Magnetic helicity: topological interpretation, relaxation and transport

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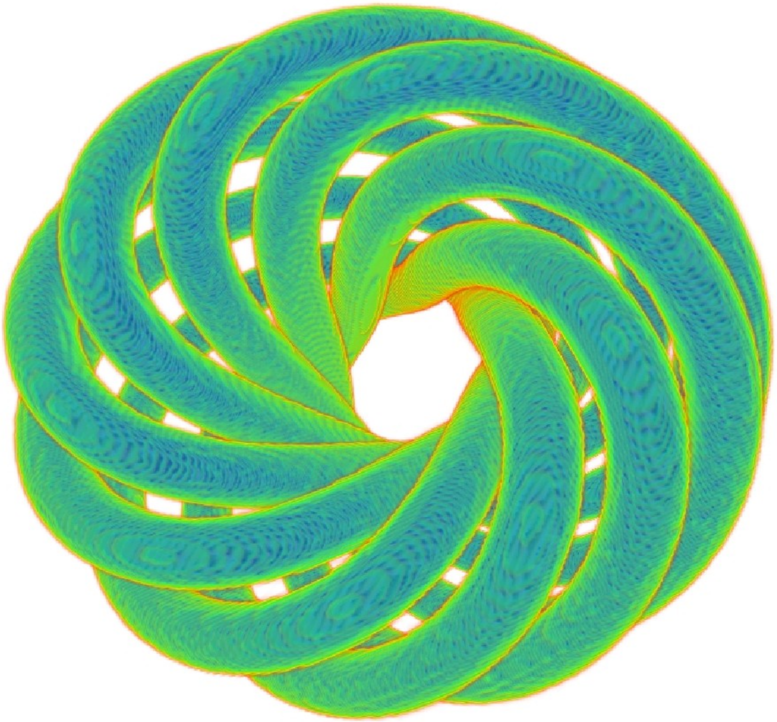


Magnetic helicity fluxes

Aim: Study the role of magnetic helicity in dynamical α quenching.

Method: 1d mean-field dynamo with helical forcing

Mean-field decomposition: $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$

Induction equation: $\partial_t \overline{\mathbf{B}} = \eta \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}})$

Electromotive force: $\overline{\boldsymbol{\mathcal{E}}} = \overline{\mathbf{u} \times \mathbf{b}}$

$$\overline{\boldsymbol{\mathcal{E}}} \underset{\alpha}{=} \alpha \overline{\mathbf{B}} - \eta_{\mathrm{t}} \boldsymbol{\nabla} \times \overline{\mathbf{B}}$$

α effect: $\alpha = \alpha_{\mathrm{K}} + \alpha_{\mathrm{M}}$

$$\alpha_{\mathrm{K}} = -\tau \overline{\boldsymbol{\omega} \cdot \mathbf{u}}/3$$

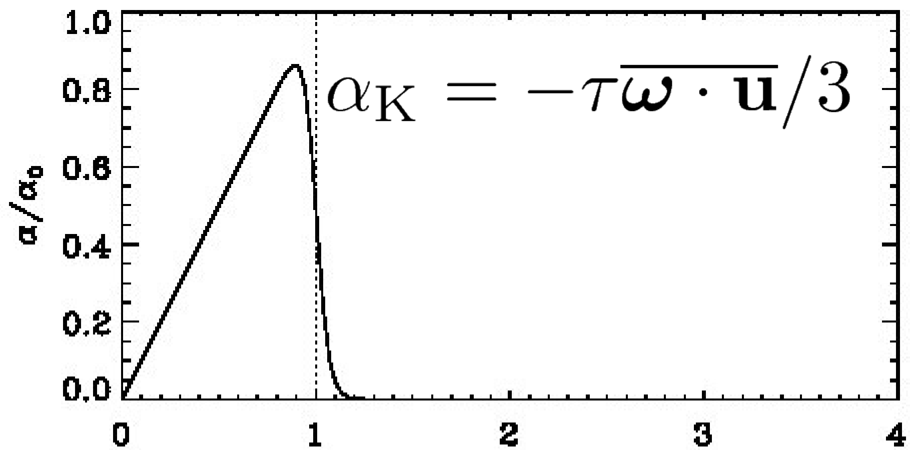
$$\alpha_{\mathrm{M}} = \tau \overline{\mathbf{j} \cdot \mathbf{b}}/(3\overline{\rho}) = \tau \overline{\mathbf{a} \cdot \mathbf{b}}/(3\overline{\rho} k^2) = \overline{h}_{\mathrm{m}}$$

Magnetic helicity fluxes

$$\frac{\partial \alpha_{\rm M}}{\partial t} = -2\eta_{\rm t} k_{\rm f}^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm M}}{R_{\rm m}} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

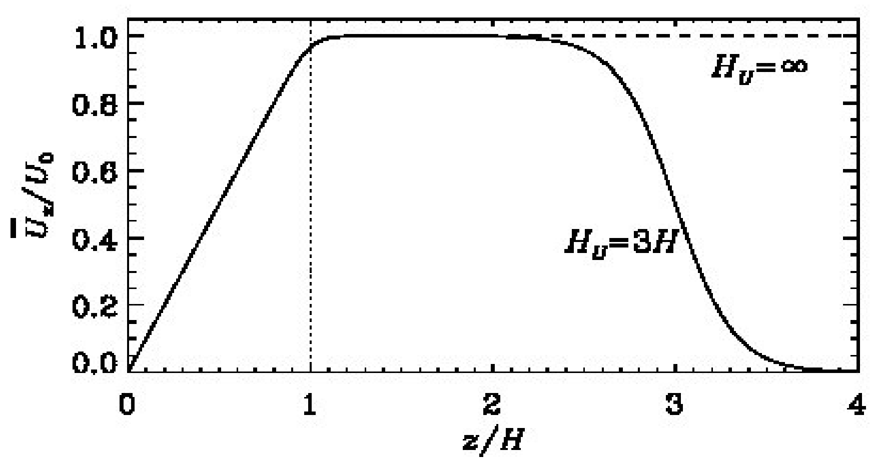
advective: $\alpha_{\rm M} \overline{\mathbf{U}}$

α diffusion $\kappa_\alpha \frac{\partial \alpha_{\rm M}}{\partial z}$



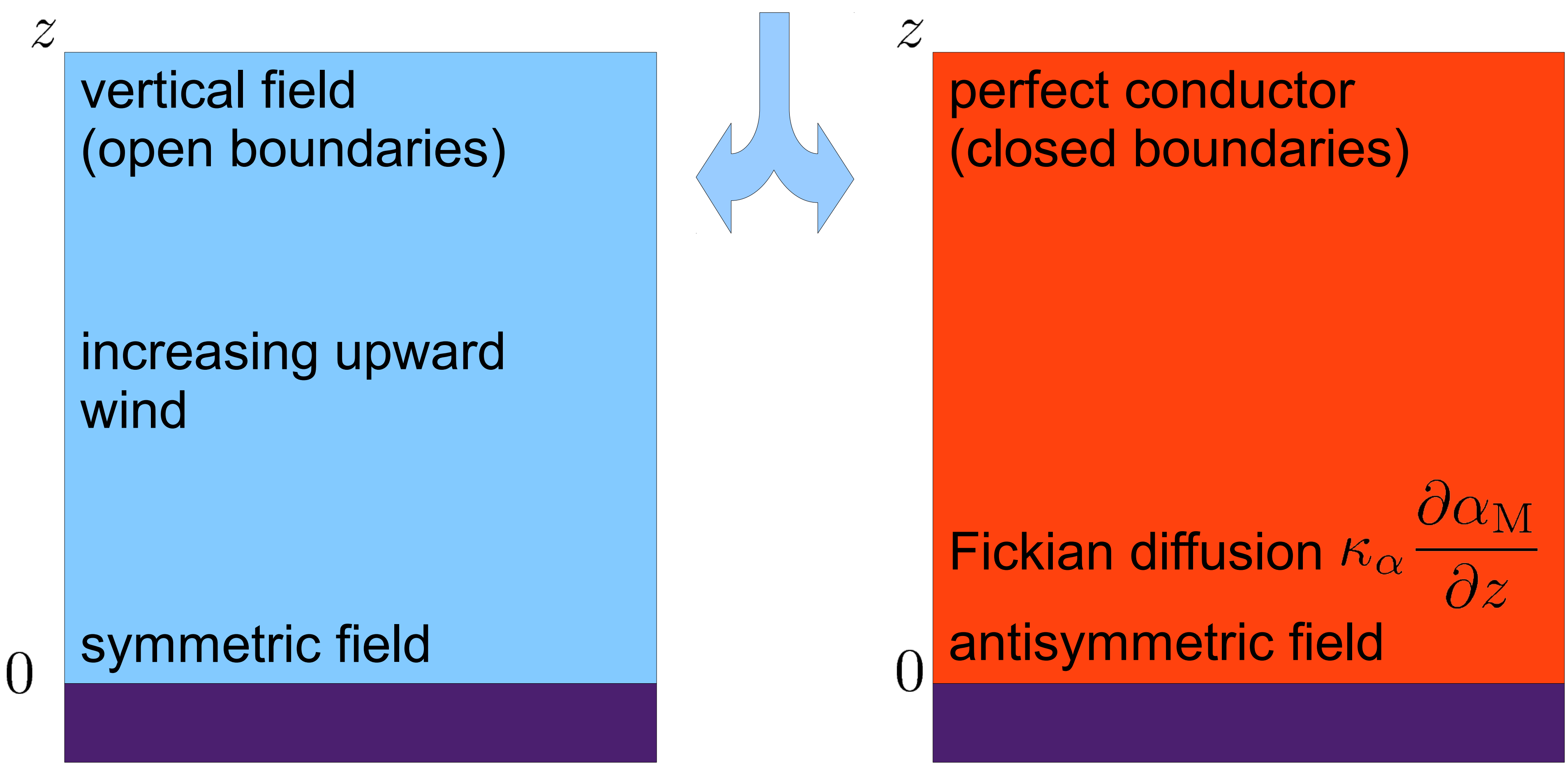
$$\frac{\partial \overline{h}_{\rm m}}{\partial t} = 2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \nabla \cdot \overline{\mathbf{F}}_{\rm m}$$

$$\frac{\partial \overline{h}_{\rm f}}{\partial t} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{j} \cdot \mathbf{b}} - \nabla \cdot \overline{\mathbf{F}}_{\rm f}$$



Magnetic helicity fluxes

Solve equations for one hemisphere.
Impose (anti)symmetric field at the equator.



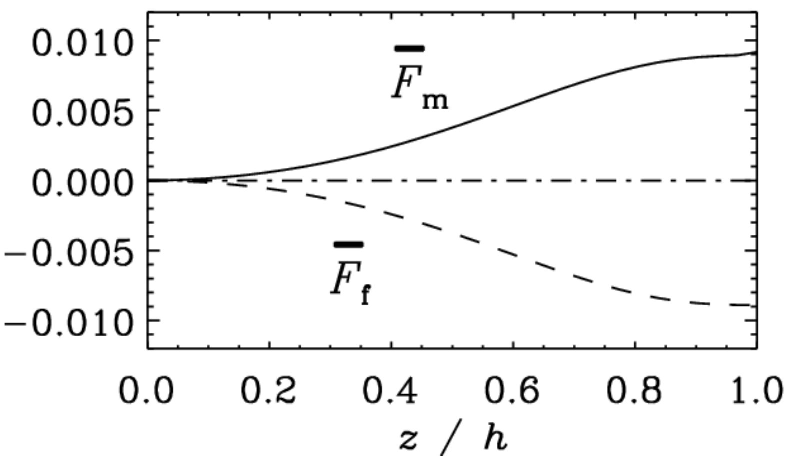
$$\mathrm{Re}_M = \frac{U_{\mathrm{rms}} L}{\eta}$$

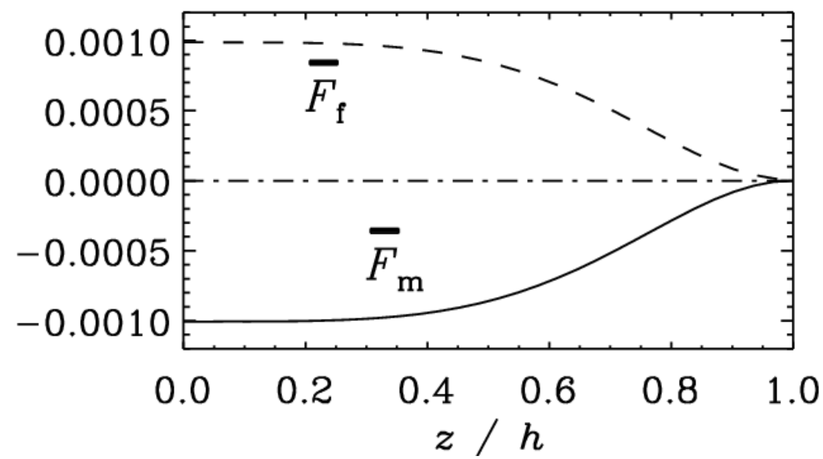
Magnetic helicity fluxes

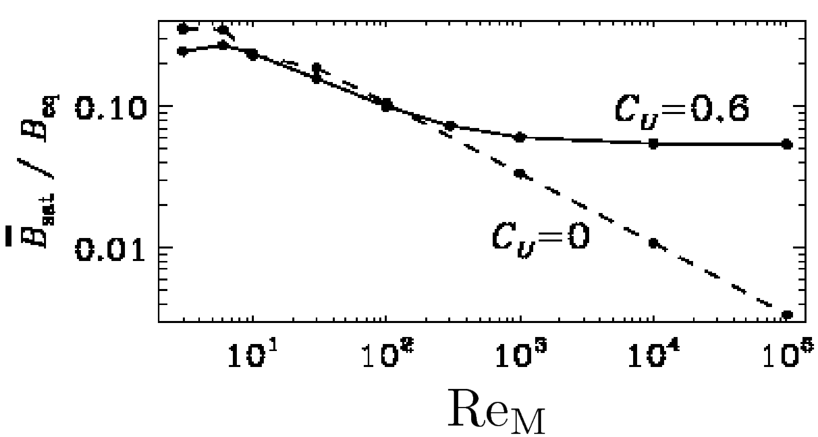
open boundary
symmetric
wind

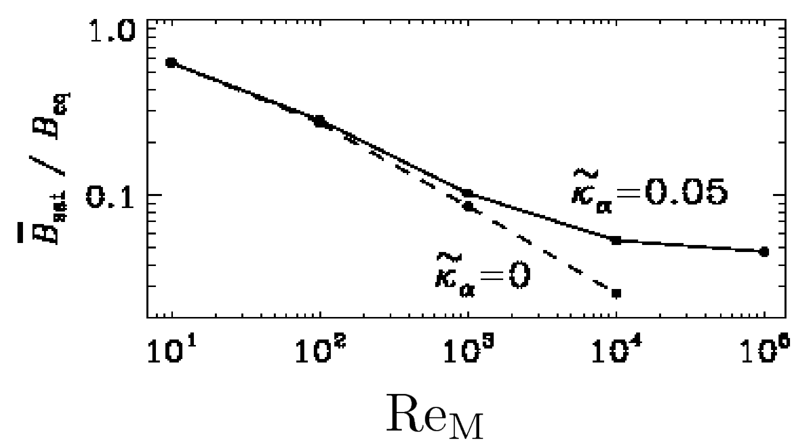
vs.

closed boundary
antisymmetric
κ_α





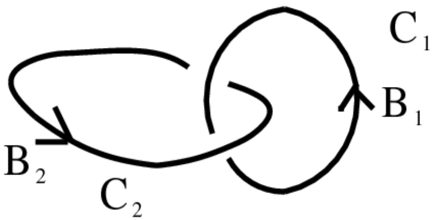




Magnetic helicity

$$H_{\mathrm{M}} = \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V = 2n\phi_1\phi_2$$

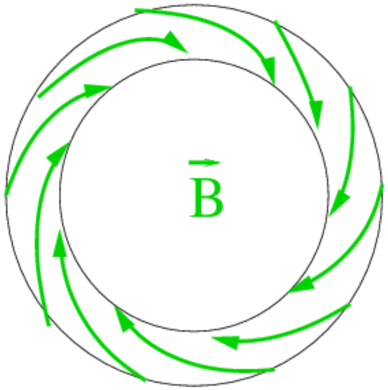
$$\phi_i = \int_{S_i} \mathbf{B} \cdot \mathrm{d}\mathbf{S}$$



Realizability condition:

$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.



twisted field

magnetic helicity conservation

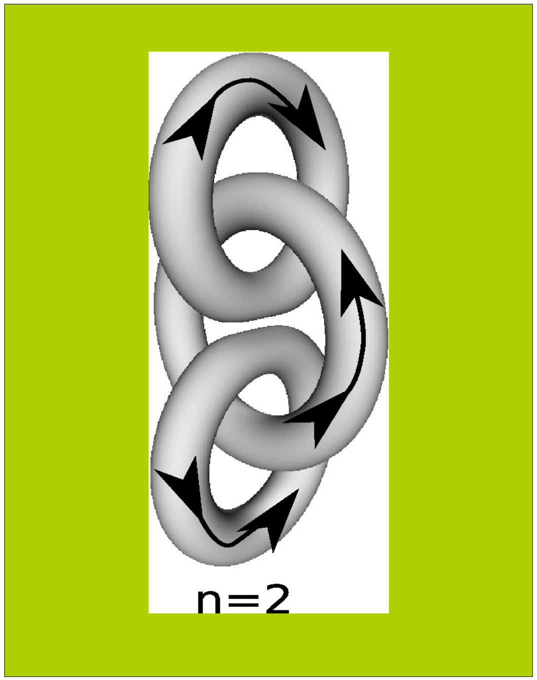
$$\mathrm{Re}_{\mathrm{M}} \to \infty$$

$$\frac{\mathrm{d}H_{\mathrm{M}}}{\mathrm{d}t} = 0$$

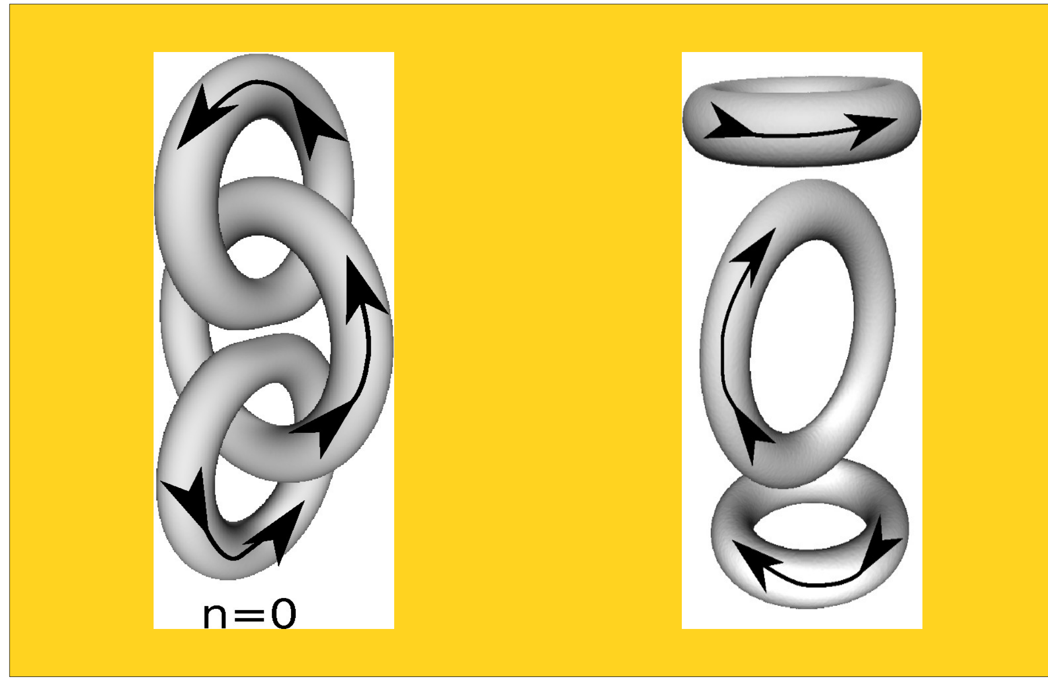

trefoil knot

Interlocked flux rings

$H_\mathrm{M} \neq 0$



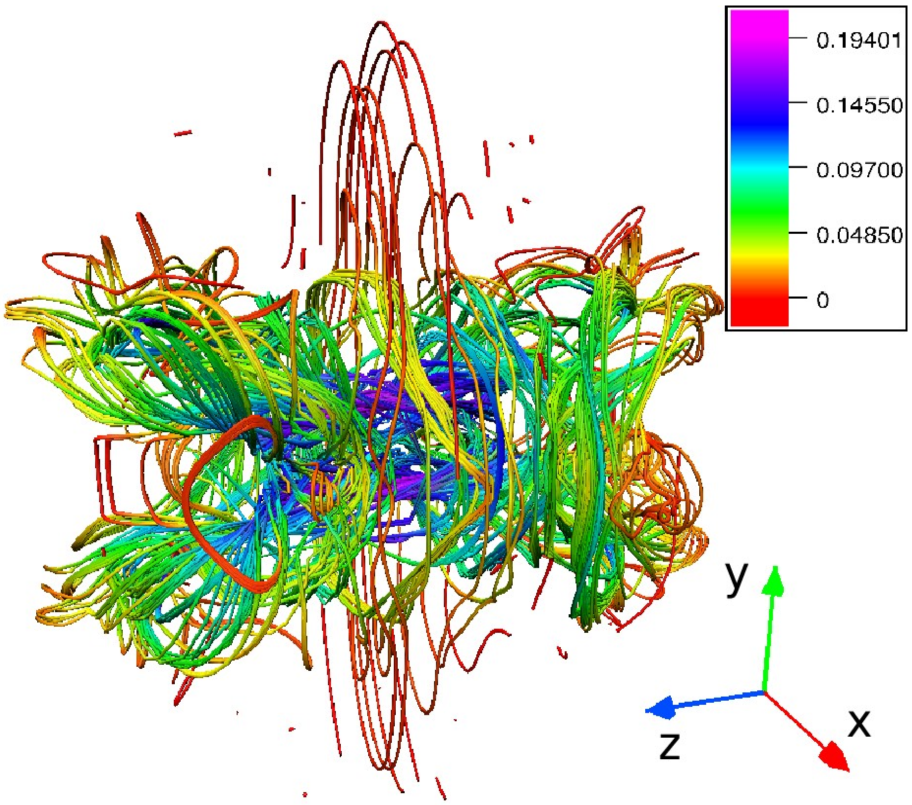
$H_\mathrm{M} = 0$



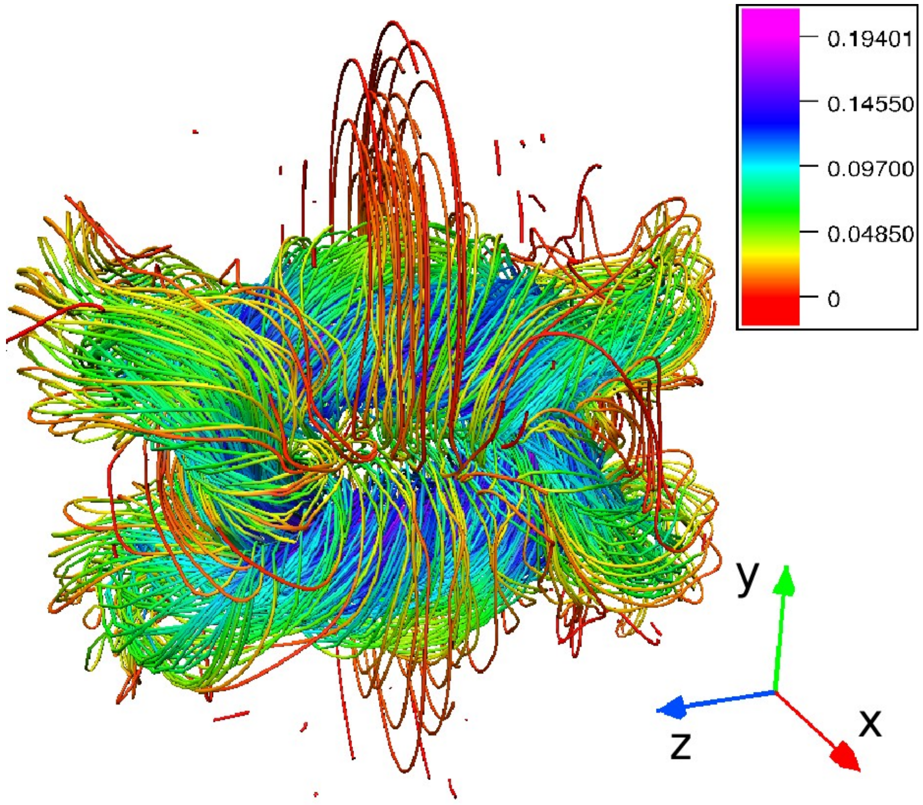
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

Interlocked flux rings

$\tau = 4$

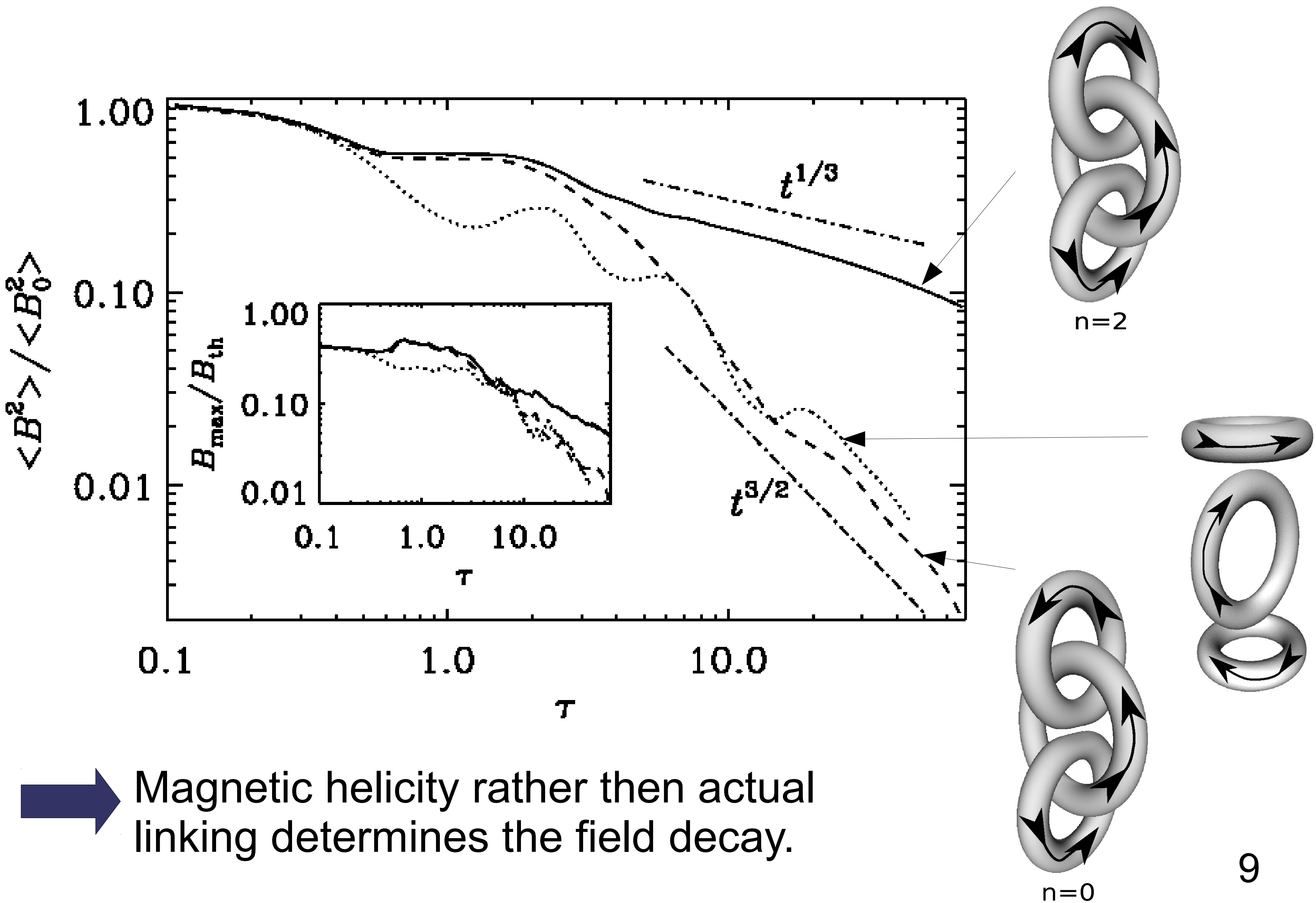


$H_{\mathrm{M}} = 0$



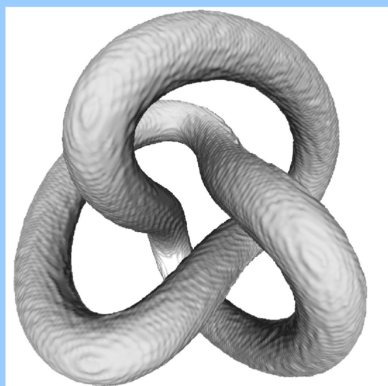
$H_{\mathrm{M}} \neq 0$

Interlocked flux rings

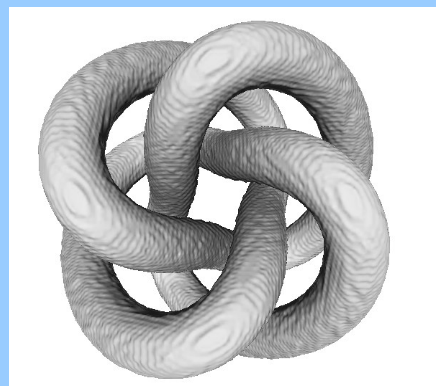


➡ Magnetic helicity rather then actual linking determines the field decay.

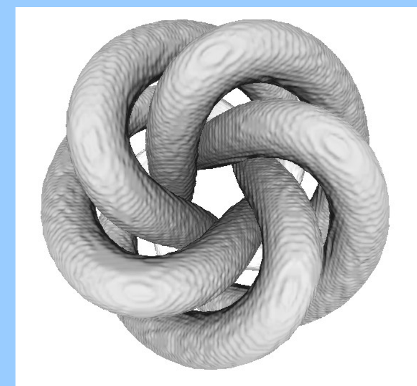
N-foil knots



3-foil



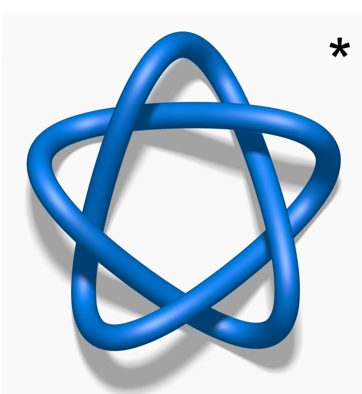
4-foil



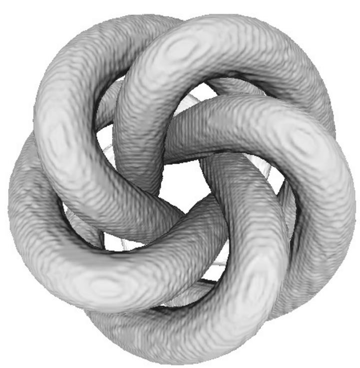
5-foil

6-foil

7-foil



$\neq$

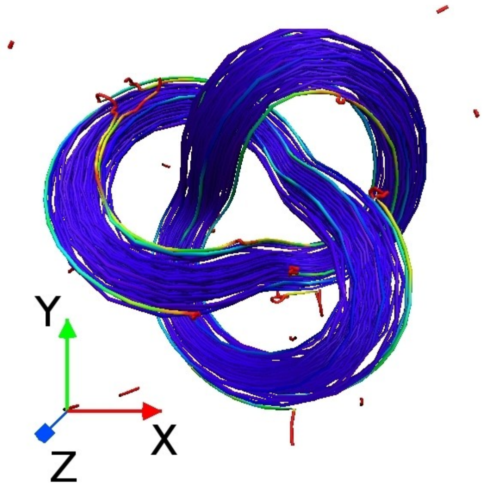


$$x(s) = \begin{pmatrix} (C + \sin s n_{\mathrm{f}}) \sin[s(n_{\mathrm{f}} - 1)] \\ (C + \sin s n_{\mathrm{f}}) \cos[s(n_{\mathrm{f}} - 1)] \\ D \cos s n_{\mathrm{f}} \end{pmatrix}$$

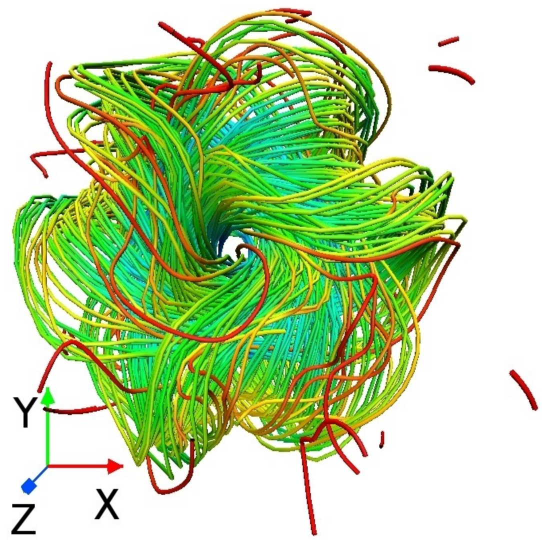
cinquefoil knot

* from Wikipedia, author: Jim.belk

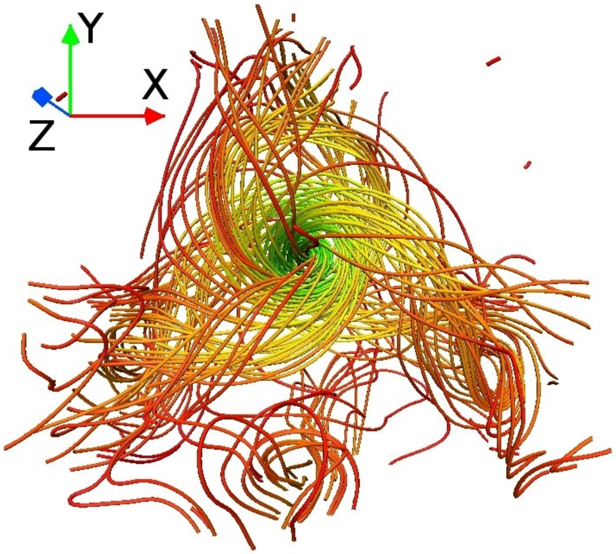
N-foil knots



$t = 0$



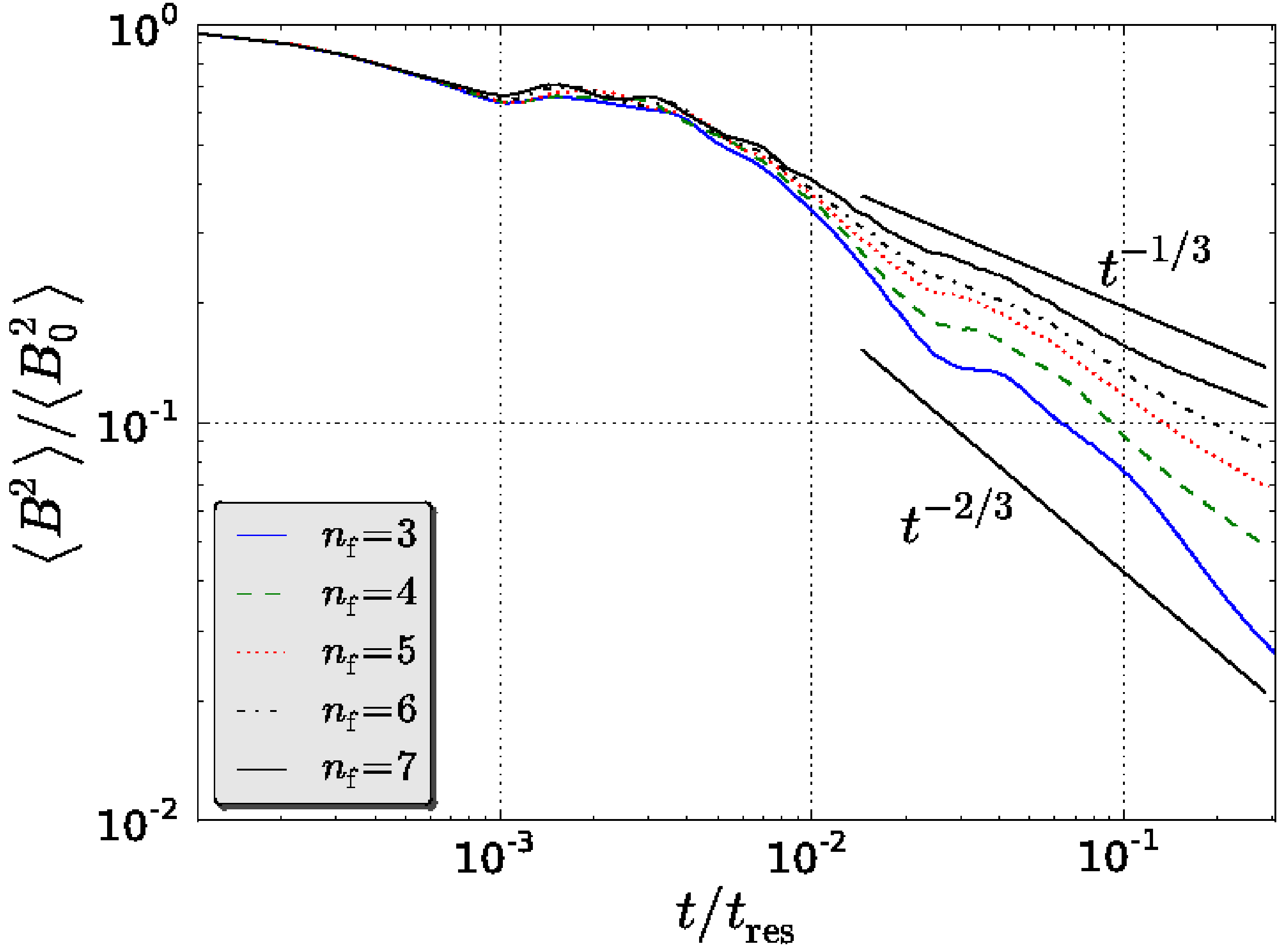
$t = 6$



$t = 39$

- Magnetic helicity is approximately conserved.
- Self-linking is transformed into twisting after reconnection.

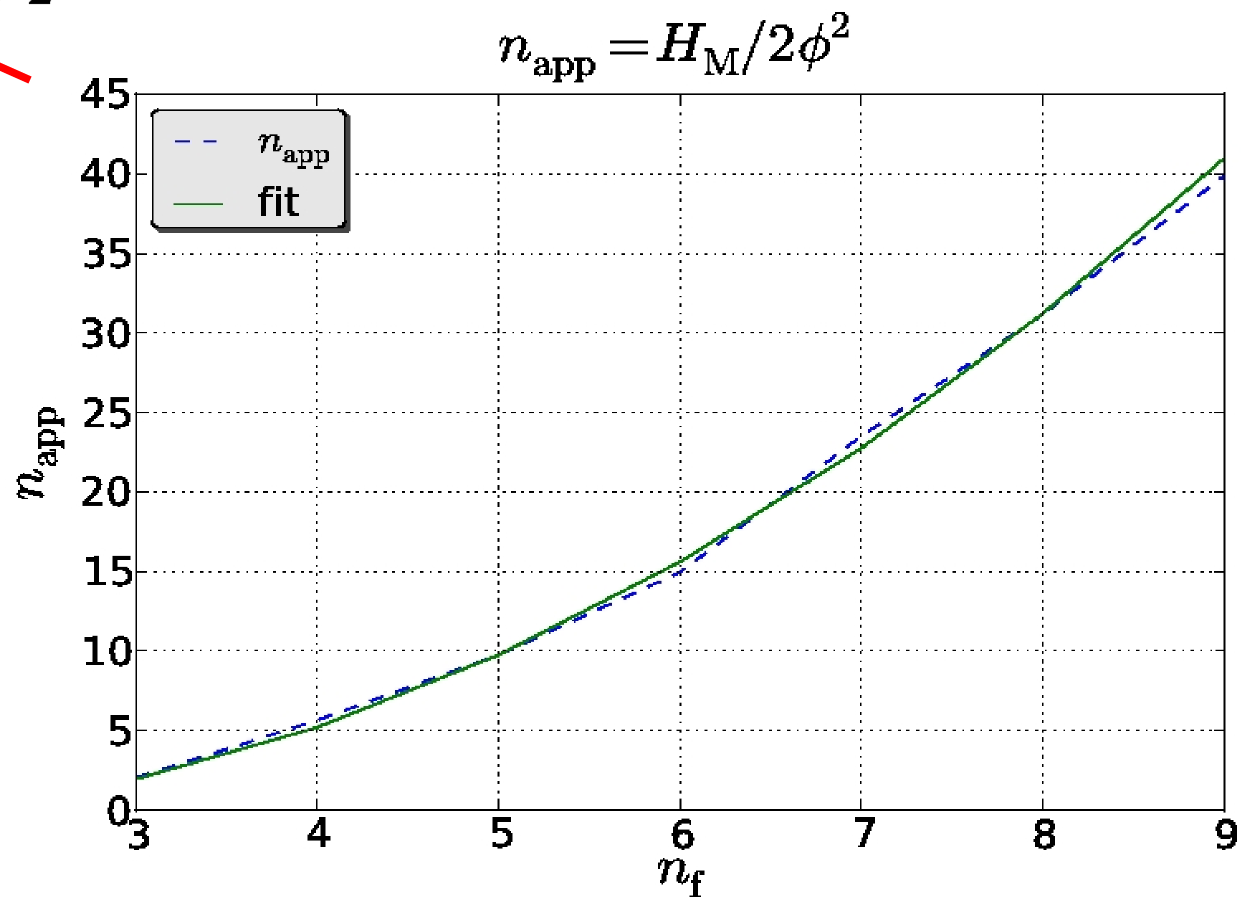
N-foil knots



Slower decay for higher $n_{\rm f}$.

N-foil knots

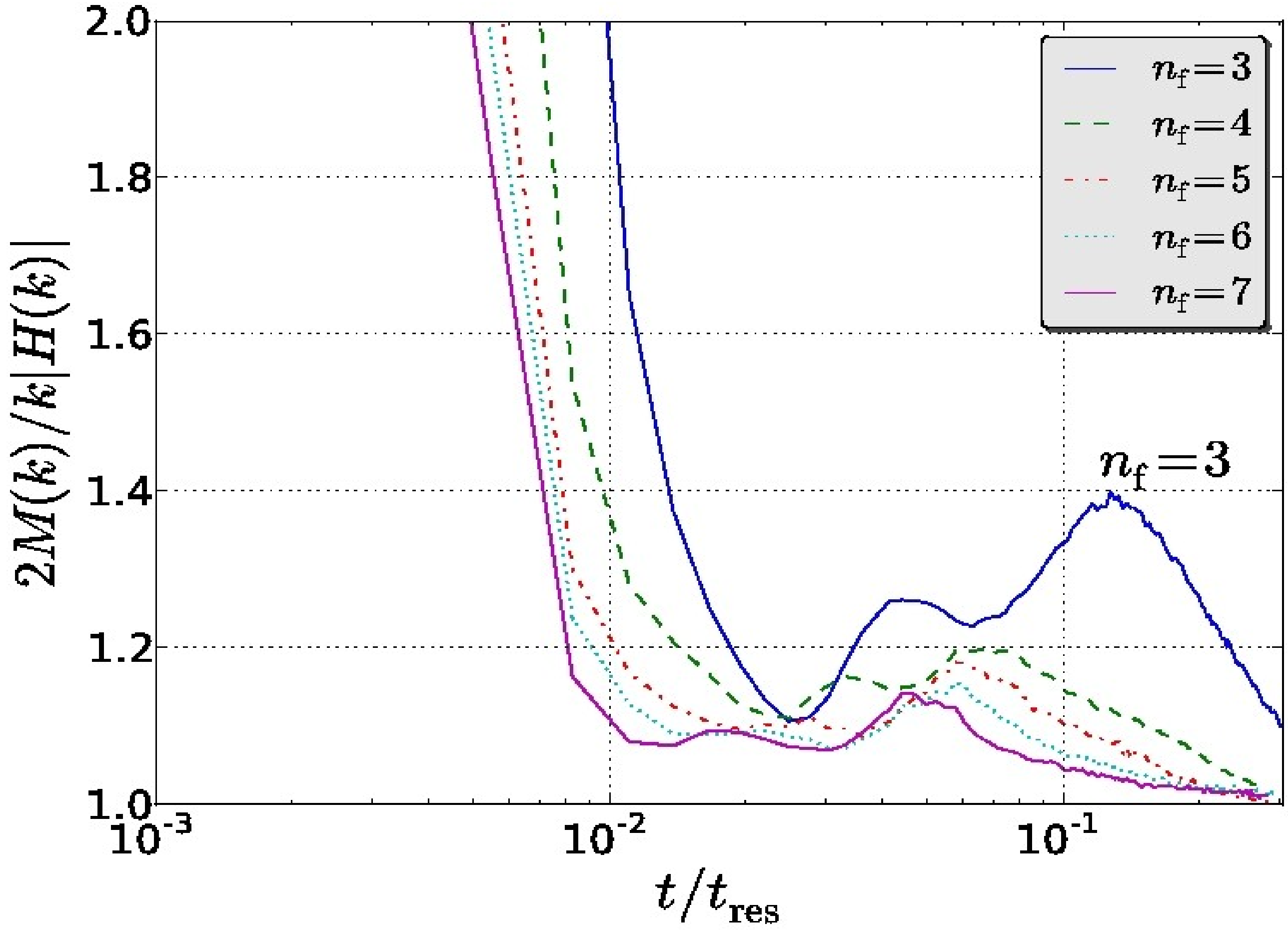
~~$H_{\mathrm{M}} = 2n\phi_1\phi_2$~~



$$H_{\mathrm{M}} = (n_{\mathrm{f}} - 2)n_{\mathrm{f}}\phi^2/2$$

N-foil knots

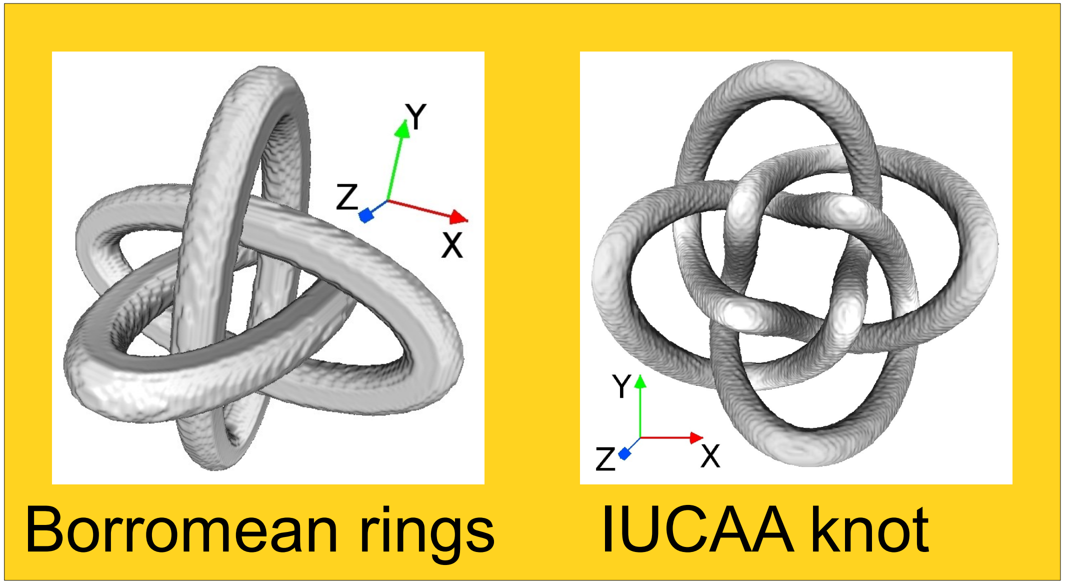
$$2M(k)/|H(k)|k$$



$k = 2$

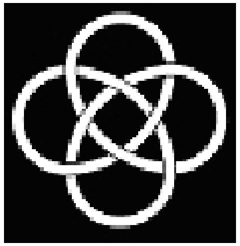
Realizability condition more important for high n_{f}.

IUCAA knot and Borromean rings



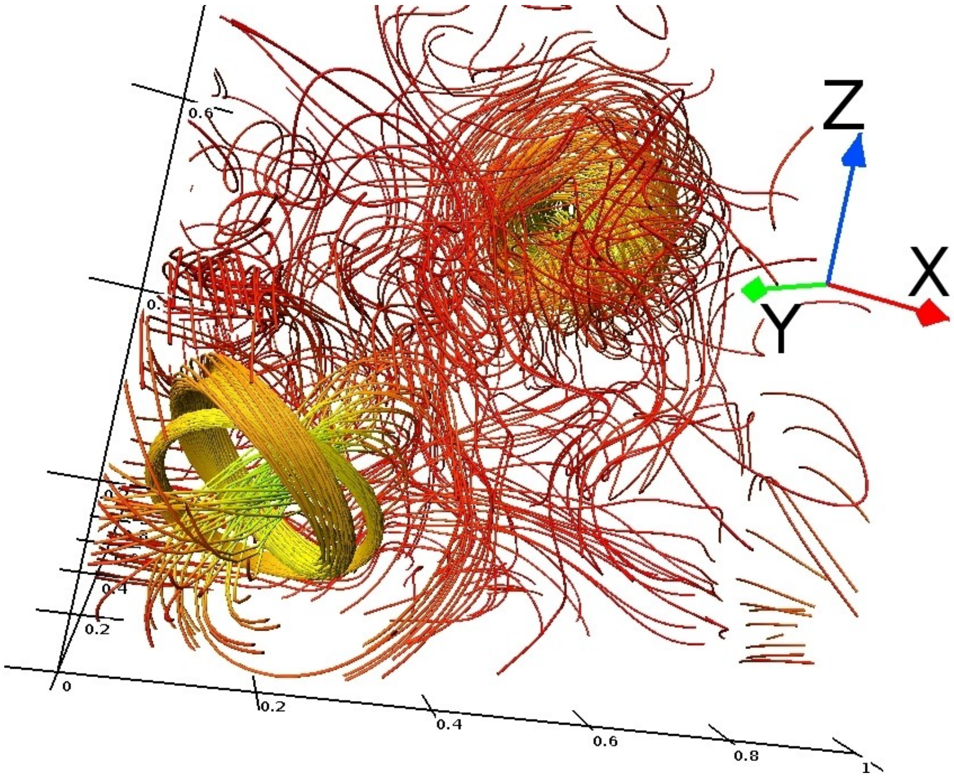
$$H_{\mathrm{M}} = 0$$

- Is magnetic helicity sufficient?
- Higher order invariants?

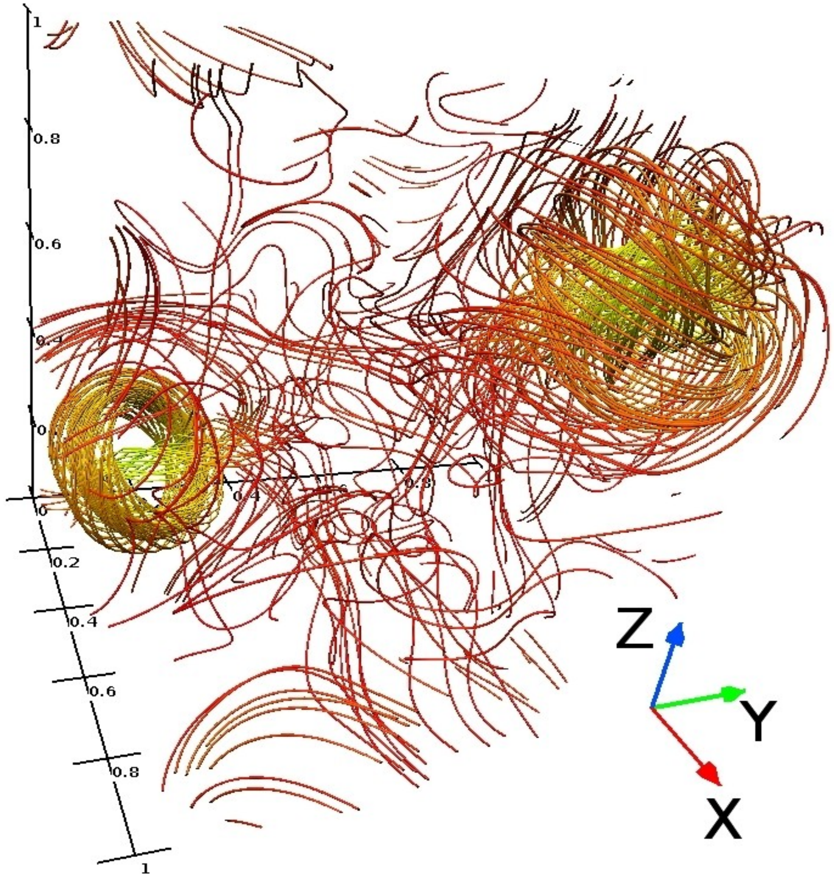


IUCAA = The Inter-University Centre for Astronomy and Astrophysics, Pune, India

Reconnection characteristics



$t = 70$

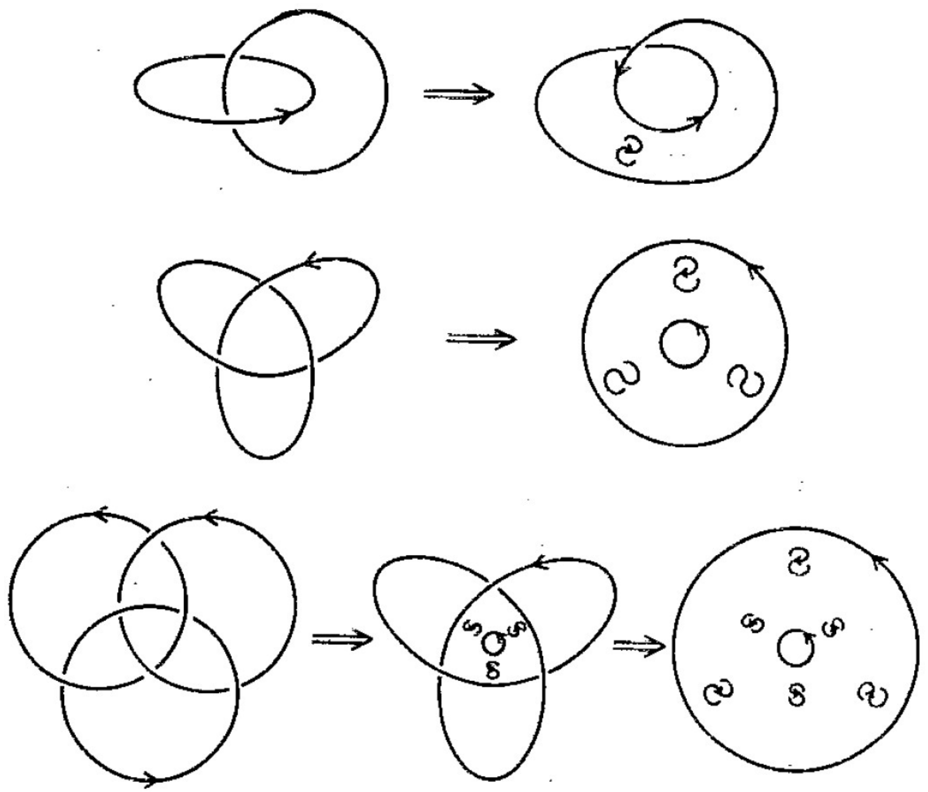


$t = 78$

3 rings → Twisted ring + interlocked rings → 2 twisted rings

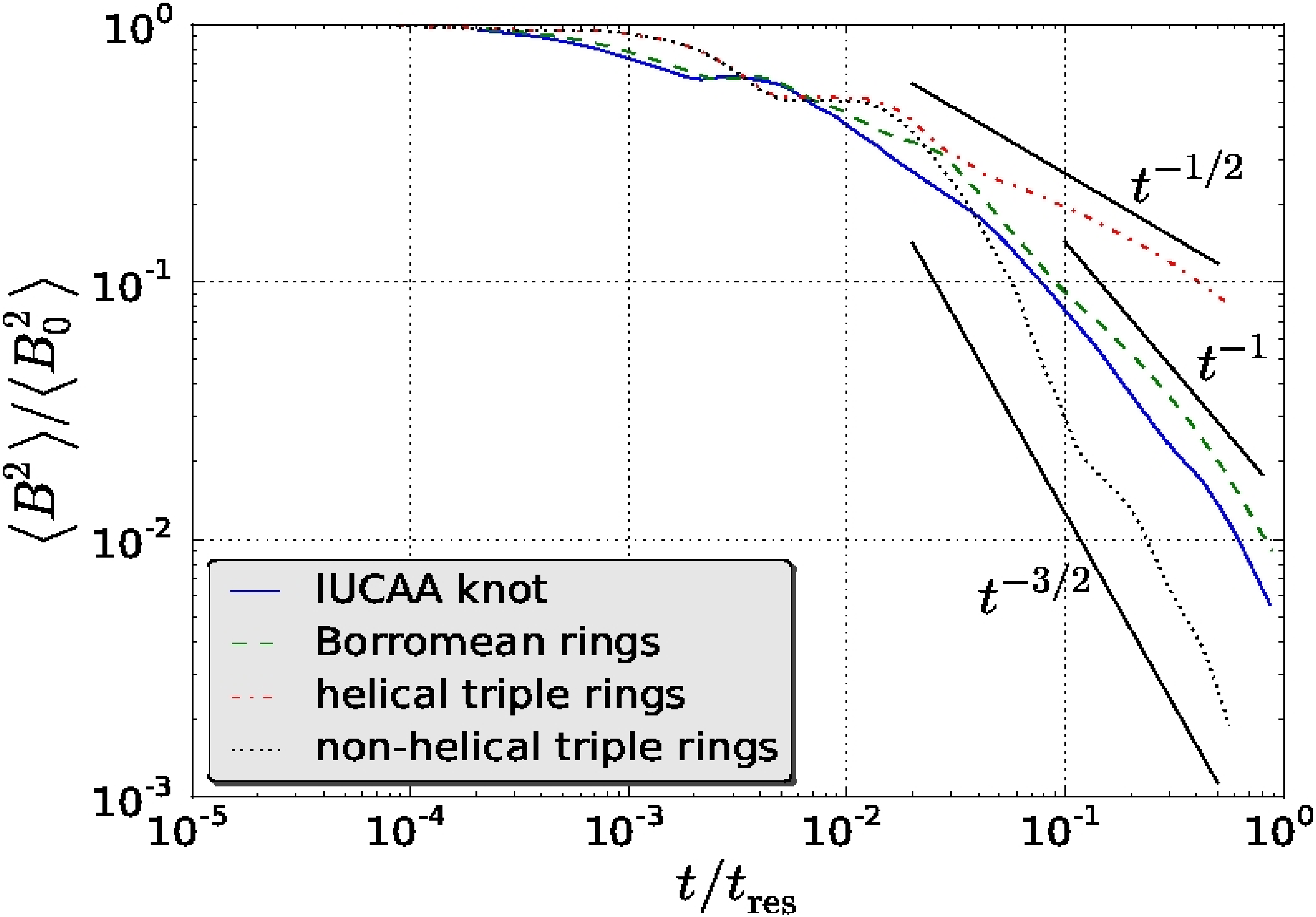
Reconnection characteristics

Conversion of linking into twisting



Ruzmaikin and Akhmetiev (1994)

Magnetic energy decay



Conclusions

- Advective magnetic helicity n_{f} fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

- Topology *can* constrain field decay.
- Stronger packing for high n_{f} leads to different decay slopes.
- Higher order invariants?
- Isolated helical structures inhibit energy decay.
- Reconsider realizability condition.

References

Brandenburg et al. 2009

Axel Brandenburg, Simon Candelaresi and Piyali Chatterjee.
Small-scale magnetic helicity losses from a mean-field dynamo.
Mon. Not. Roy. Astron. Soc., 398:1414-1422, September 2009.

Candelaresi and Brandenburg 2011

Simon Candelaresi, and Axel Brandenburg.
Decay of helical and non-helical magnetic knots.
Phys. Rev. E, Phys. Rev. E, 84(1):016406, July 2011.

Del Sordo et al. 2010

Fabio Del Sordo, Simon Candelaresi, and Axel Brandenburg.
Magnetic-field decay of three interlocked flux rings with zero linking number.
Phys. Rev. E, 81:036401, March 2010.

Ruzmaikin and Akhmetiev 1994

A. Ruzmaikin and P. Akhmetiev.
Topological invariants of magnetic fields, and the effect of reconnections.
Phys. Plasmas, vol. 1, pp. 331–336, 1994.

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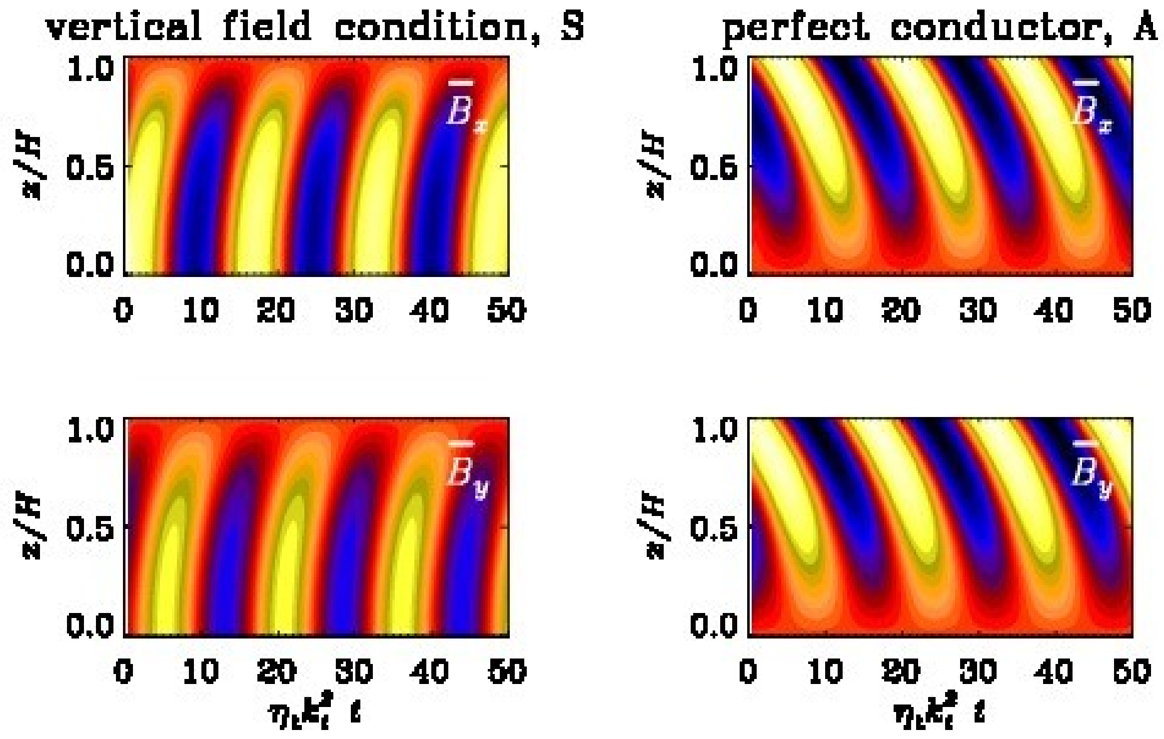
Magnetic helicity fluxes

Random helical forcing:

$$\mathbf{f}(\mathbf{x},t)=\frac{i\mathbf{k}\times(\mathbf{k}\times\mathbf{e})-\sigma|\mathbf{k}|(\mathbf{k}\times\mathbf{e})}{\sqrt{1+\sigma^2\mathbf{k}^2}\sqrt{1-(\mathbf{k}\cdot\mathbf{e})^2/\mathbf{k}^2}}e^{i(\mathbf{k}(t)\cdot\mathbf{x}+\phi(t))}$$



Simulations

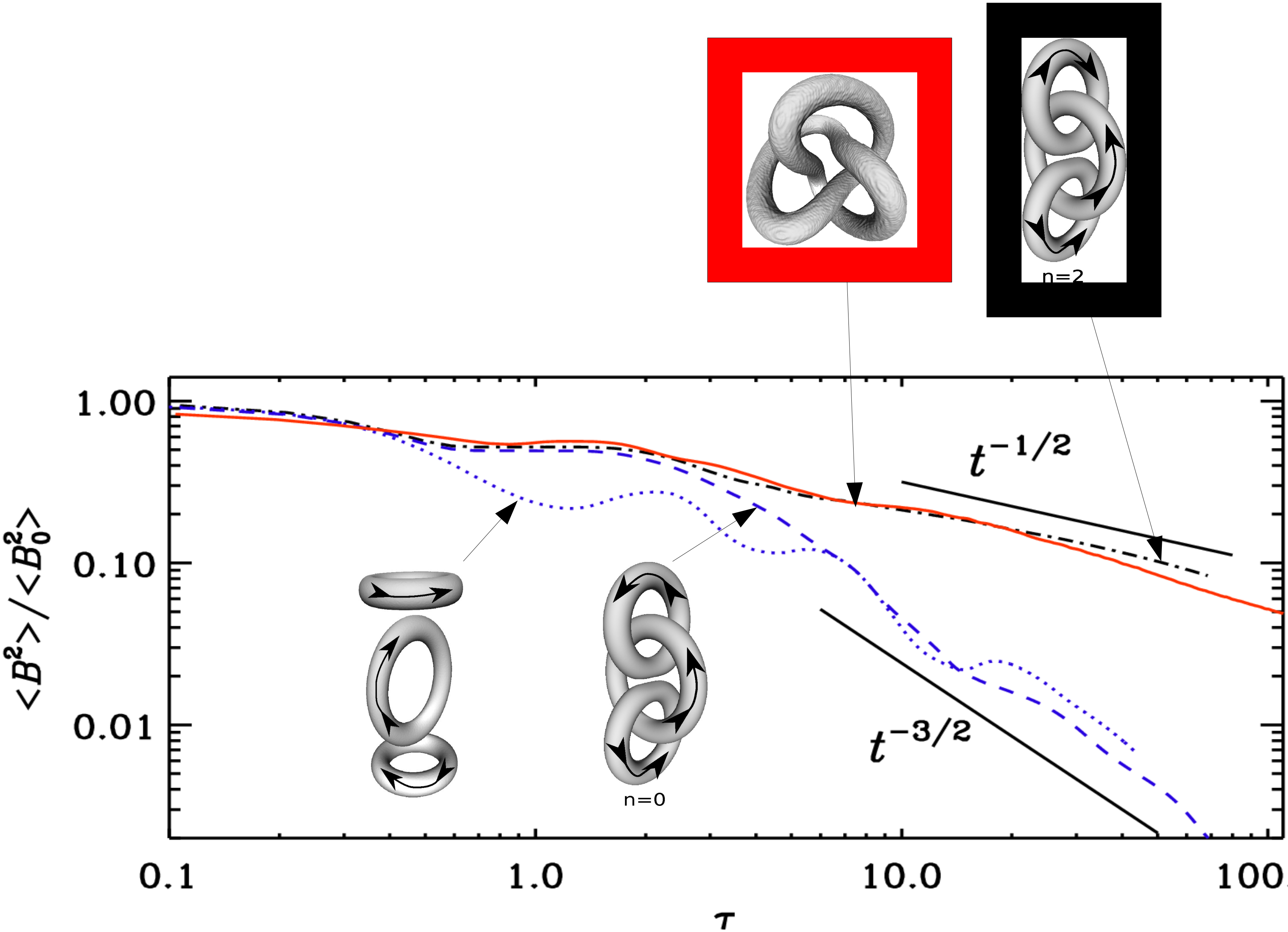
- 256^3 mesh point
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}$$

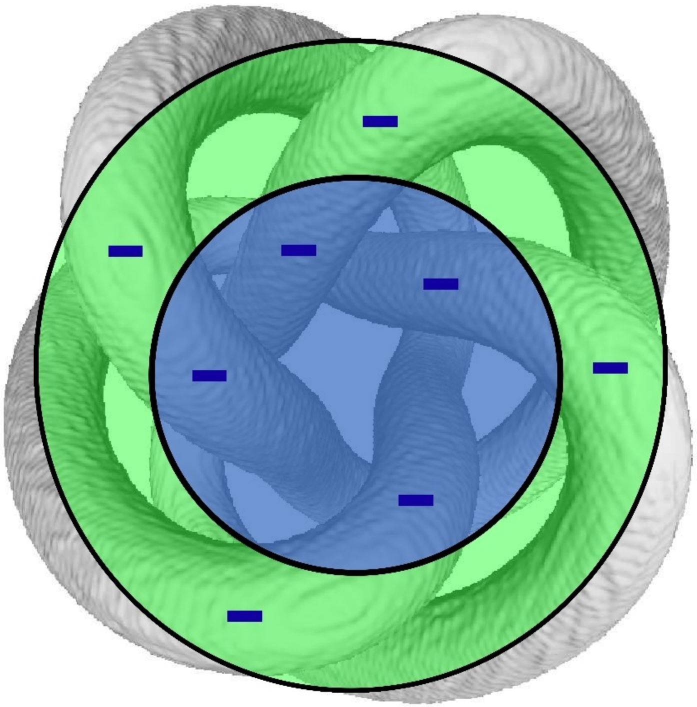
$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \mathbf{J} \times \mathbf{B}/\rho + \mathbf{F}_{\mathrm{visc}}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \mathbf{U}$$

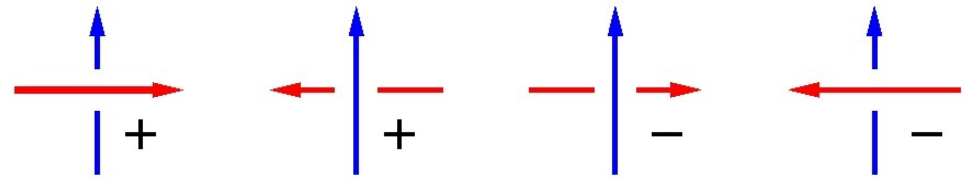
Magnetic energy decay



Linking number



Sign of the crossings
for the 4-foil knot



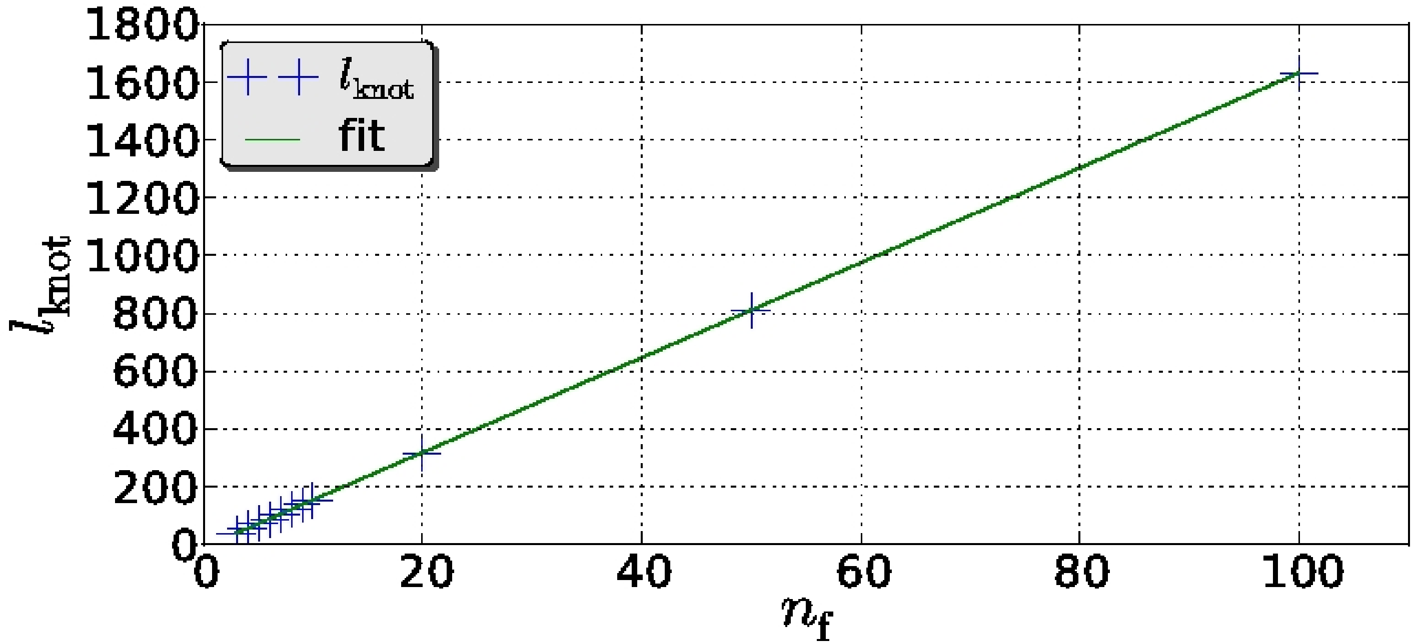
$$n_{\text{linking}} = (n_+ - n_-)/2$$

Number of crossings increases like n_{f}^2

$$H_{\text{M}} \propto n_{\text{linking}}$$

$$H_{\text{M}} \propto n_{\text{f}}^2$$

Helicity vs. energy



$$E_{\mathrm{M}} \propto l_{\mathrm{knot}} \propto n_{\mathrm{f}}$$

$$H_{\mathrm{M}} \propto n_{\mathrm{f}}^2$$

Knot is more strongly packed with increasing n_{f}.

Magnetic energy is closer to its lower limit for high n_{f}.