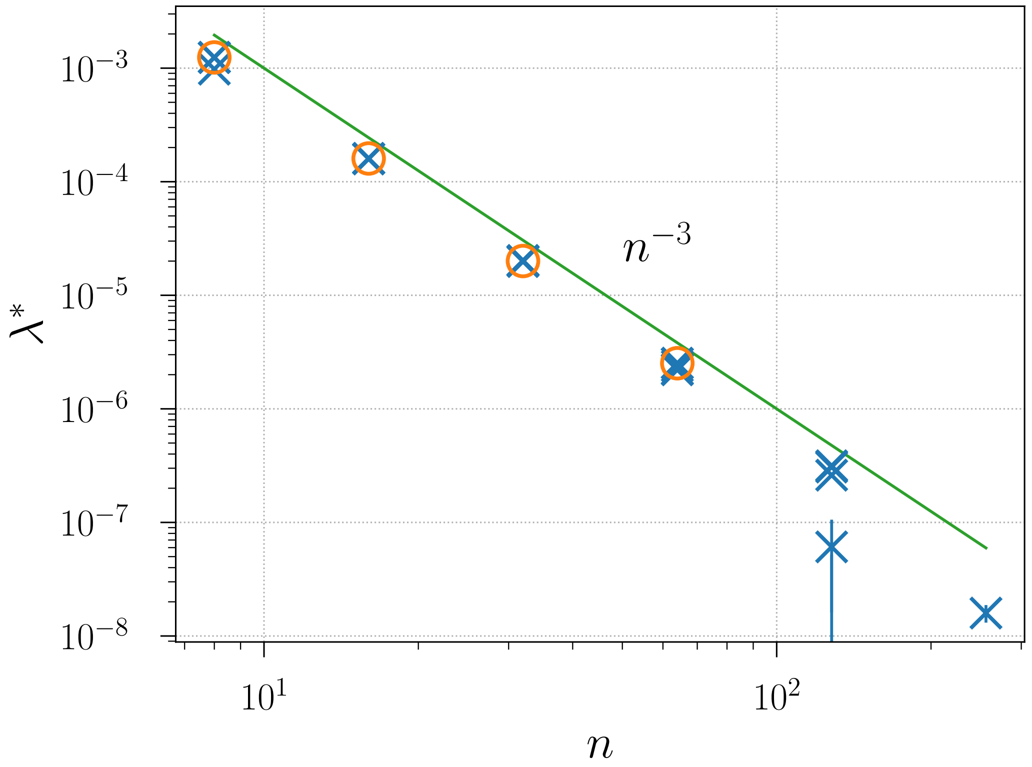
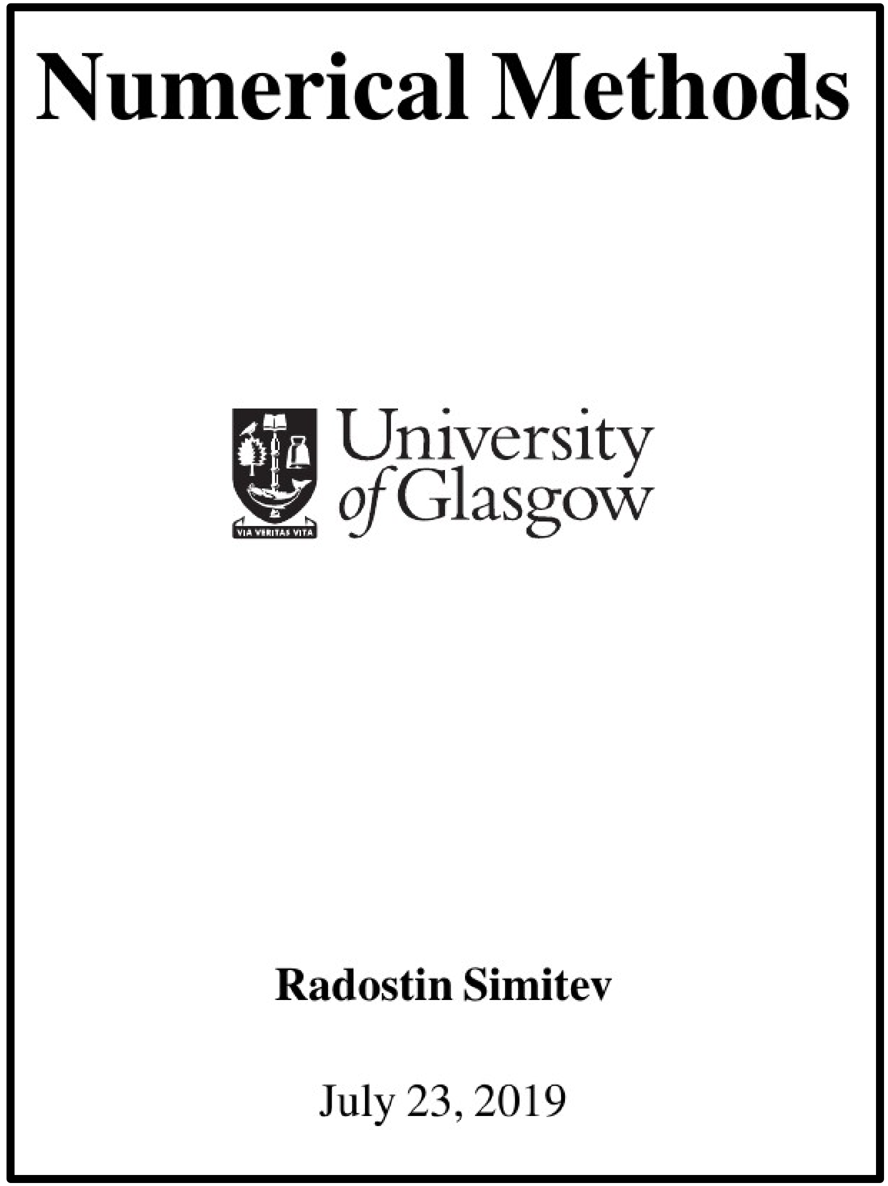


What is Numerical Diffusion?

Everyone is talking about it,
but no one knows what is really is.

Analytical Approach


Numerical Methods

University of Glasgow

VIA VERITAS VITA

Radostin Simitev

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Local Truncation Error

discretized → exact $\mathcal{L}[\hat{u}] = 0$

Definition 7.4. *The quantity*

$$\tau_{(h)} = \mathcal{A}_{(h)}[\hat{u}] - \mathcal{L}[\hat{u}] = \mathcal{A}_{(h)}[\hat{u}] = A_{(h)}\hat{u} - F_{(h)}.$$

is called the local truncation error (local residual) of the numerical scheme $\mathcal{A}_{(h)}[] = 0$.

Example 7.5. *Find the local truncation error of the numerical scheme*

$$\mathcal{A}_{(h)}[u] = \frac{u_{k-1} - 2u_k + u_{k+1}}{h^2} - f_k = 0$$

for the solution of

$$u'' - f = 0.$$

(see: Naveen)

This is solved exactly.

Solution. Now

$$\mathcal{A}_{(h)}[\hat{u}] = \frac{\hat{u}_{k-1} - 2\hat{u}_k + \hat{u}_{k+1}}{h^2} - f_k = (\hat{u}_k'' + O(h^2)) - f_k,$$

but

$$\mathcal{L}[\hat{u}] = \hat{u}_k'' - f_k = 0,$$

so using the definition directly

$$\tau_{(h)} = \mathcal{A}_{(h)}[\hat{u}] - \mathcal{L}[\hat{u}] = O(h^2).$$

Numerical Diffusion

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot\boldsymbol{u} + \tau_\rho$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -c_\mathrm{S}^2\boldsymbol{\nabla}\ln\rho + \boldsymbol{J}\times\boldsymbol{B}/\rho + \boldsymbol{F}_\mathrm{visc} + \tau_{\boldsymbol{u}}$$

$$\frac{\partial\boldsymbol{A}}{\partial t} = \boldsymbol{u}\times\boldsymbol{B} + \eta\nabla^2\boldsymbol{A} + \tau_{\boldsymbol{A}}$$

numerical diffusion: $\tau_{\boldsymbol{A}} = \eta^*\nabla^2\boldsymbol{A}$

How does the numerical diffusion depend on the grid resolution?

Alfvén Wave Damping

wave propagation: $e^{i\boldsymbol{k}\cdot\boldsymbol{x}+i\omega t}$

$\mathrm{Im}(\omega) \neq 0$ ⇨ wave damping

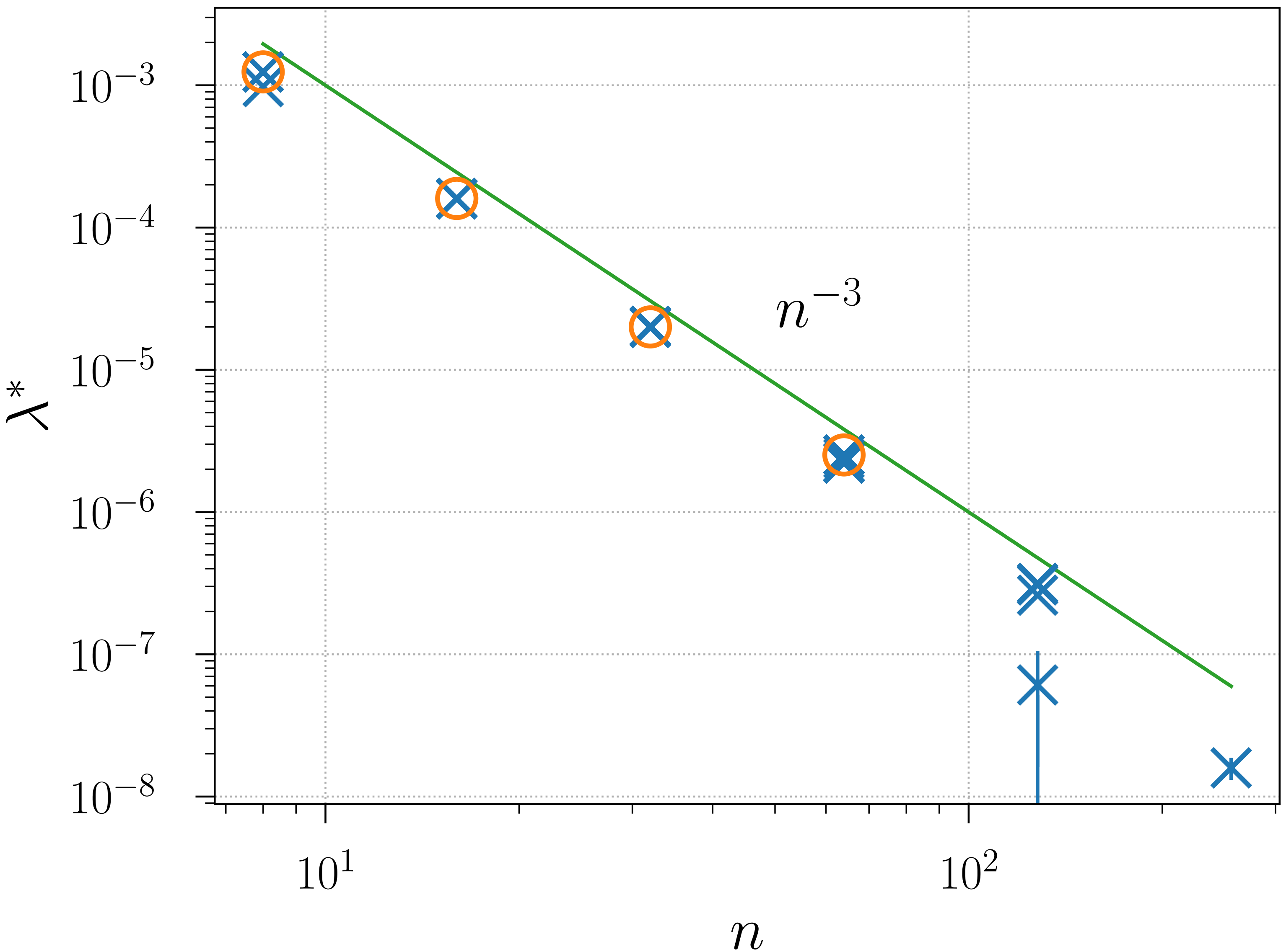
damping parameter: $\lambda = \mathrm{Im}(\omega) = \frac{1}{2}(\nu + \eta)k^2$

numerical damping: $\lambda^* = \frac{1}{2}(\nu^* + \eta^*)k^2$

⇨ Perform linear Alfven wave simulations and measure the numerical damping.

$$\nu = \eta \in [10^{-1}, 10^{-6}] \qquad n \in [8, 256]$$

Inviscid Navier-Stokes 3d



Numerical Experiments

THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 230:18 (32pp), 2017 June https://doi.org/10.3847/1538-4365/aa6254

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On the Measurements of Numerical Viscosity and Resistivity in Eulerian MHD Codes

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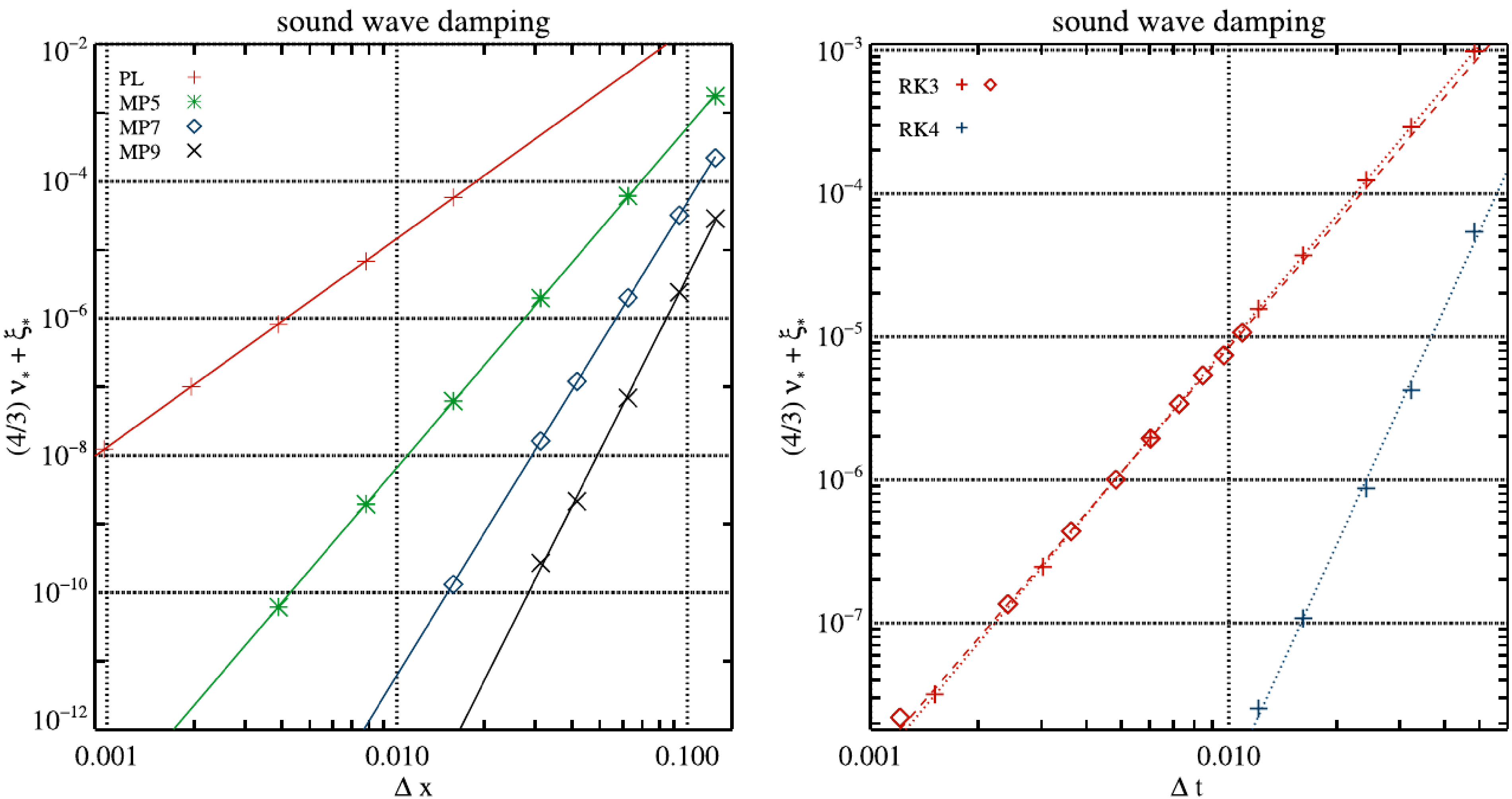
Table 1
Wave Damping Simulations I

Series	Wave	Reco	Riemann	Time	CFL	Resolution	$\mathfrak{N}^{\Delta x}_{\rm tot}$	r	$\mathfrak{N}^{\Delta t}_{\rm tot}$	q
#S1	sound	PL	HLL	RK4	0.01	64...1028	14.3 ± 0.7	3.049 ± 0.009	...	...
#S2	sound	MP5	LF	RK4	0.01	8...256	42.9 ± 2.3	4.957 ± 0.013	...	...
#S3	sound	MP5	HLL	RK4	0.01	8...256	43.4 ± 2.5	4.961 ± 0.014	...	...
#S4	sound	MP5	HLLD	RK4	0.01	8...256	42.7 ± 2.2	4.956 ± 0.013	...	...
#S5	sound	MP7	HLL	RK4	0.01	8...64	302 ± 20	6.897 ± 0.021	...	...
#S6	sound	MP9	HLL	RK4	0.01	8...32	830 ± 340	8.42 ± 0.15	...	...
#S7	sound	MP9	HLL	RK3	0.5	8...256	...	...	1.492 ± 0.013	2.985 ± 0.002
#S8	sound	MP9	HLL	RK3	0.1...0.9	64	...	...	2.45 ± 0.17	2.95 ± 0.01
#S9	sound	MP9	HLL	RK4	0.5	8...32	...	...	71 ± 32	5.5 ± 0.2
#A1	Alfvén	MP5	LF	RK4	0.01	8...256	42 ± 3	4.95 ± 0.02	...	...
#A2	Alfvén	MP5	HLL	RK4	0.01	8...256	42.6 ± 2.1	4.96 ± 0.01	...	...
#A3	Alfvén	MP5	HLLD	RK4	0.01	8...256	42 ± 3	4.95 ± 0.02	...	...
#A4	Alfvén	MP7	HLL	RK4	0.01	8 ...128	44 ± 53	6.19 ± 0.03	...	...
#A5	Alfvén	MP9	HLL	RK4	0.01	8...64	1190 ± 190	8.57 ± 0.06	...	...
#A6	Alfvén	MP9	HLL	RK3	0.8	16...128	...	...	0.86 ± 0.08	2.949 ± 0.022
#A7	Alfvén	MP9	HLL	RK4	0.8	8...64	...	...	7.6 ± 2.5	5.18 ± 0.10
#A8	Alfvén	MP5	HLL	RK3	0.5	5...1024	...	...	...	...
#MS1	magnetosonic	MP5	HLL	RK4	0.01	8...128	40 ± 3	4.95 ± 0.02	...	...
#MS2	magnetosonic	MP7	HLL	RK4	0.01	8...64	288 ± 20	6.903 ± 0.023	...	...
#MS3	magnetosonic	MP9	HLL	RK4	0.01	8...32	1970 ± 160	8.82 ± 0.03	...	...
#MS4	magnetosonic	MP9	HLL	RK3	0.1...0.9	64	...	...	1.77 ± 0.06	2.977 ± 0.007
#MS5	magnetosonic	MP9	HLL	RK4	0.2...0.9	64	...	...	4.3 ± 0.8	4.834 ± 0.013

Wave Damping



Conclusions

- Numerical viscosity and diffusion can be calculated analytically.
- Find numerical diffusion in numerical experiments.
- Power law of -3.

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