

Twisted Magnetic Knots and Links and their Current Alignment

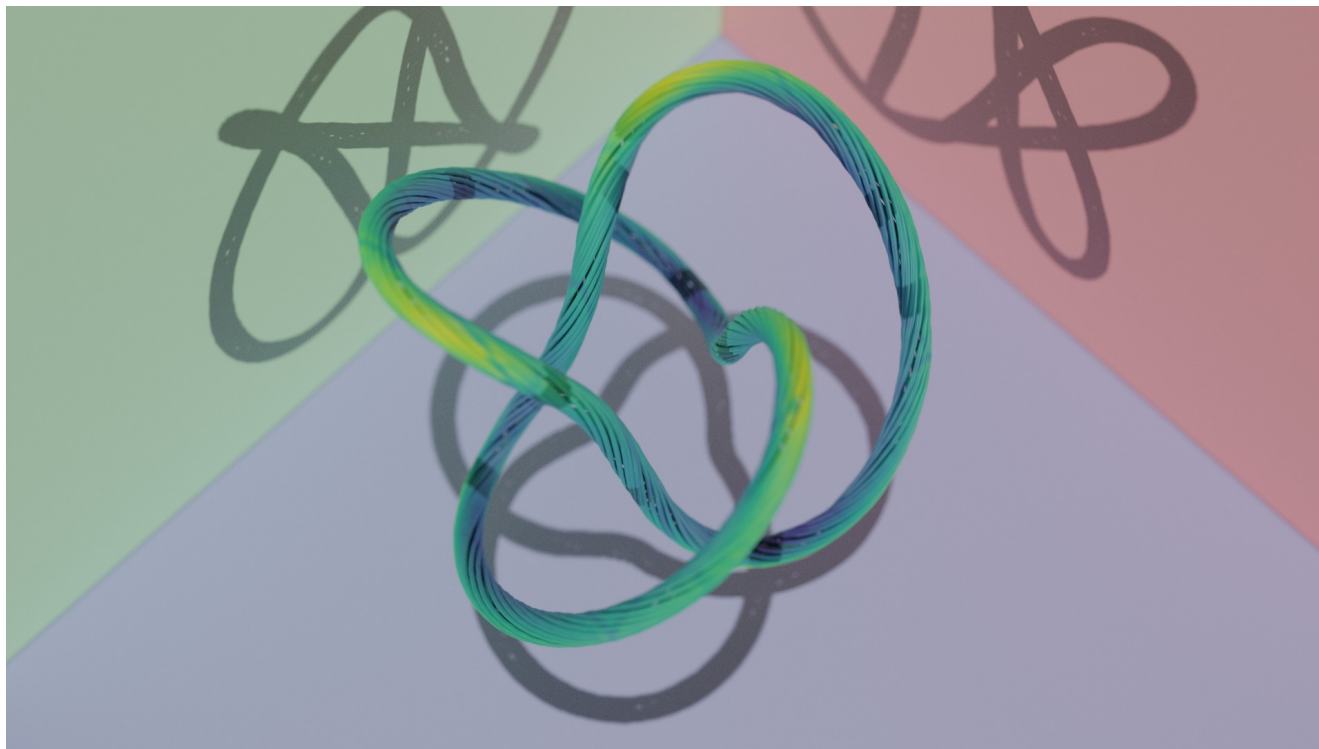
Simon Candelaresi, Celine Beck

H L R I S

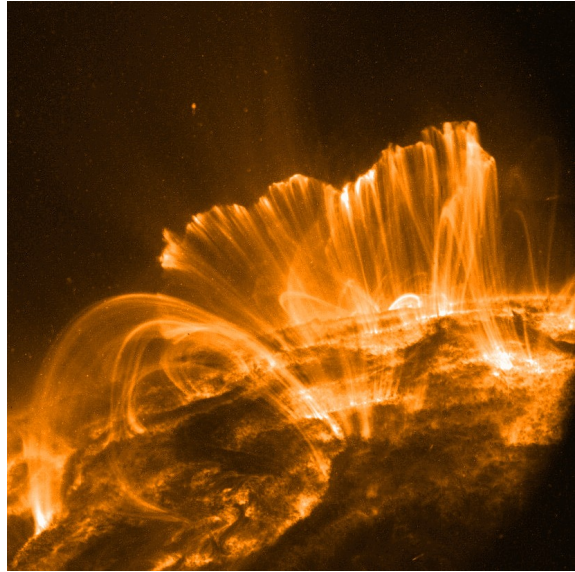
High-Performance Computing Center Stuttgart



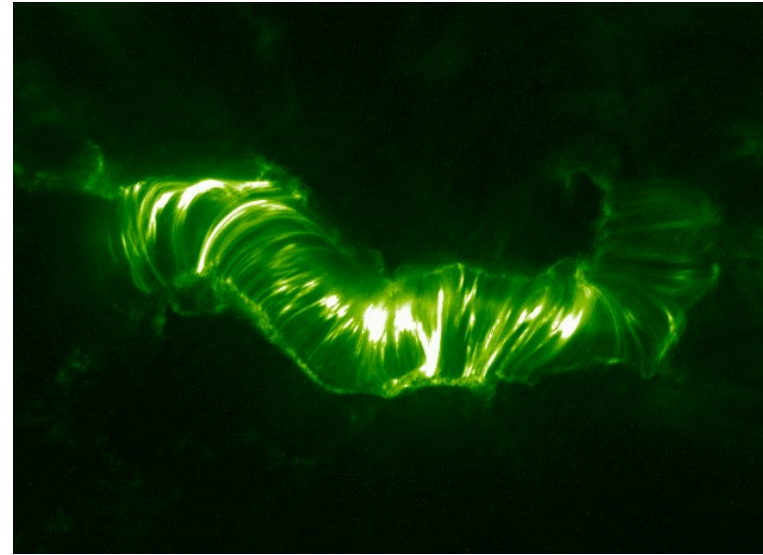
University
of Glasgow



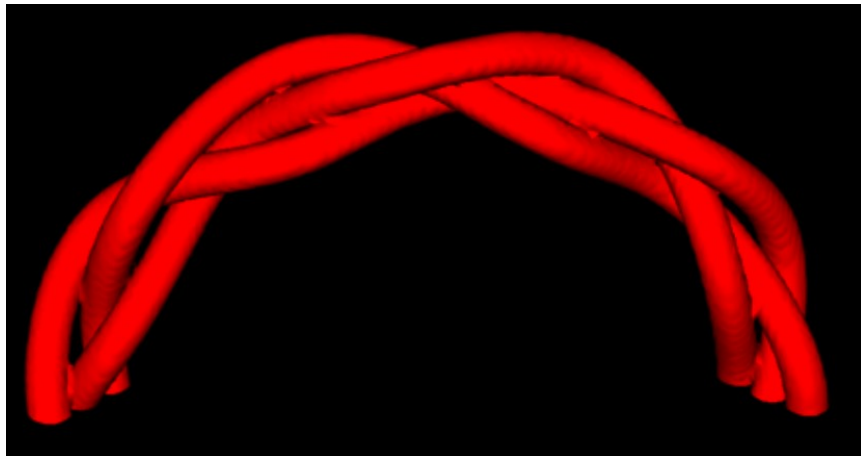
Solar Magnetic Field



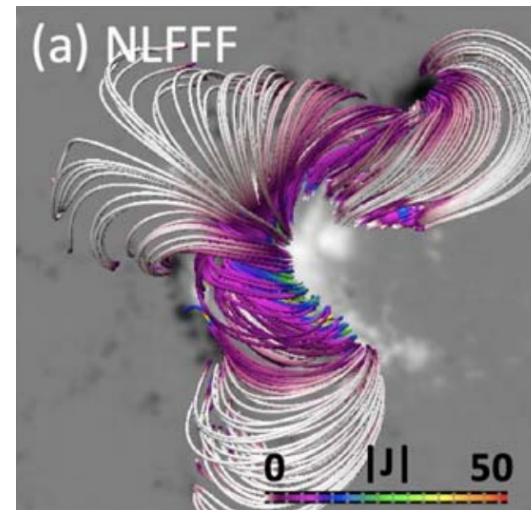
(Trace)



(Trace)



(Prior and MacTaggart 2016)



(Yamasaki et al. 2021)

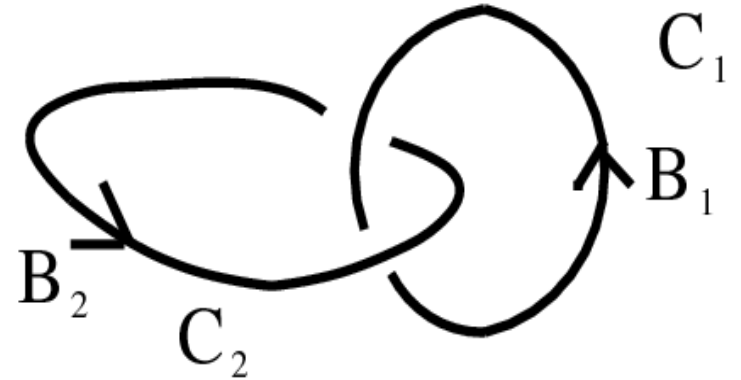
Magnetic Helicity

Measure for the topology:

$$H_M = \int_V \mathbf{A} \cdot \mathbf{B} \, dV = 2n\phi_1\phi_2$$

$$\nabla \times \mathbf{A} = \mathbf{B} \quad \phi_i = \int_{S_i} \mathbf{B} \cdot d\mathbf{S}$$

n = number of mutual linking



Conservation of magnetic helicity:

$$\lim_{\eta \rightarrow 0} \frac{\partial}{\partial t} \langle \mathbf{A} \cdot \mathbf{B} \rangle = 0 \quad \eta = \text{magnetic resistivity}$$

$$\frac{\partial}{\partial t} \int_V \mathbf{A} \cdot \mathbf{B} \, dV = -2\eta \int_V \mathbf{J} \cdot \mathbf{B}$$

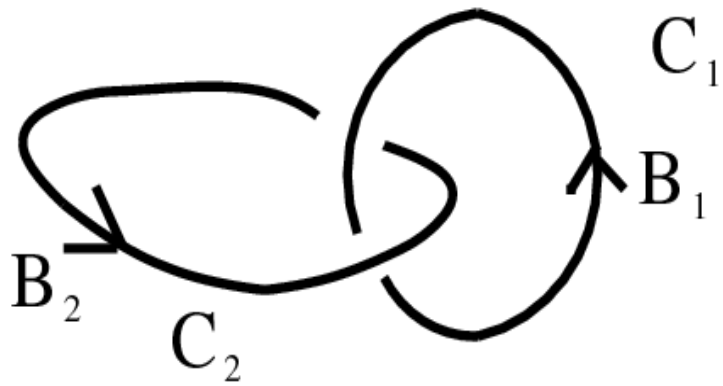
Realizability condition:

$$E_m(k) \geq k|H(k)|/2\mu_0$$

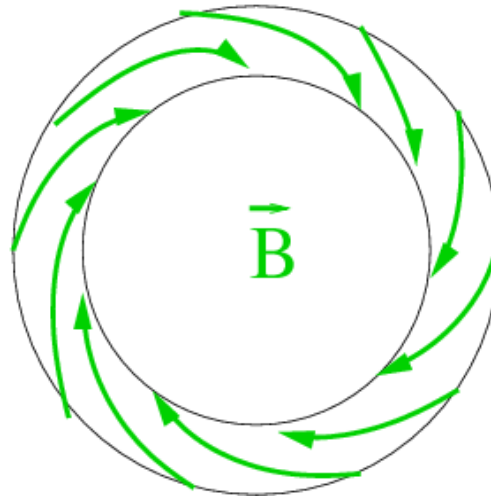


Magnetic energy is bound from below by magnetic helicity.

Topologies of Magnetic Fields



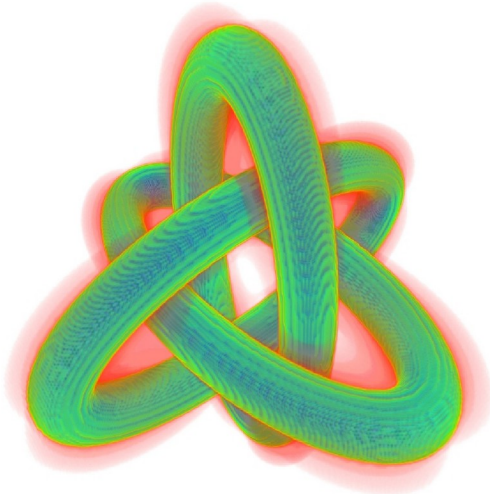
Hopf link



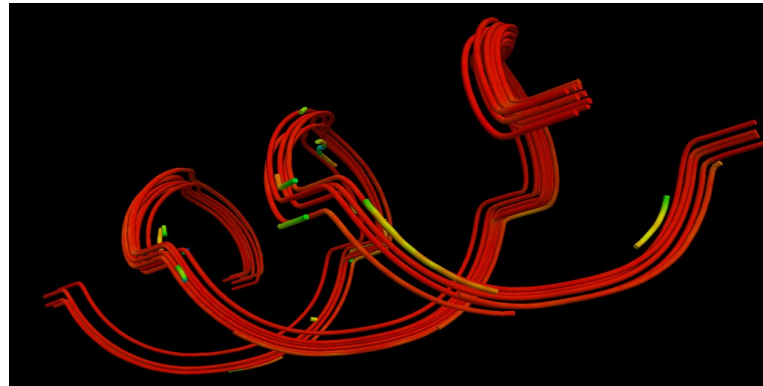
twisted field



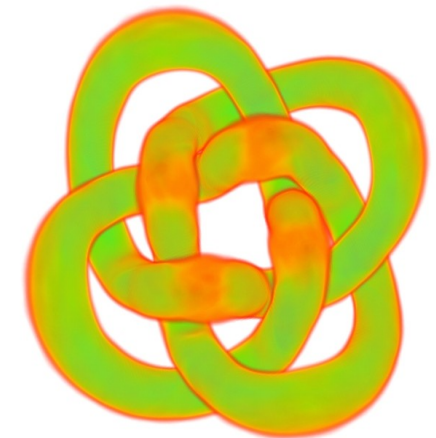
trefoil knot



Borromean rings



magnetic braid

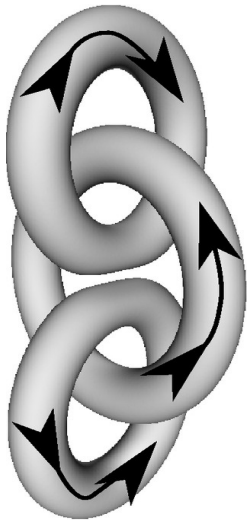


IUCAA (8_18) knot

Interlocked Flux Rings

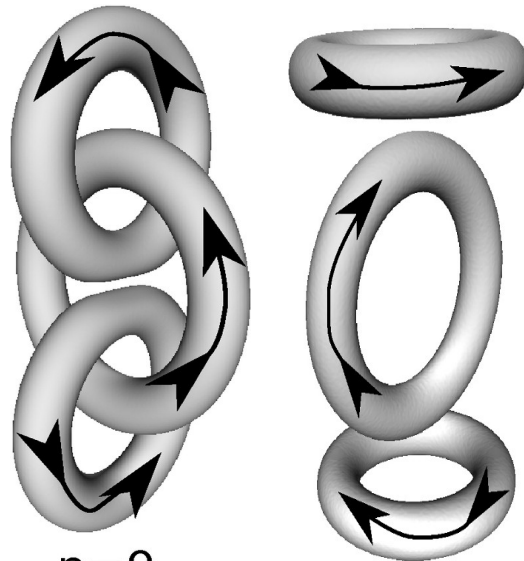
actual linking vs. magnetic helicity

$$H_M \neq 0$$



$n=2$

$$H_M = 0$$



$n=0$

- initial condition: flux tubes
- isothermal compressible gas
 - viscous medium
 - periodic boundaries

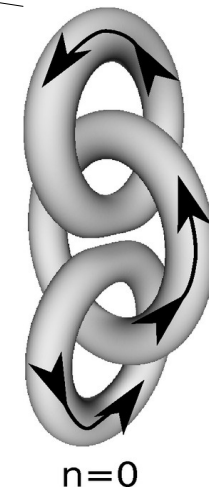
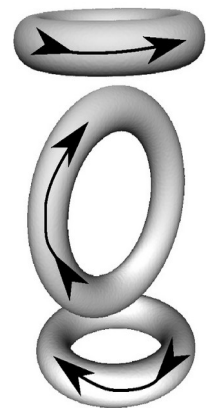
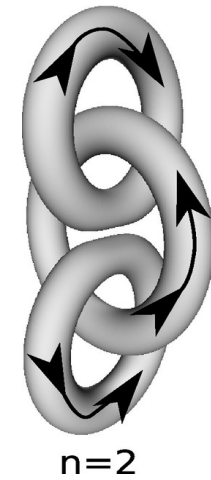
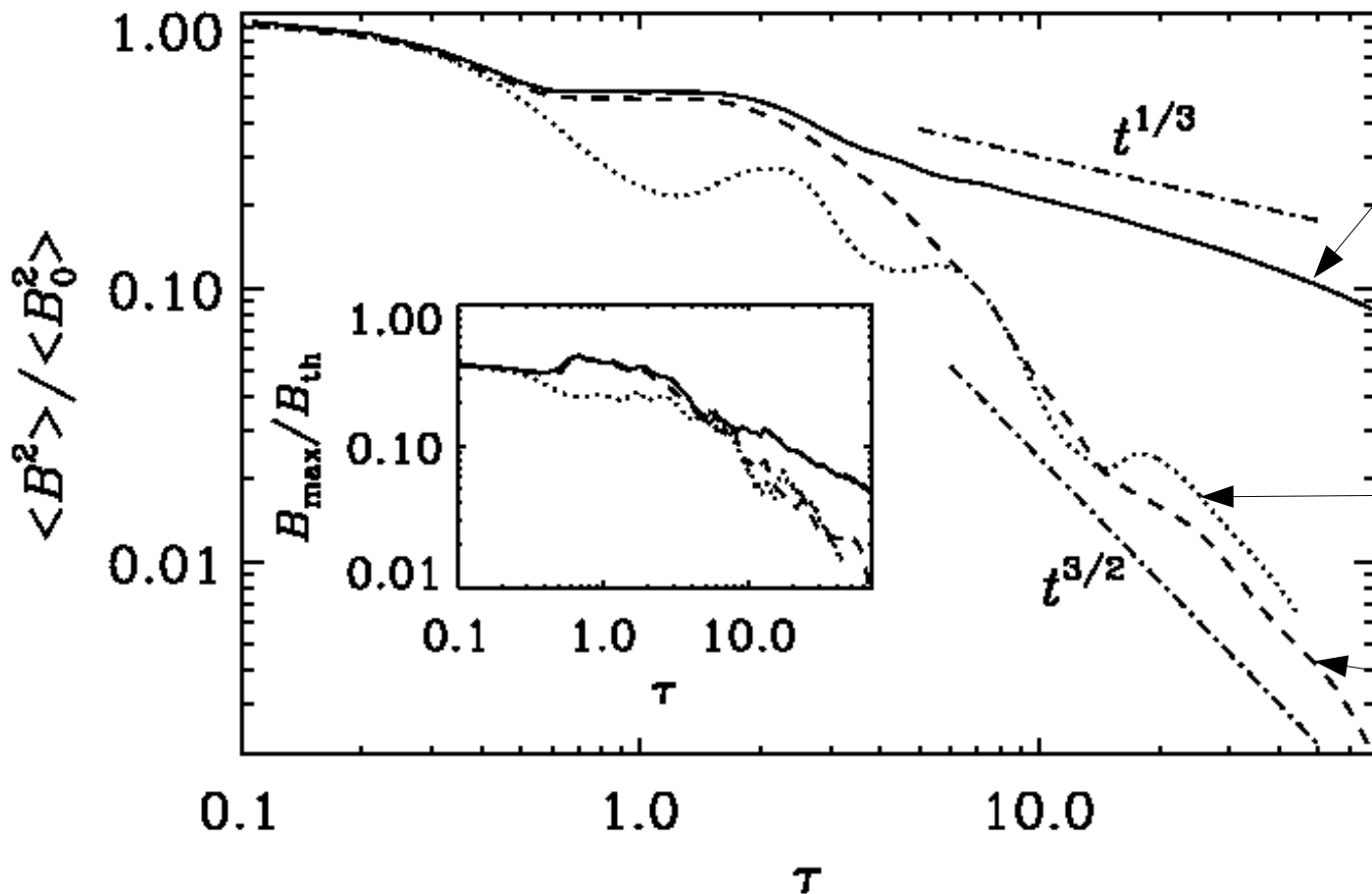
(Del Sordo et al. 2010)

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}$$

$$\frac{D\mathbf{U}}{Dt} = -c_S^2 \nabla \ln \rho + \mathbf{J} \times \mathbf{B} / \rho + \mathbf{F}_{\text{visc}}$$

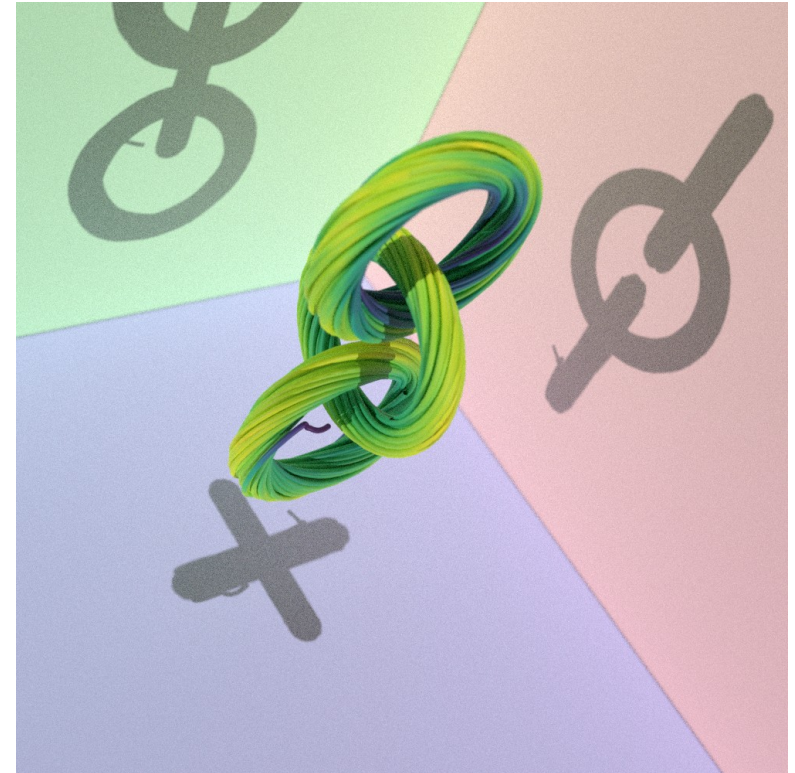
Interlocked Flux Rings



Magnetic helicity rather than actual linking determines the field decay.

Magnetic Fields with a Twist

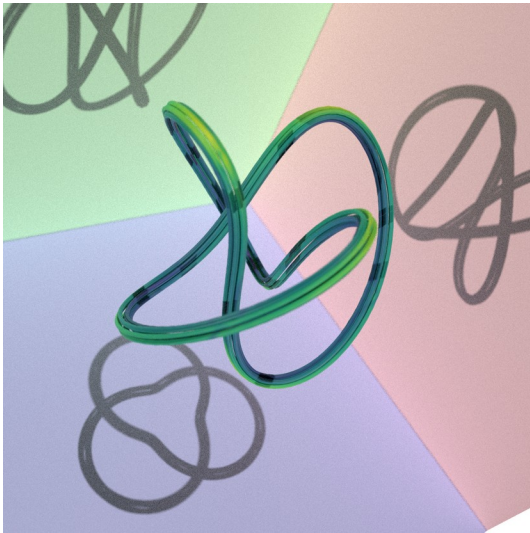
- ➔ Helical fields can be made non-helical by twisting the field lines.
- ➔ Non-helical fields can be made helical by twisting the field lines.
- ➔ Simulated twisted knots and links in MHD (Pencil Code).



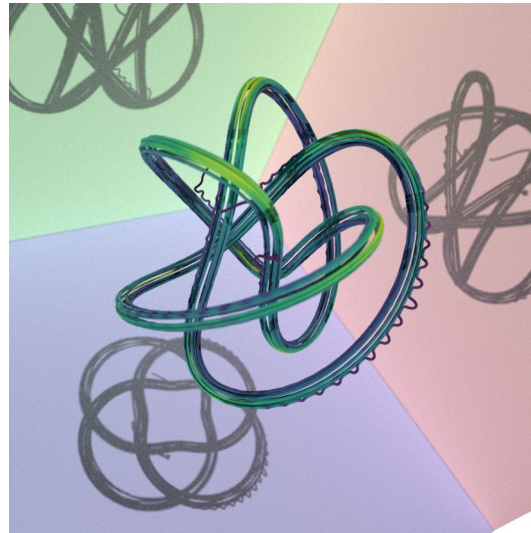
$$E_M(t) =? \quad \frac{d}{dt} H_m =? \quad \int_V J \cdot B \, dV =?$$

Knots and Links

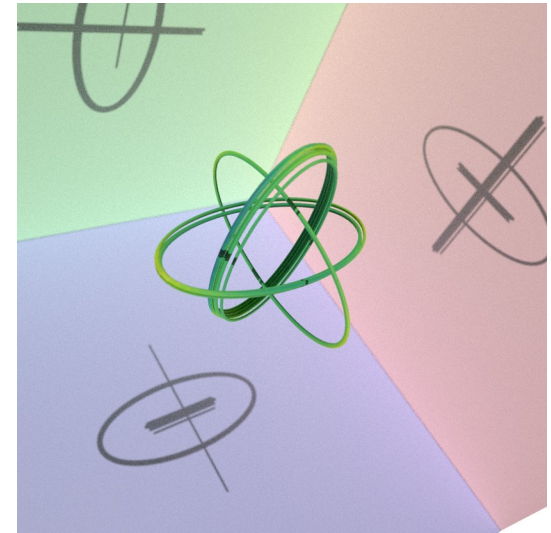
trefoil



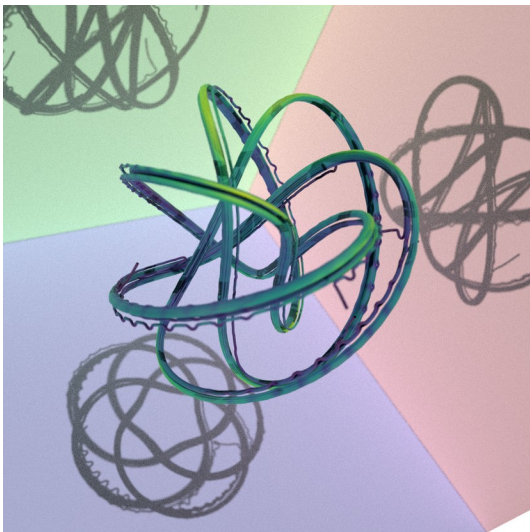
4-foil



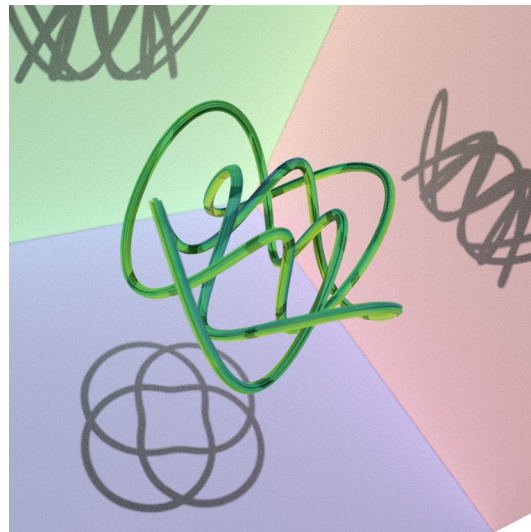
Borromean rings



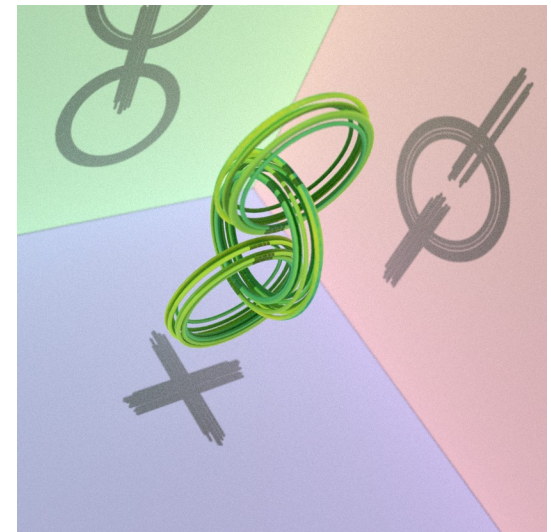
5-foil



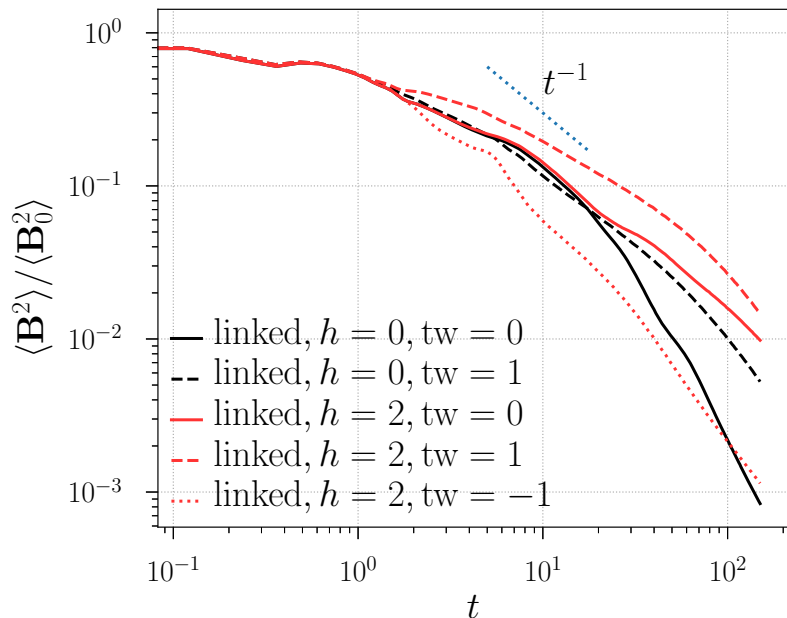
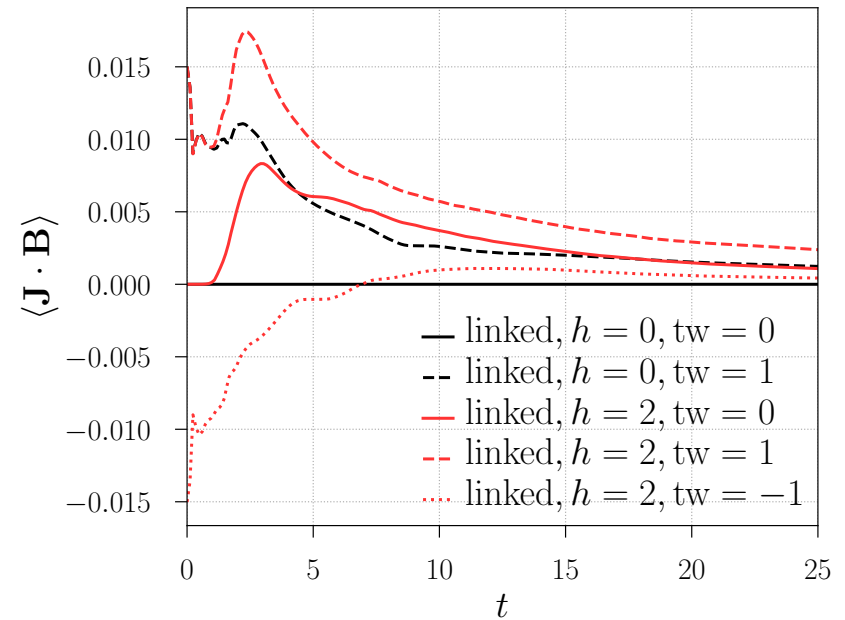
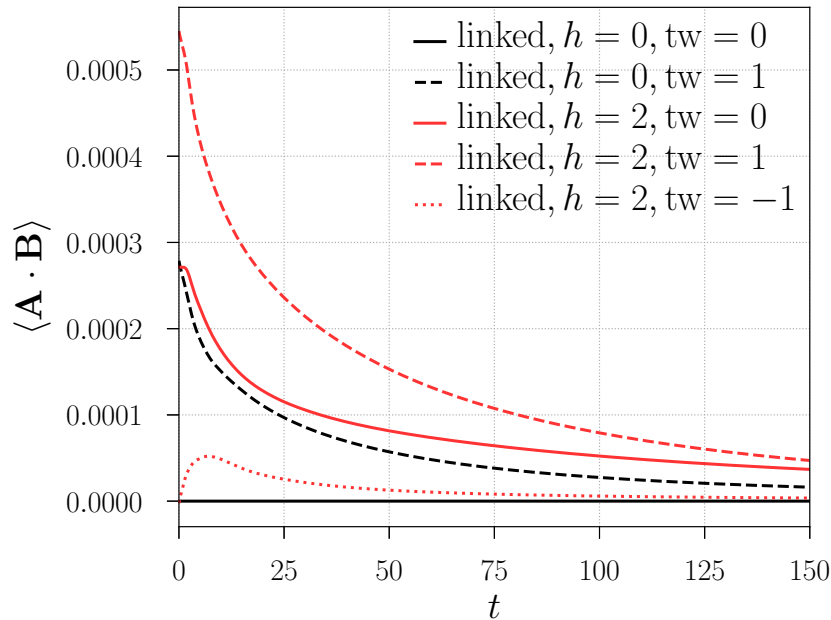
IUCAA (8_18)



triple rings



Triple Rings

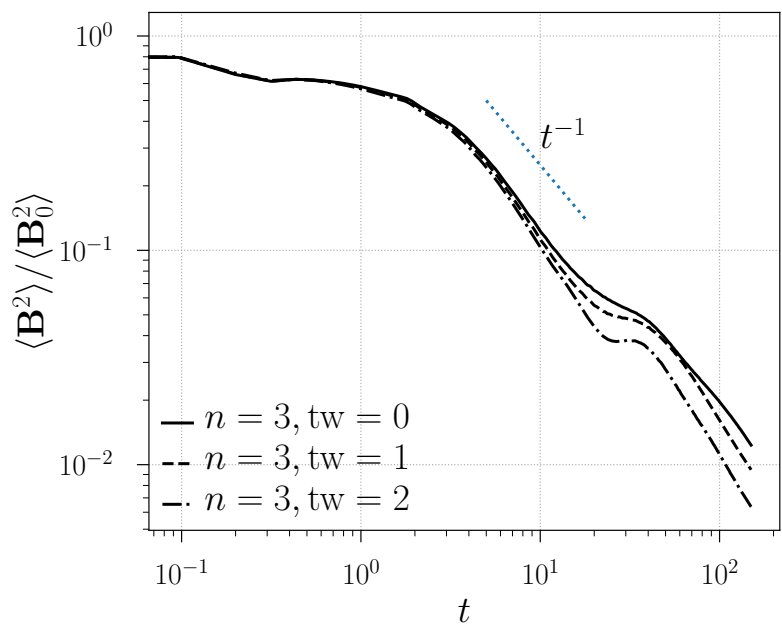
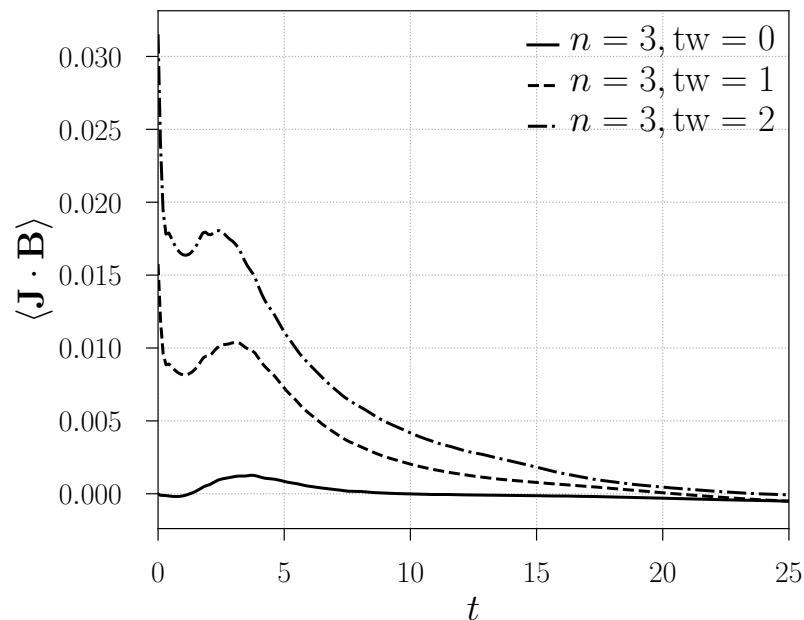
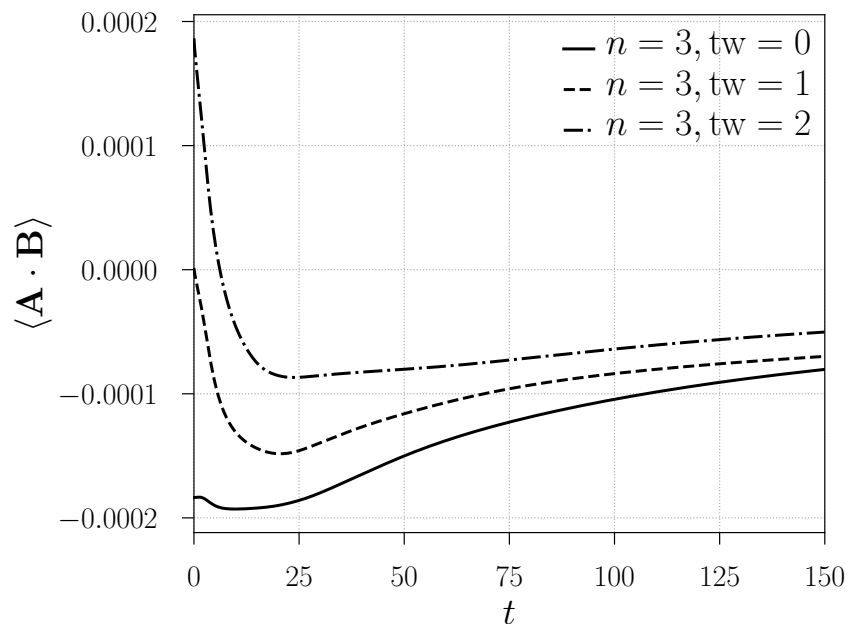


Helicity restricts decay.



Small helicity production in twisted non-helical field.

Knots

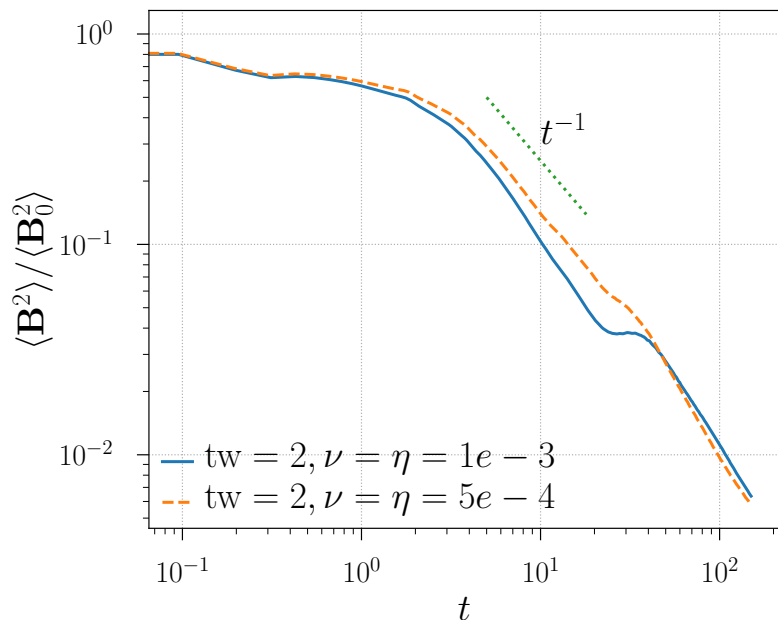
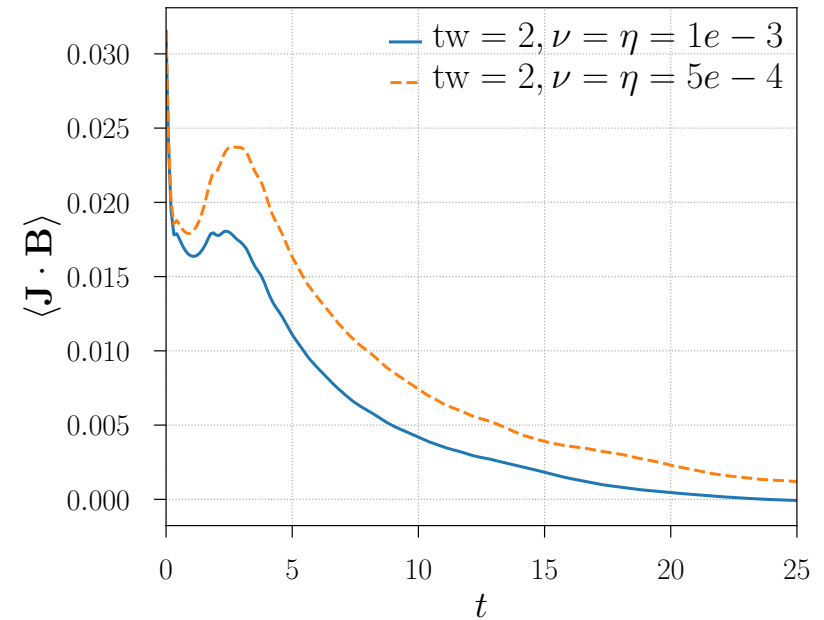
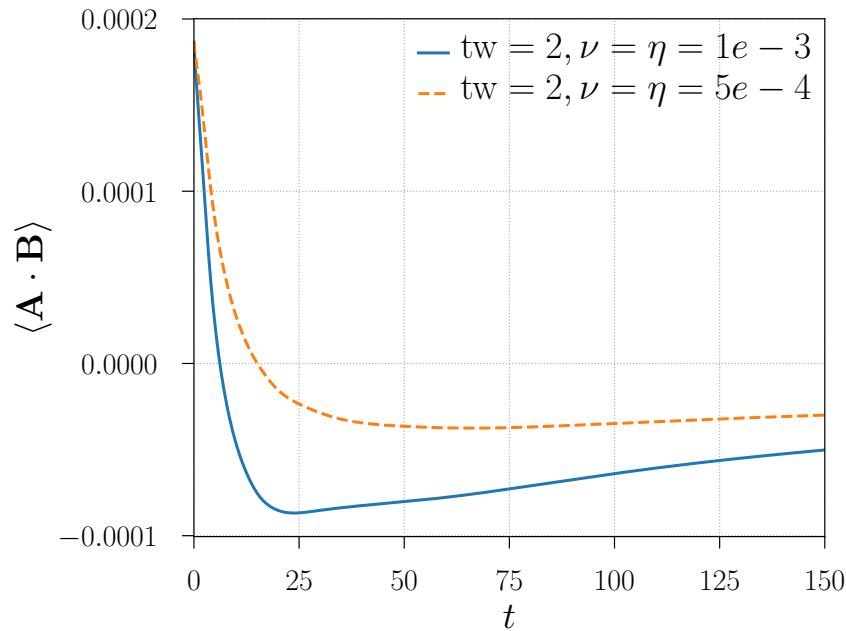


Significant helicity production.



Initial helicity is not a good predictor on dynamics.

Low Resistivity Twisted Trefoil Knot



➔ Stronger alignment of J and B.

➔ Lower resistivity partially compensated by stronger alignment.

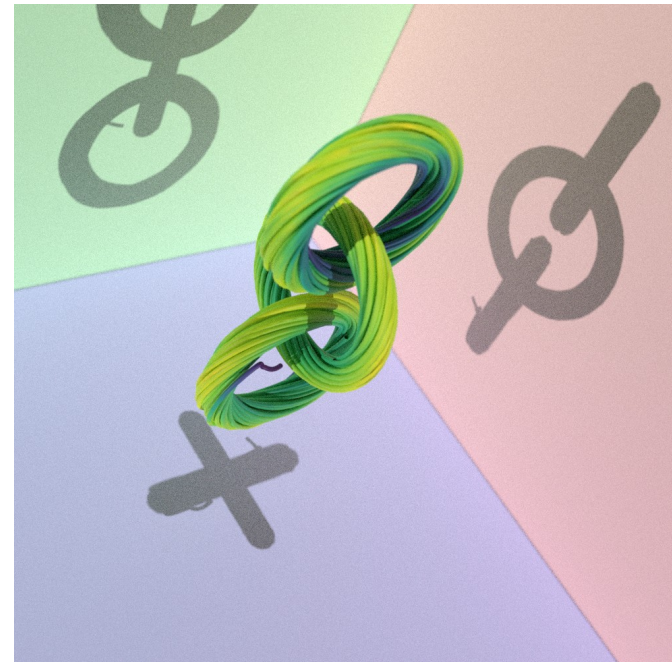
Conclusions

- Helicity alone not a good indicator.
- Consider helicity production (current – magnetic field alignment)
- Increased turbulent effects at lower resistivity leads to stronger J-B alignment and significant helicity production.



Candelaresi and Beck
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BlenDaViz:
github.com/SimonCan/BlenDaViz