Polynomial knot invariants in the dynamics of braided magnetic fields

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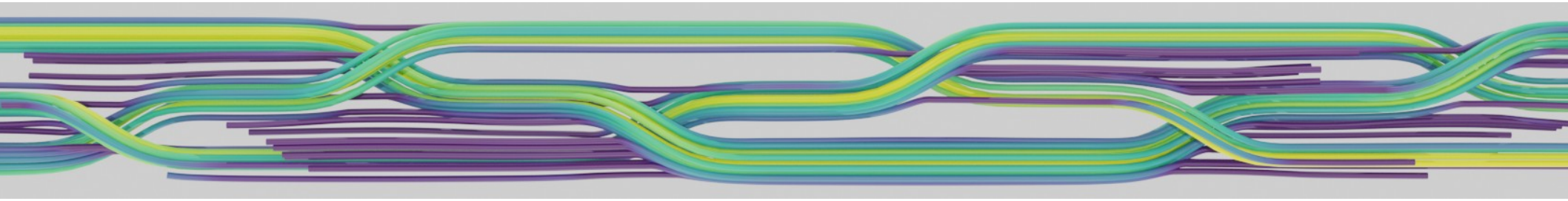
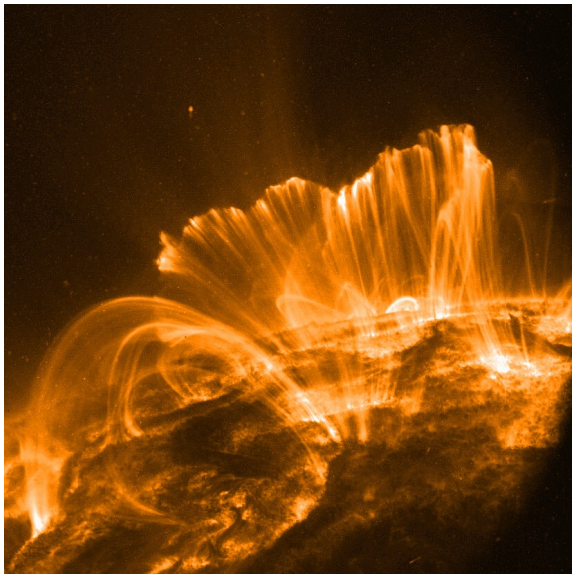
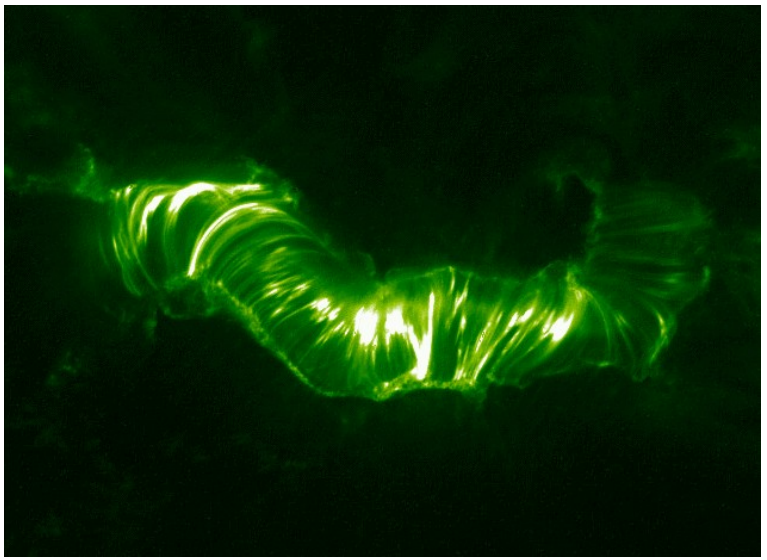


Solar Magnetic Field



(Trace)



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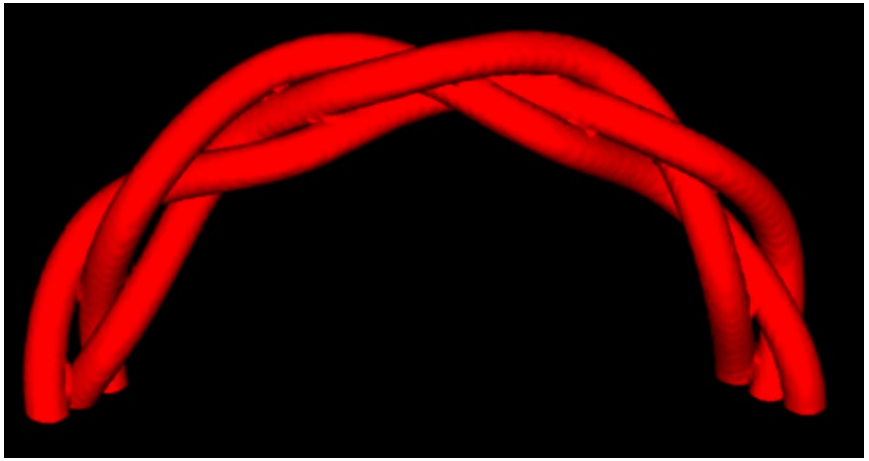
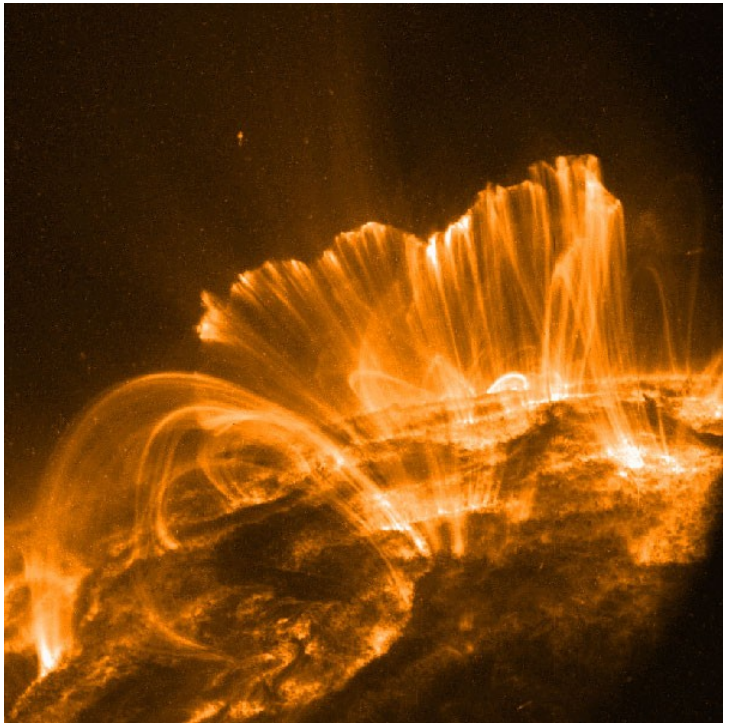
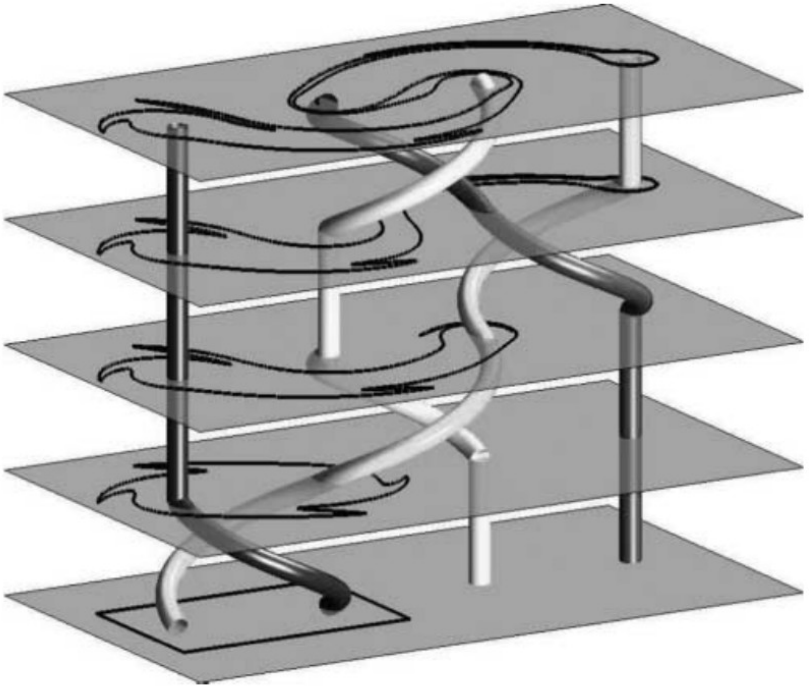


- Twisted flux tubes may rise to the corona. *(Prior and MacTaggart 2016)*.

Coronal Magnetic Fields

NASA





(Thiffeault et al. 2006)

Field line tangling in solar magnetic fields.

Magnetic Helicity

$$H_{\mathrm{m}} = \int \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 0$$

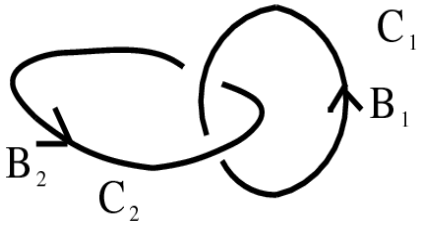
Conservation of magnetic helicity:

$$\lim_{\eta \to 0} \frac{\partial}{\partial t} \int \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 0 \qquad \eta = \text{ magnetic resistivity}$$

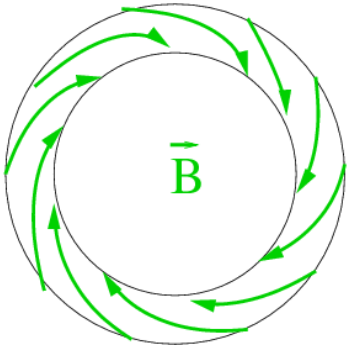
Realizability condition:

$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$

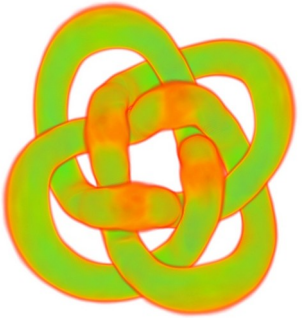
Magnetic energy is bound from below by magnetic helicity.



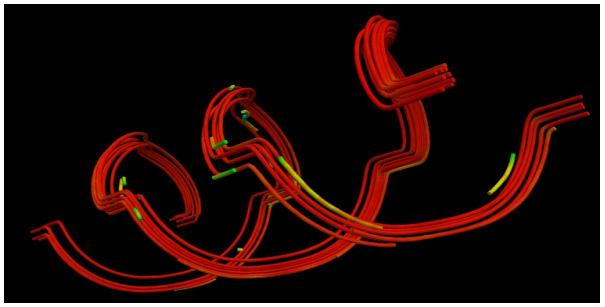
link



twist



knot



braid

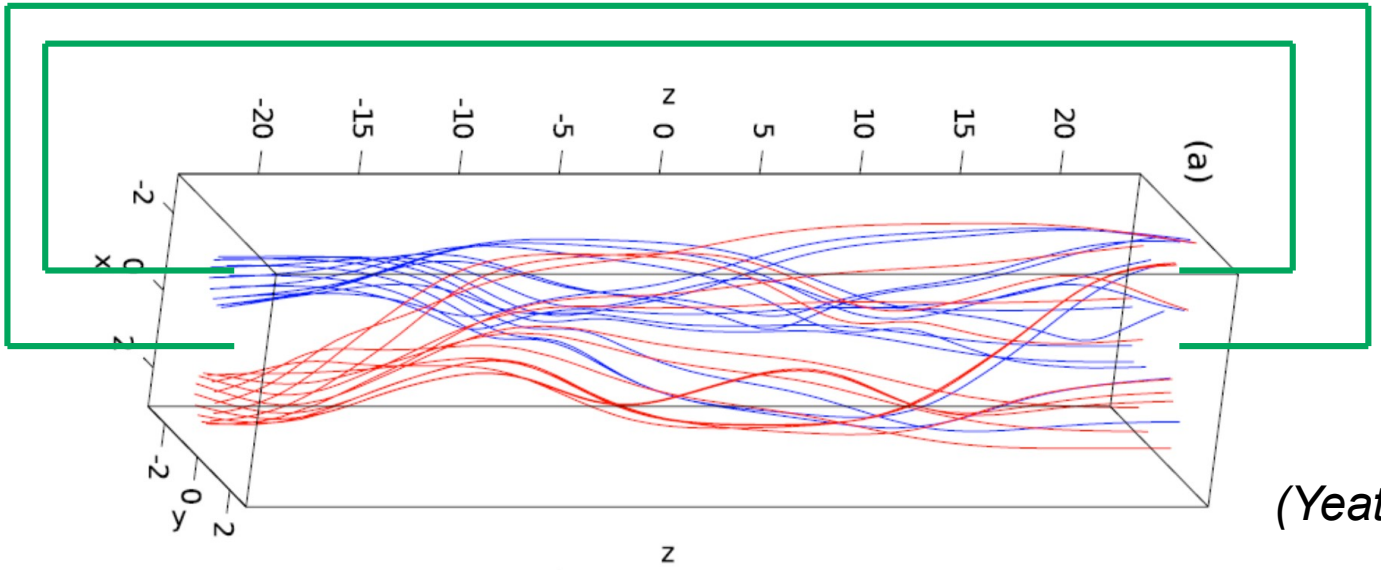
Beyond Magnetic Helicity

Describe knots, braids and links using knot polynomials:

Jones polynomials for the trefoil knot:

$$q - 1 + q^{-1}$$

closure

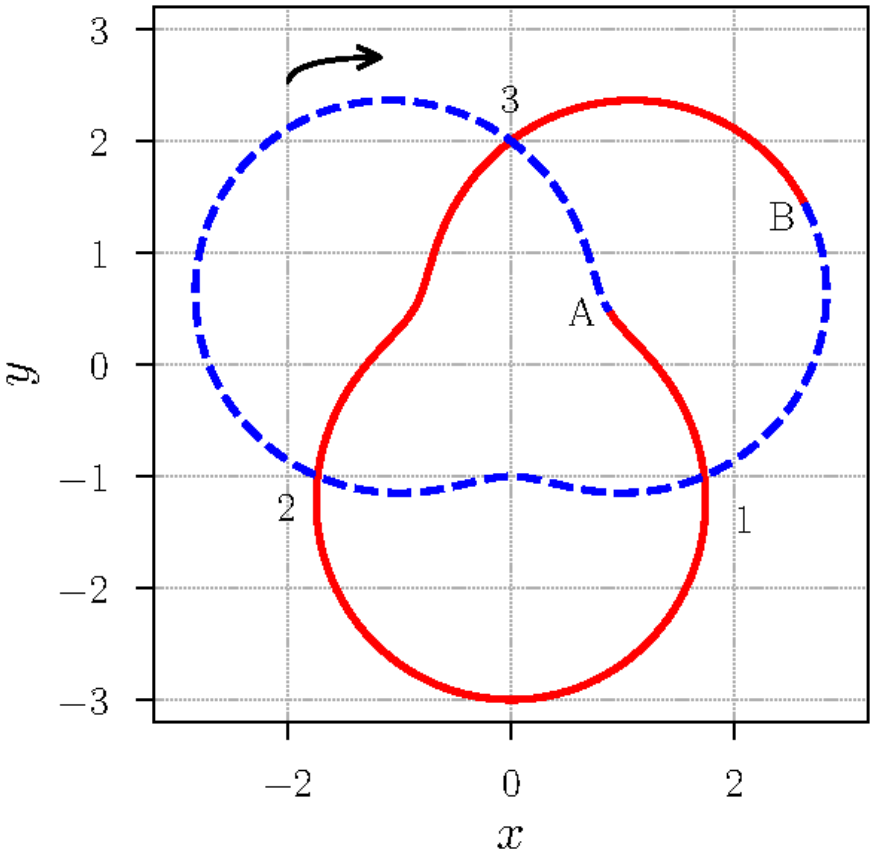


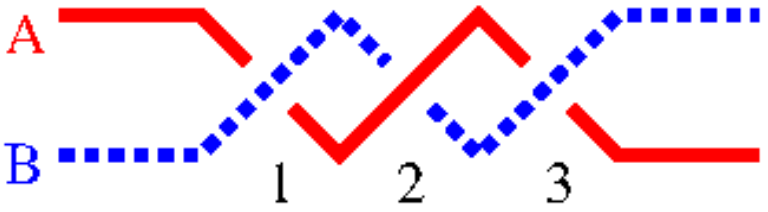
(Yeates 2011)

Use Python package Topoly to find polynomials.

(Dabrowski-Tumanski et al. (2020))

Knots and Links as Braids

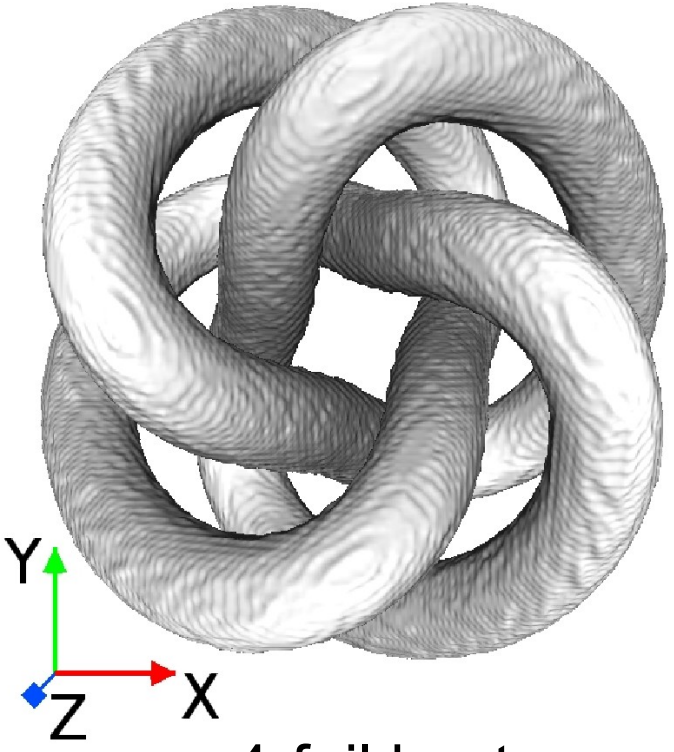




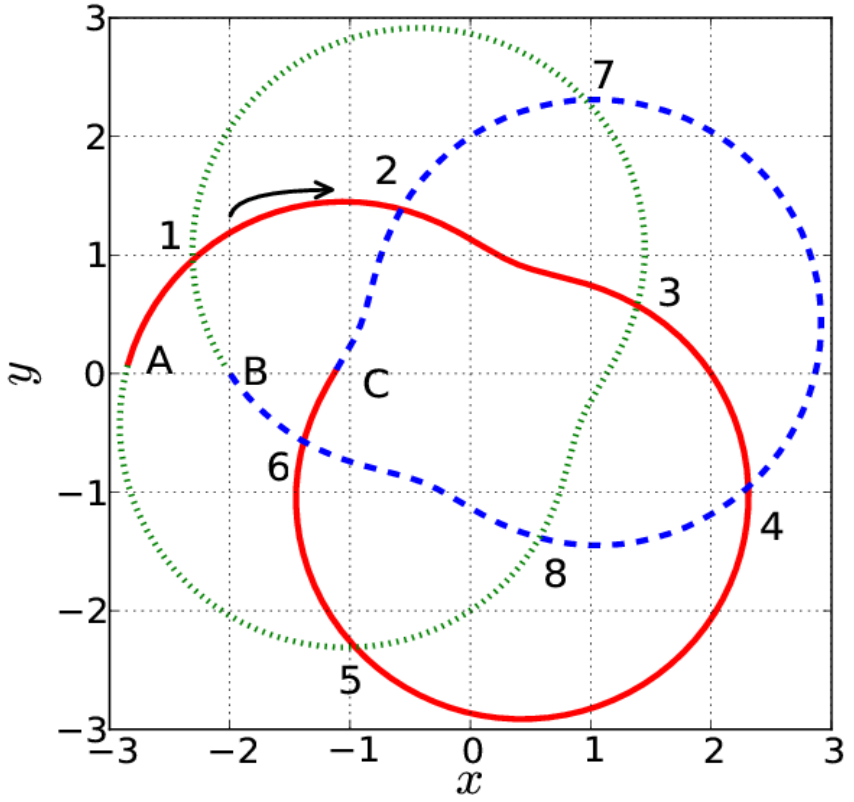
Periodic boundaries.

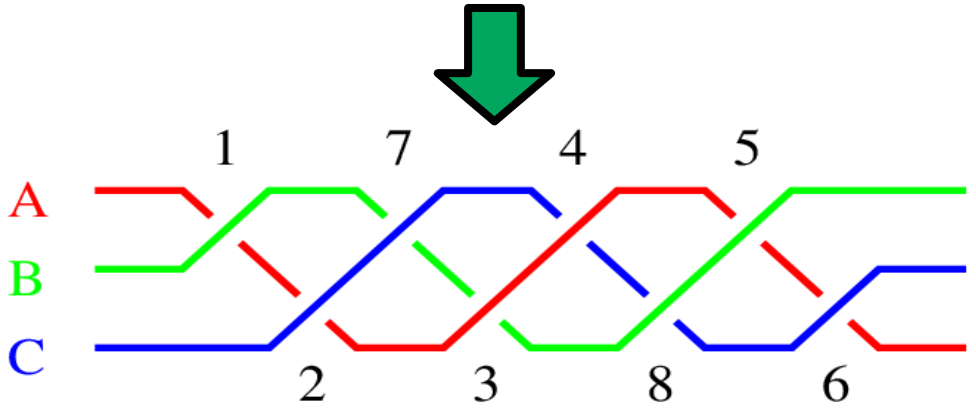
Knots and Links as Braids

need $B_z > 0$ ⇨ braid representation of knots and links



4-foil knot

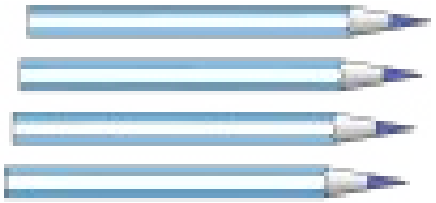




MHD Simulations

- initial condition: braids
- isothermal compressible gas
- viscous medium
- periodic in z

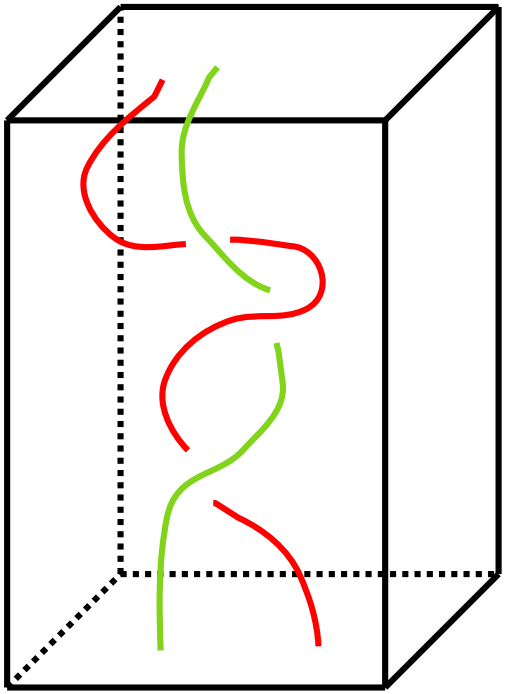
Pencil Code



$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A}$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

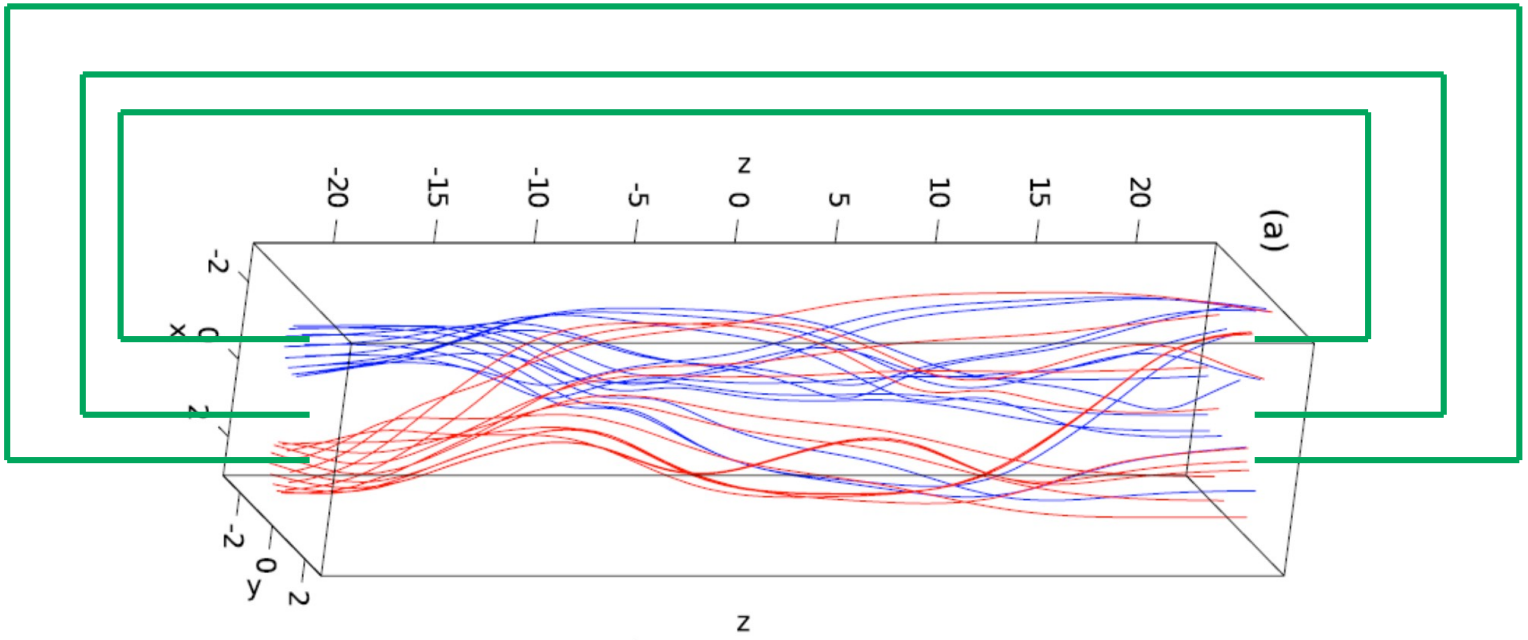
$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$



$B_z > 0$

Link Spectrum

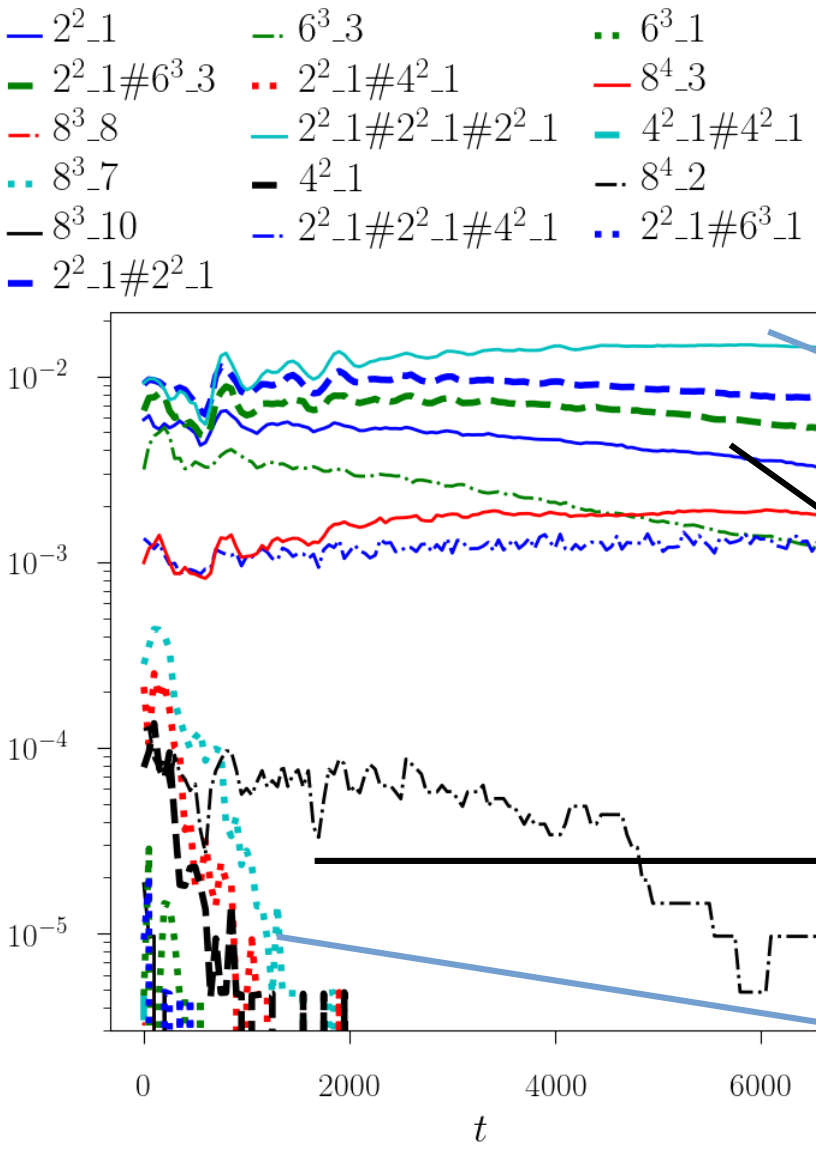
Pick a few random field lines and determine the link type.

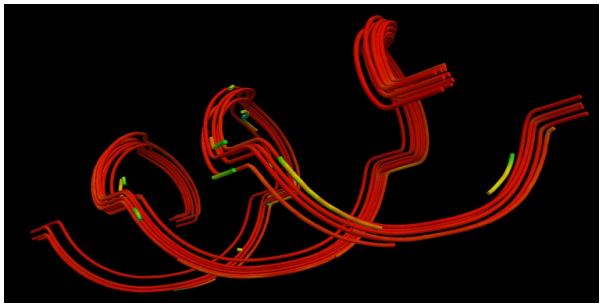


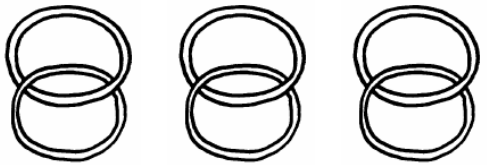
Repeat ca. 200,000 times for each snapshot.

Time dependent spectrum of links.

Trefoil Knot



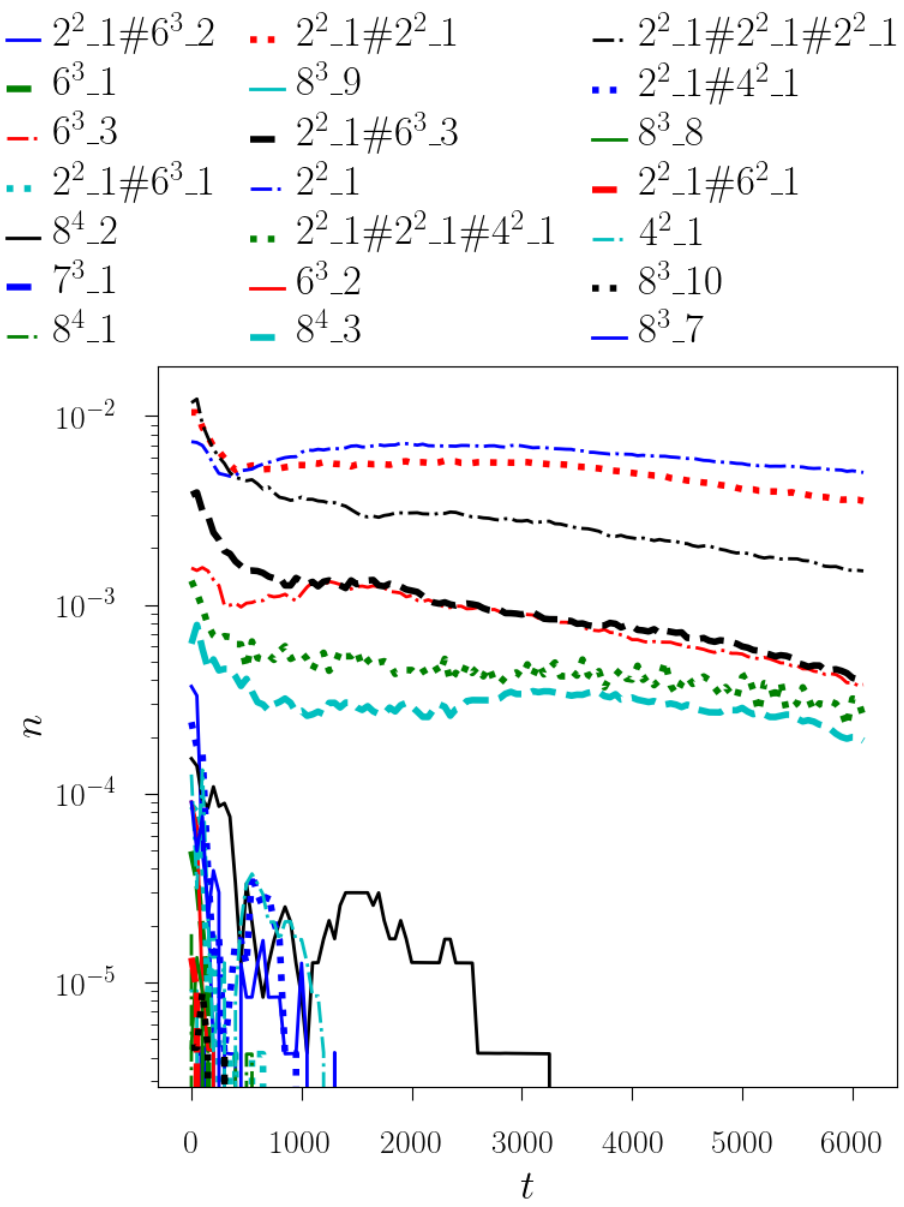


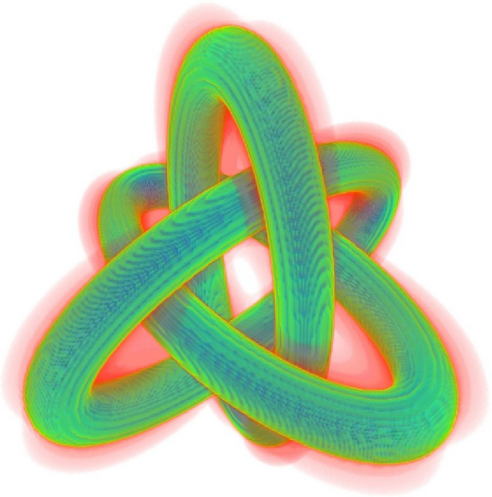


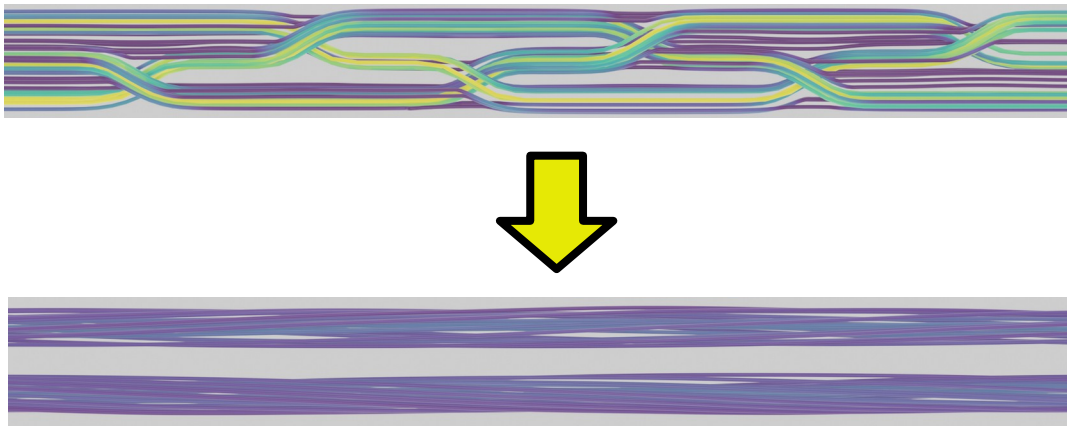
generally simple

generally complex

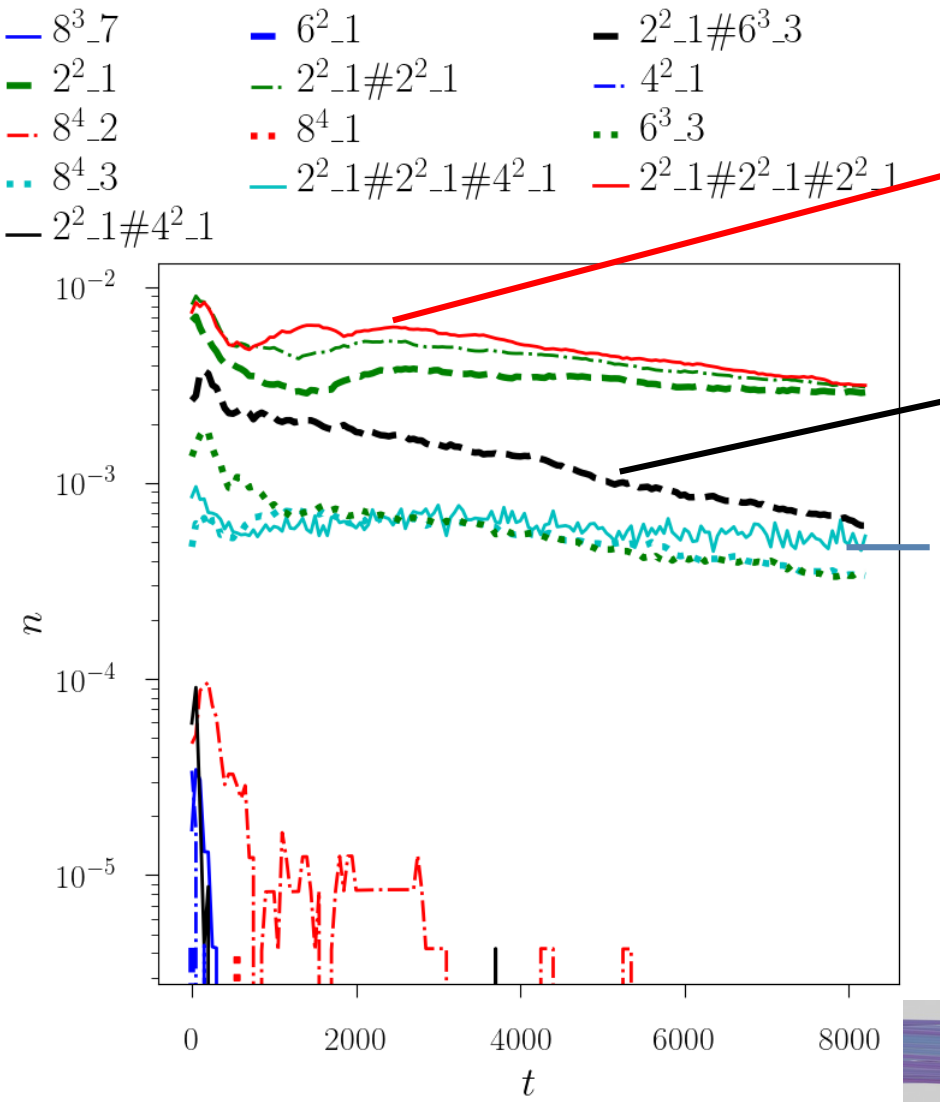
Borromean Rings

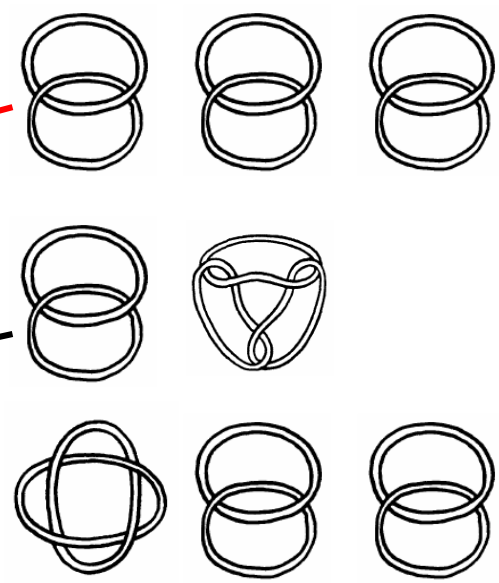


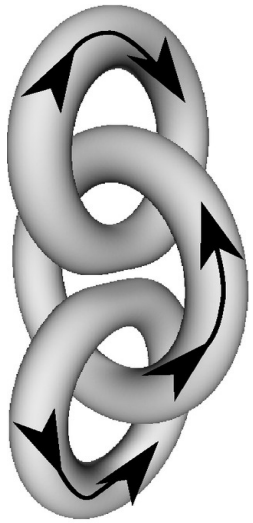


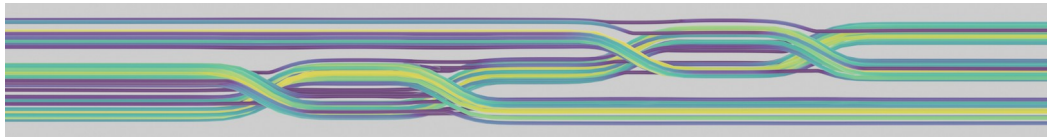


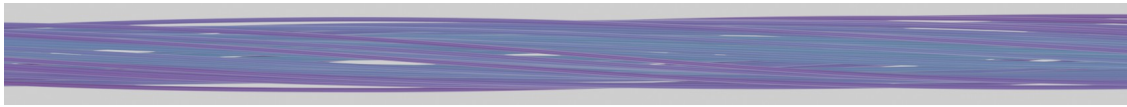
Helical 3 Rings



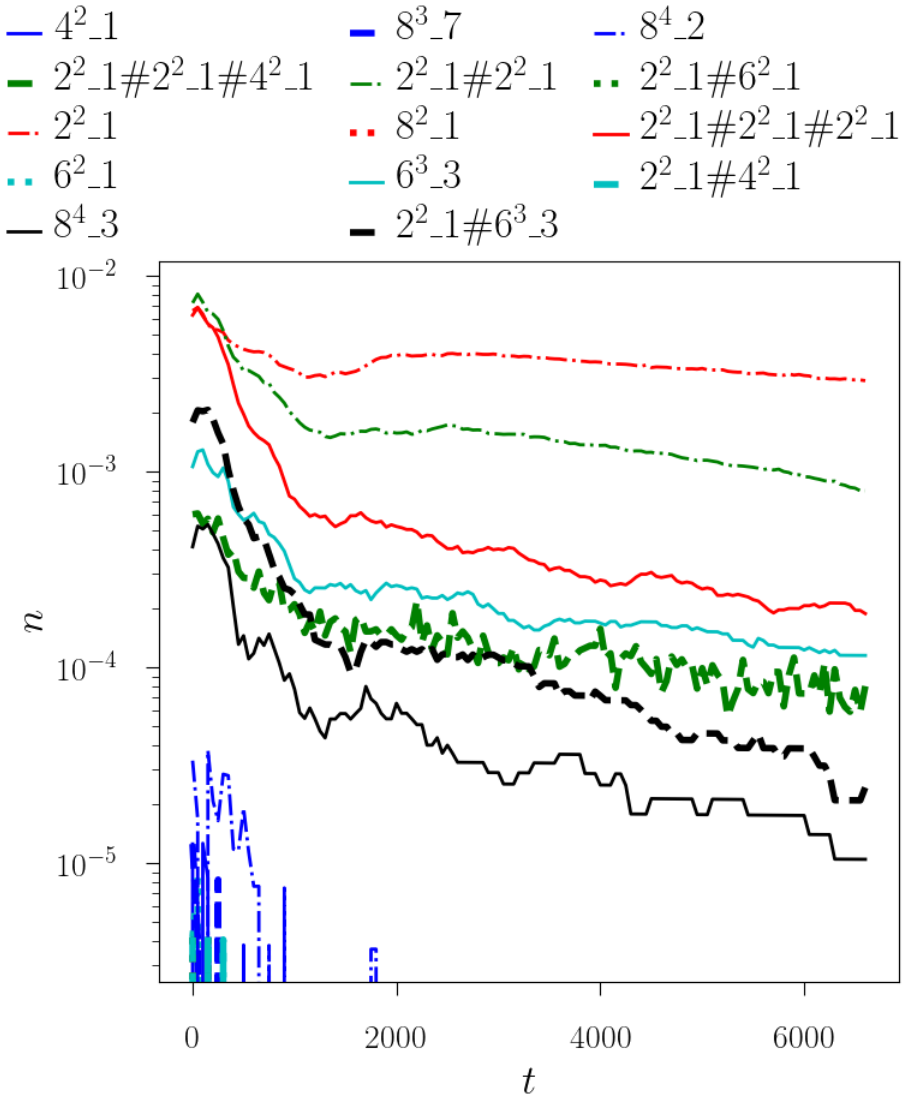


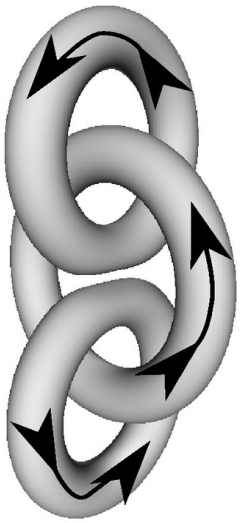


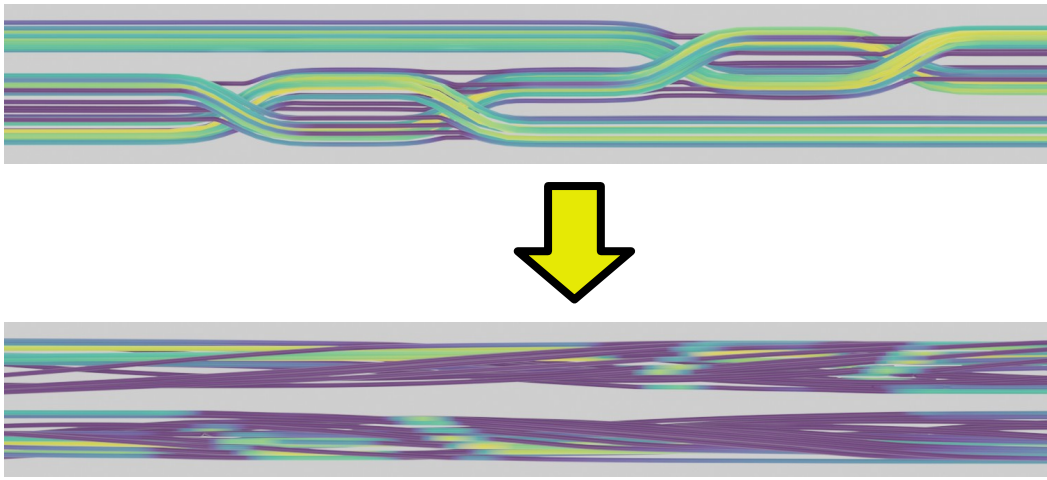




Non-Helical 3 Rings







Conclusions

- Knot polynomials for braids (coronal magnetic loops).
- Reconnection leads to simplification.
- Simple knots/links preserve more easily.

- Single number from link distribution?
- Solar atmosphere: potential field closure?
- Sudden changes in spectrum related to violent solar events?

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