Magnetic Field Line Tangling and Topological Entropy

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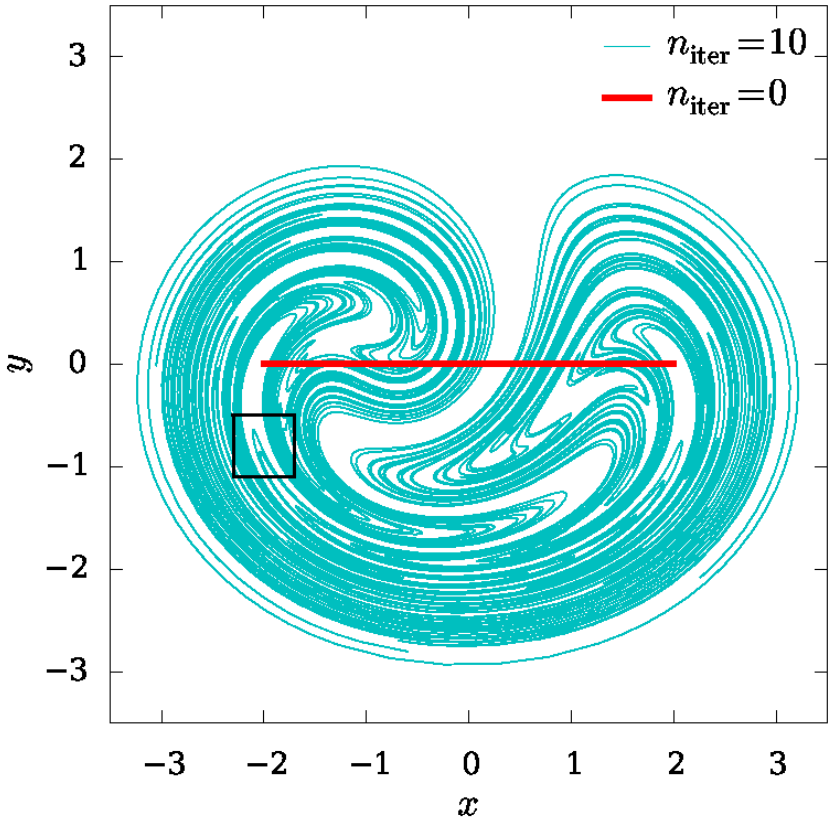
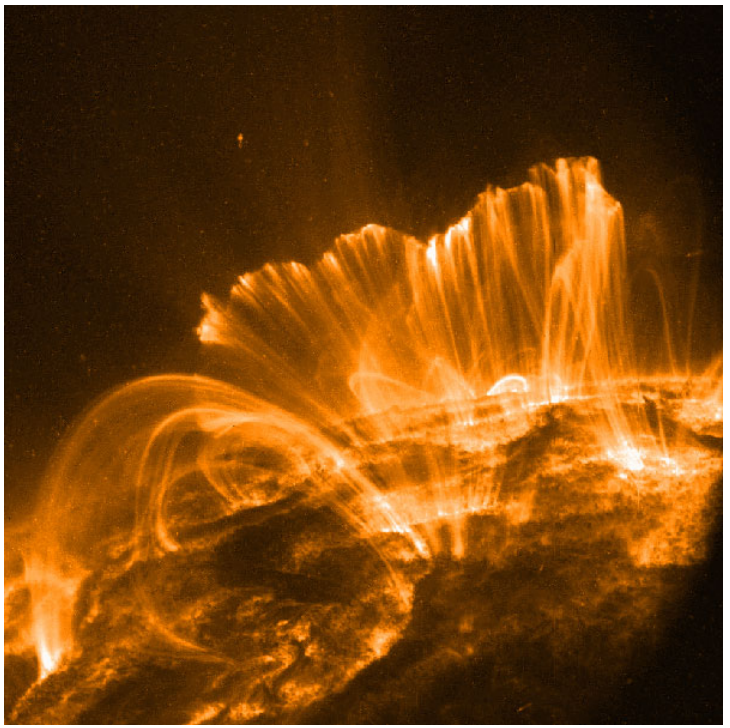
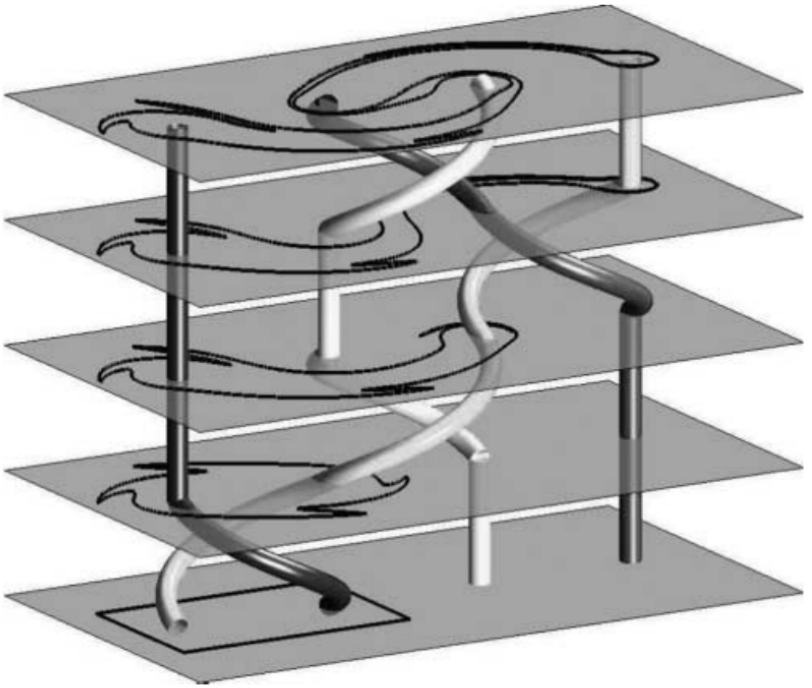


Magnetic Fields

NASA

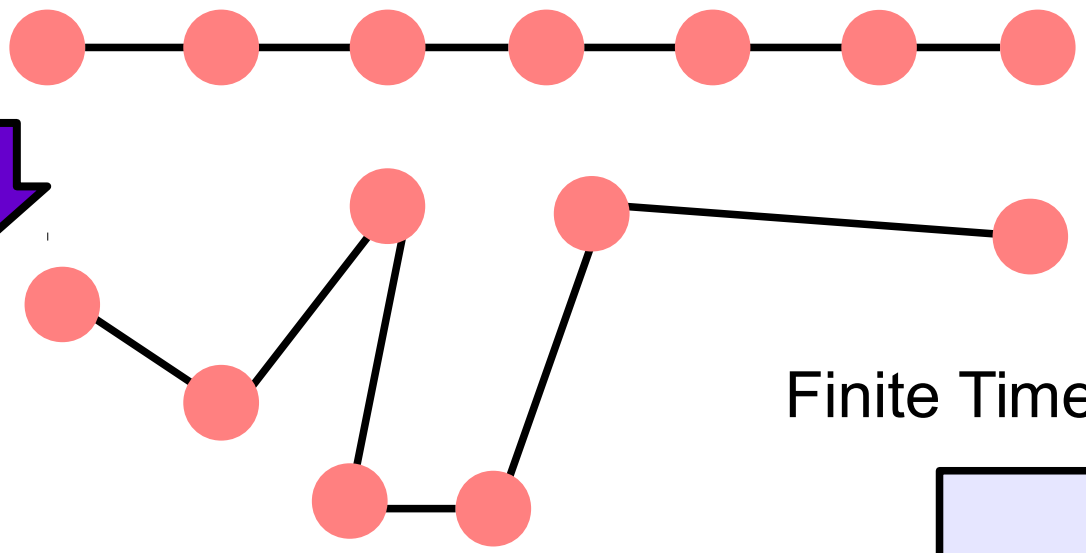




(Thiffeault et al. 2006)

- Field line tangling in solar and laboratory magnetic fields.
- Study the tangling of magnetic field lines.

Topological Entropy



$n_{\text{iter}} = 0 \qquad l_0$

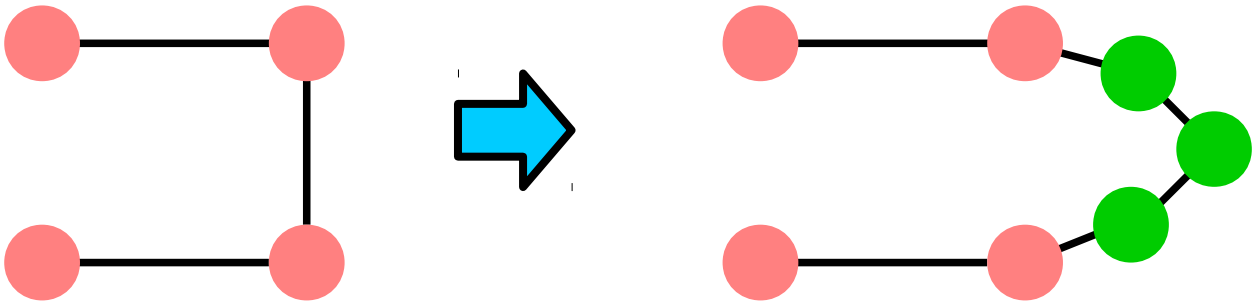
$n_{\text{iter}} > 0 \qquad l$

Finite Time Topological Entropy (FTTE):

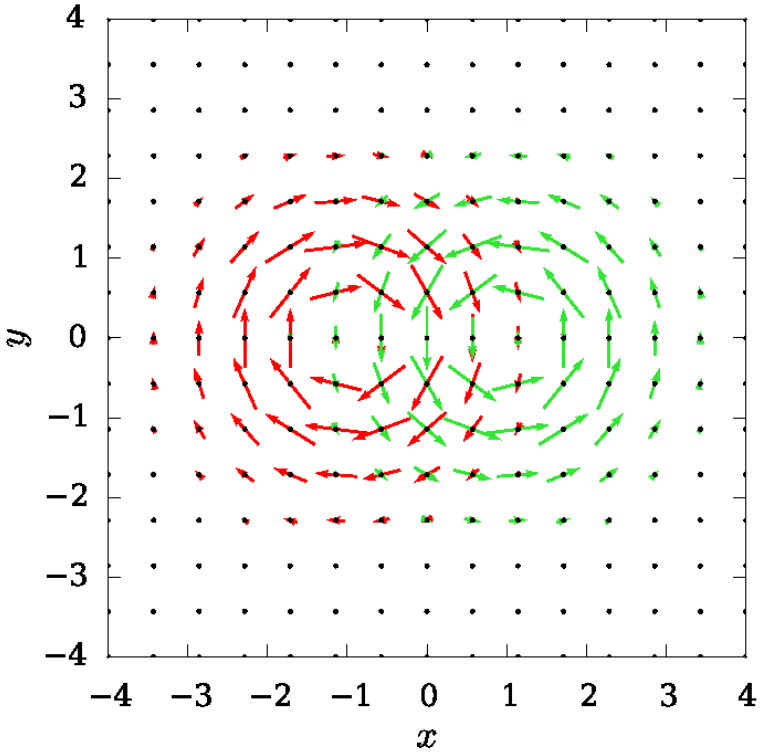
$$h = \frac{1}{n_{\text{iter}}} \ln(l/l_0)$$

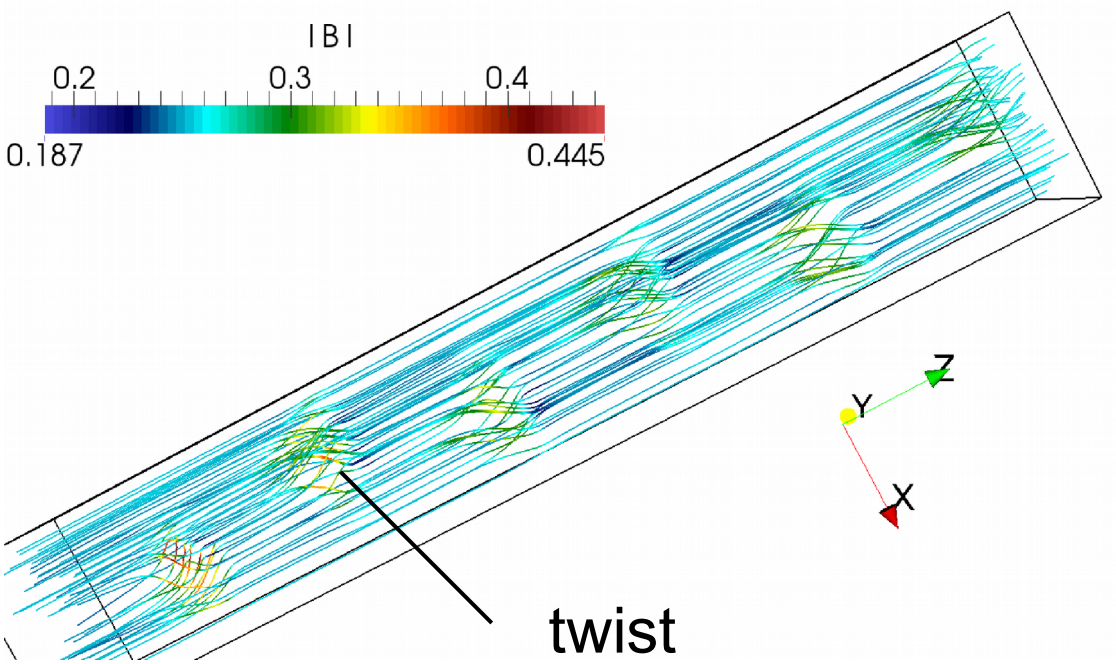
Number of necessary points large!

Adaptive refinement:



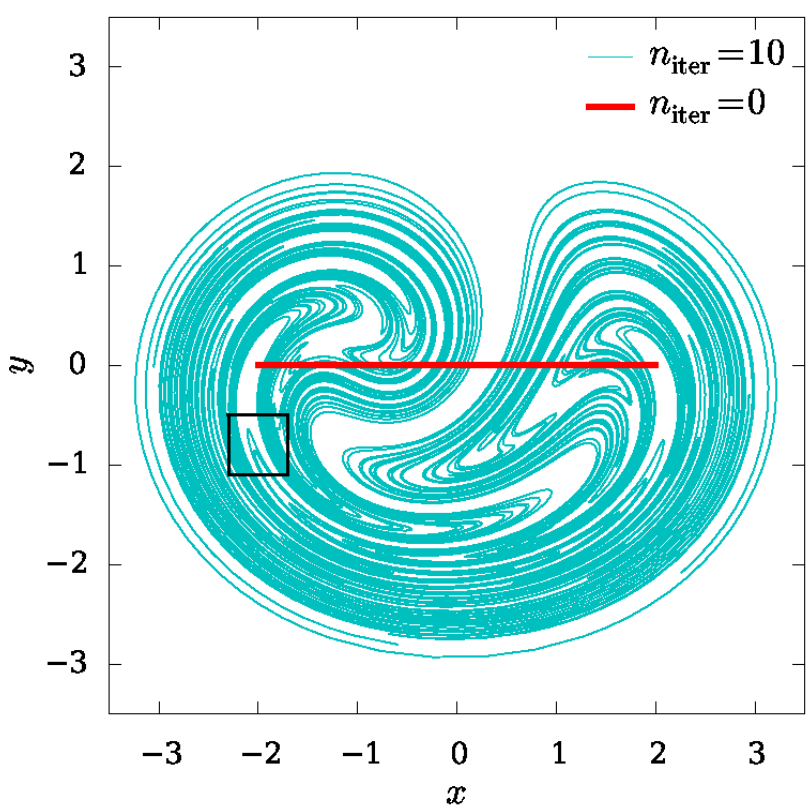
Blinking Vortex Experiments

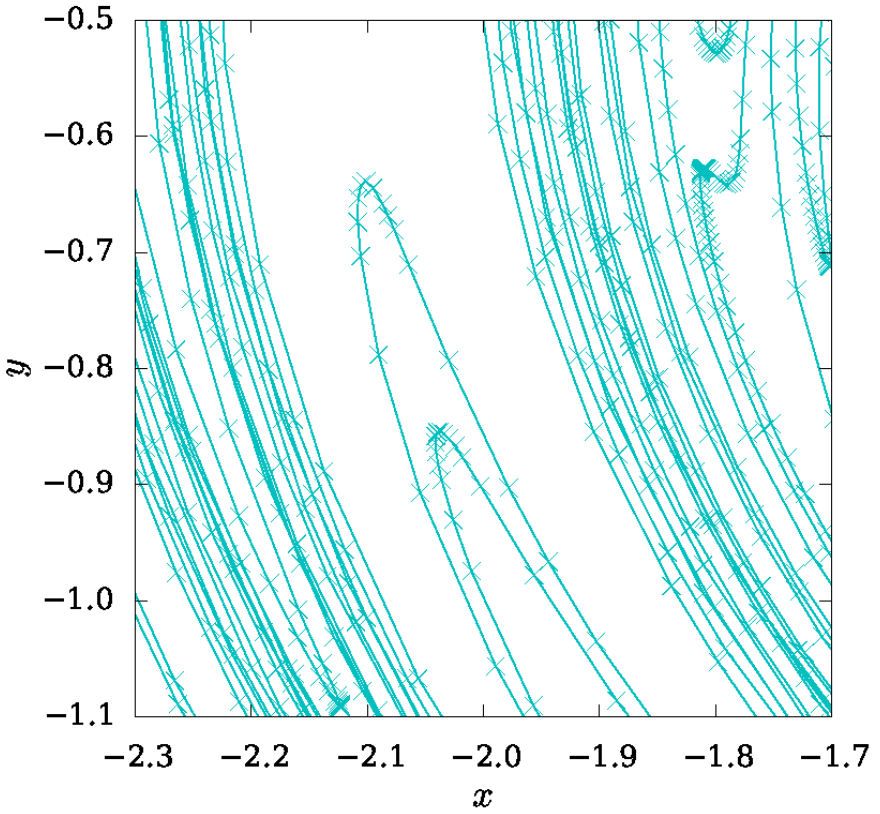




- Repeated applications of the blinking vortex motion.
- World lines correspond to 3d braided magnetic field.

Blinking Vortex Experiments

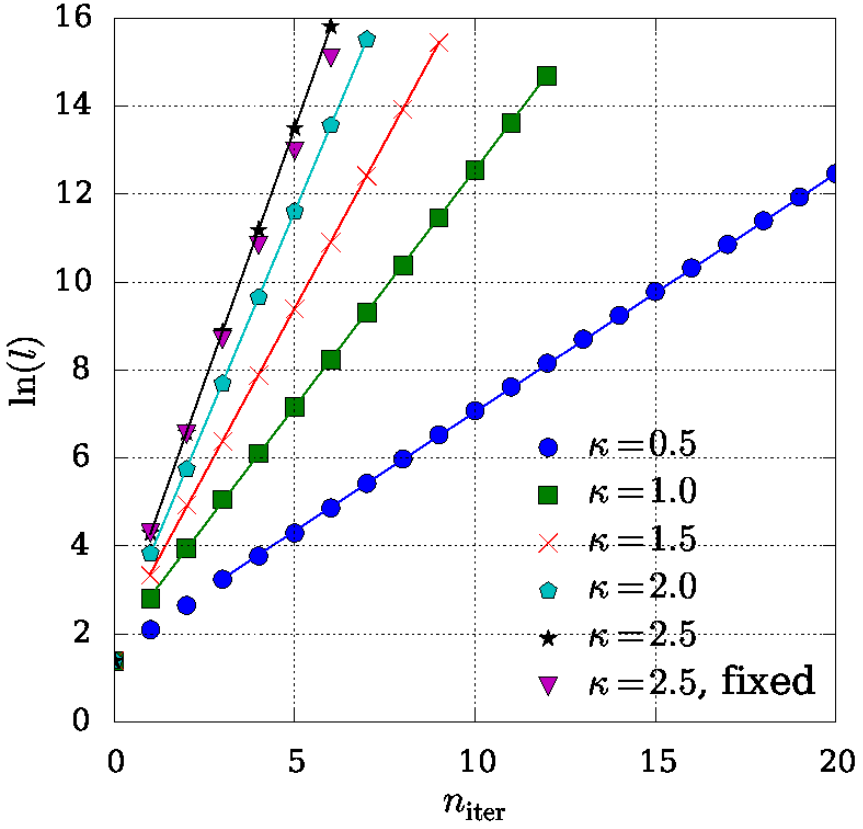


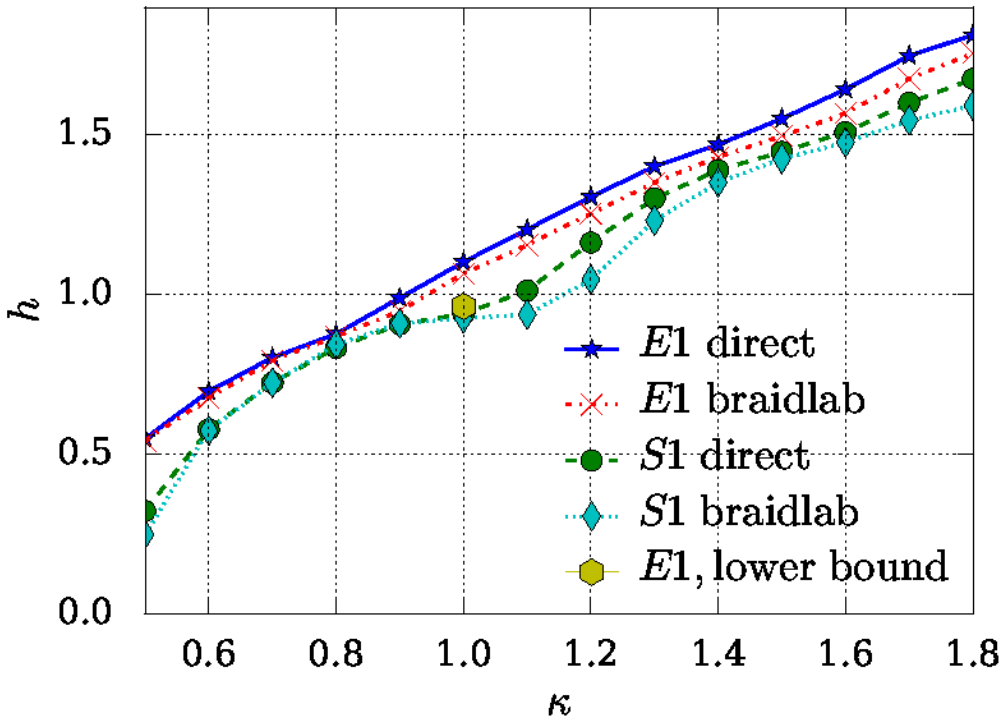


(Candelaresi et al. 2017)

- Adaptive refinement successfully increases resolution where needed.
- Greatly decrease number of points.

Blinking Vortex Experiments

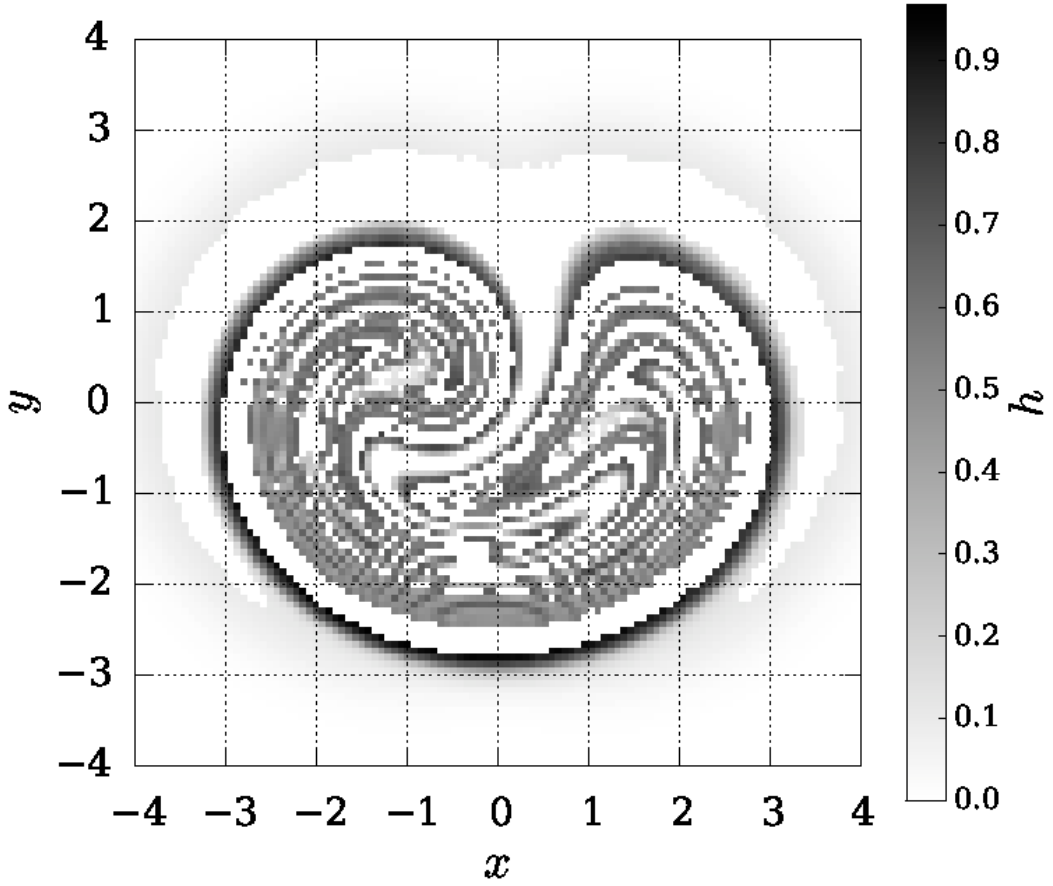




(Candelaresi et al. 2017)

- Speed up of 450x compared to previous methods.
- Accurately compute the FTTE
- See talk by David Pontin

Topological Entropy Distribution



- Map circles and measure their exponential stretching.
- FTTE distribution shows areas of chaotic behavior.

Conclusions

- Measure field line tangling through topological entropy.
- Estimate the entropy through material line stretching.
- Adaptively refine calculations.
- Speed up of 450x.
- Spatial topological entropy distribution.

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