Magnetic helicity conservation and fluxes in astrophysical dynamos

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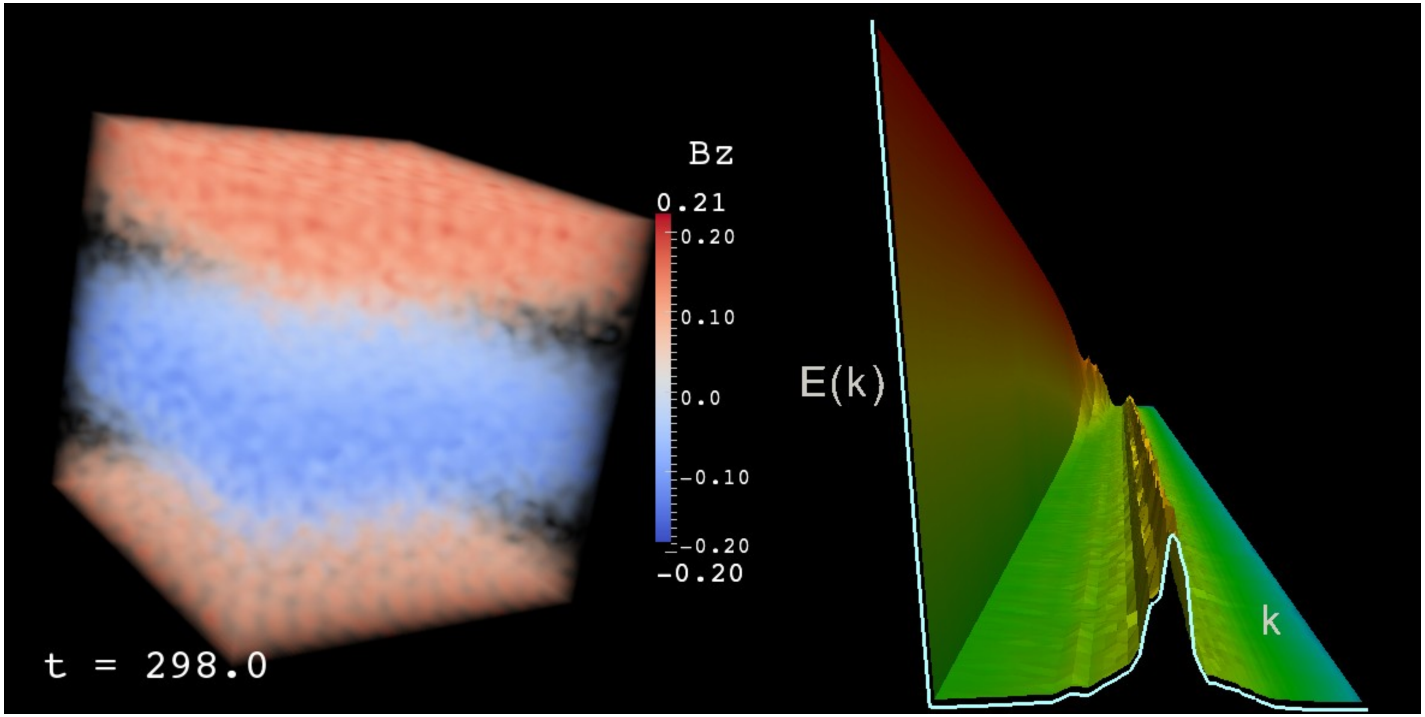
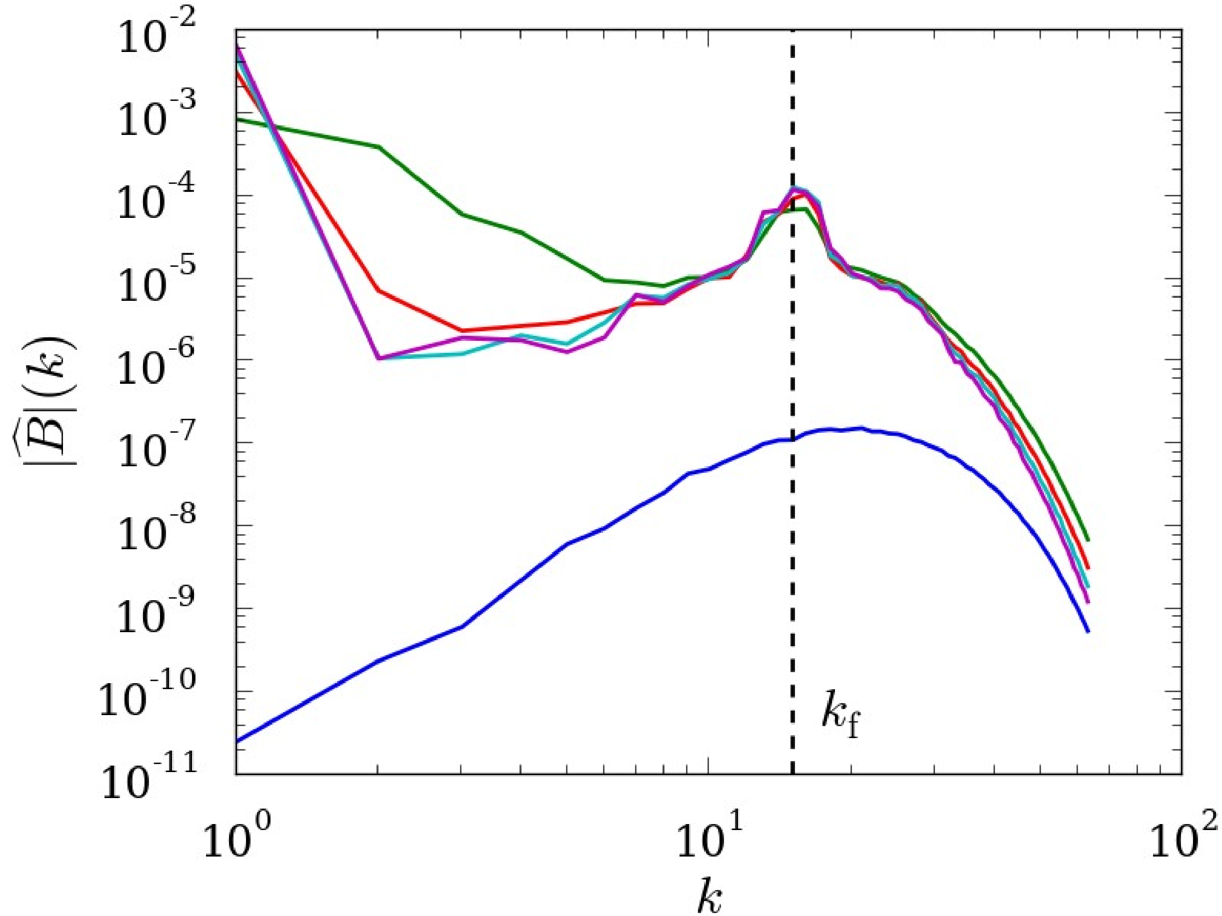


Helical Dynamos

kinetic helicity $\boldsymbol{\omega} \cdot \boldsymbol{u}$ → helical magnetic fields $\overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ ↑ $\overline{\boldsymbol{A}} \cdot \overline{\boldsymbol{B}}$ ↓

α effect → growth of large-scale fields



Closed alpha^2 Dynamo

Momentum equation:

forcing function

$$\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{U} = -c_\mathrm{s}^2 \boldsymbol{\nabla} \ln\rho + \frac{1}{\rho}\boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{F}_\mathrm{visc} + \boldsymbol{f}$$

Helical forcing $\boldsymbol{f}$ on scale k_f

Helical motions $\langle\boldsymbol{\omega}\cdot\boldsymbol{u}\rangle \approx \epsilon_\mathrm{f} k_\mathrm{f} \langle\boldsymbol{u}^2\rangle$

Helical magnetic field $\langle\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}\rangle = \epsilon_\mathrm{m} k_\mathrm{m} \langle\overline{\boldsymbol{B}}^2\rangle$

$$= (\epsilon_\mathrm{m} k_\mathrm{m})^2 \langle\overline{\boldsymbol{A}}\cdot\overline{\boldsymbol{B}}\rangle \qquad \boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$$

(Frisch et. al. 1975, Seehafer 1996)

$\epsilon_\mathrm{f}, \epsilon_\mathrm{m}$ = normalized helicities

$$t_\mathrm{sat} = t_\mathrm{res} = (2\eta\epsilon_\mathrm{m}^2 k_1^2)^{-1}$$

resistive growth for large-scale field $\overline{\boldsymbol{B}}$ *(Brandenburg, Subramanian 2005)*

Predictions from the General Theory

mean-field interpretation

Saturation magnetic field strength:

$$\overline{B}_{\rm sat}^2/B_{\rm eq}^2 = (C_\alpha/\epsilon_{\rm m} - 1)\iota$$

(Blackman, Brandenburg 2002)

$$\iota = \eta_{\rm T}/\eta_{\rm t} = (1 + 3/{\rm Re_M})$$

$${\rm Re_M} = \frac{u_{\rm rms}}{\eta k_{\rm f}} \qquad B_{\rm eq} = u_{\rm rms}(\mu_0\overline{\rho})^{1/2}$$

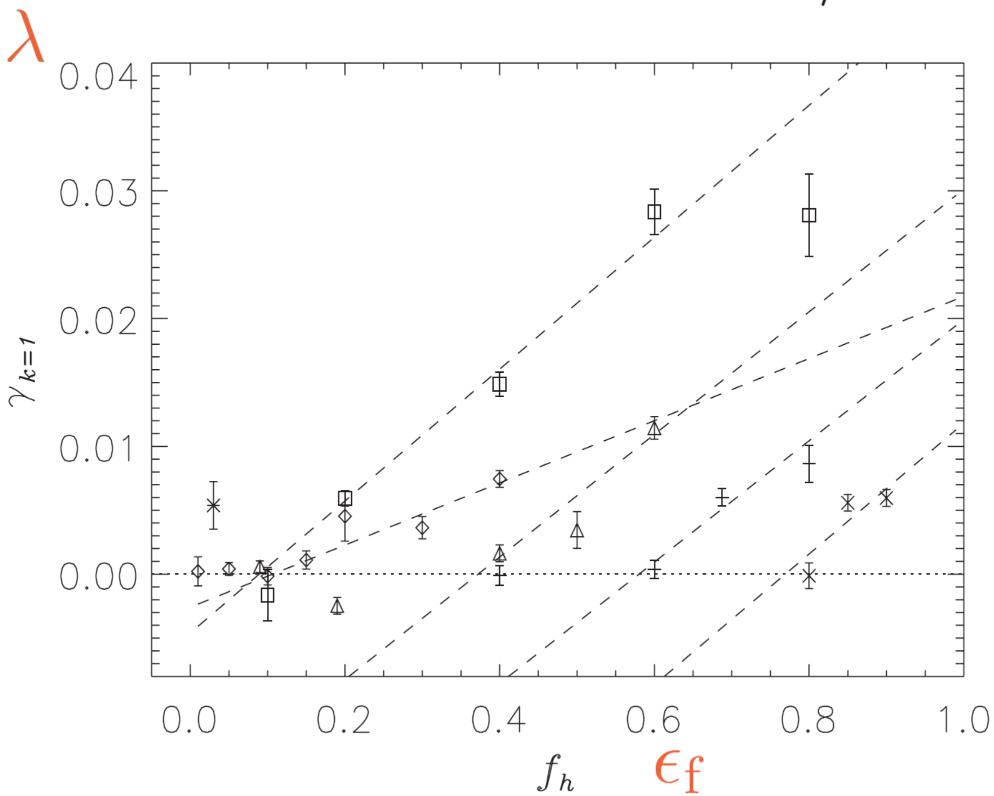
For the man magnetic field to grow: $|C_\alpha^{\rm crit}| = \epsilon_{\rm m}$

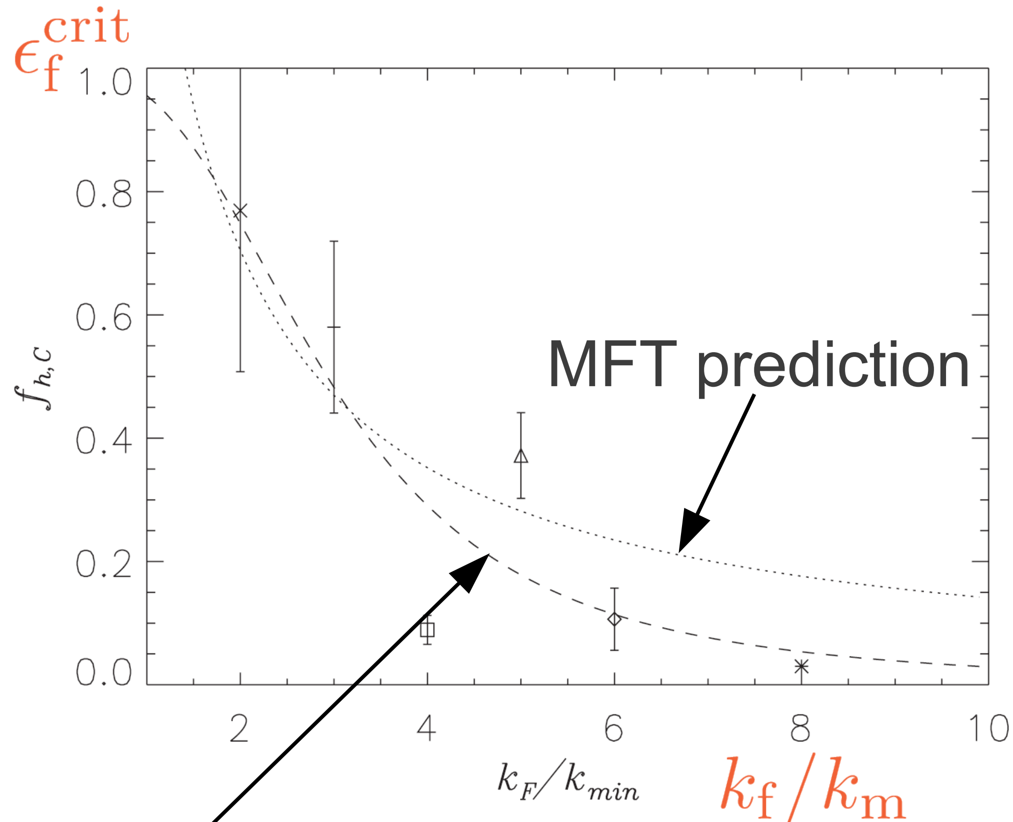
$$C_\alpha = -\frac{\langle \boldsymbol{\omega}\cdot\boldsymbol{u}\rangle}{\iota k_{\rm f} u_{\rm rms}^2} = -\frac{\epsilon_{\rm f} k_{\rm f}}{\iota k_{\rm m}} \quad \Rightarrow \quad \epsilon_{\rm f}^{\rm crit} = \iota\epsilon_{\rm m}\left(\frac{k_{\rm f}}{k_{\rm m}}\right)^{-1}$$

$$\epsilon_{\rm f} = \frac{\langle \boldsymbol{\omega}\cdot\boldsymbol{u}\rangle}{k_{\rm f} u_{\rm rms}^2}$$ = normalized kinetic helicity

What Pietarila Graham Finds

Parameters: ϵ_{f} and $k_{\mathrm{f}}/k_{\mathrm{m}}$





Fit formula: $f_{\mathrm{h,C}} = 1/\left(1 + C^2 (k_{\mathrm{f}}/k_{\mathrm{m}})^{2\xi+2}\right) \qquad \xi \approx 0.46$

(*Pietarila Graham, et. al. 2012*)

$$\left(\frac{k_{\mathrm{f}}}{k_{\mathrm{m}}}\right)^{-1} \neq \left(\frac{k_{\mathrm{f}}}{k_{\mathrm{m}}}\right)^{-3}$$

Reproduction of the Predictions

Consider the resistive phase well after the kinematic phase.

$$\frac{\partial}{\partial t}\boldsymbol{A} = \boldsymbol{U} \times \boldsymbol{B} - \eta\mu_0\boldsymbol{J}$$

forcing function

$$\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{U} = -c_\mathrm{s}^2\boldsymbol{\nabla}\ln\rho + \frac{1}{\rho}\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{F}_\mathrm{visc} + \boldsymbol{f}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\ln\rho = -\boldsymbol{\nabla}\cdot\boldsymbol{U}$$

triply periodic BC → magnetic helicity is conserved

helical forcing $\boldsymbol{f}$ → helical motions $\langle\boldsymbol{\omega}\cdot\boldsymbol{u}\rangle \approx \epsilon_\mathrm{f} k_\mathrm{f}\langle\boldsymbol{u}\rangle$

→ helical magnetic field (Beltrami field) $\langle\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}\rangle = k_\mathrm{m}\langle\overline{\boldsymbol{B}}^2\rangle$

Parameters: ϵ_f and $k_\mathrm{f}/k_\mathrm{m}$

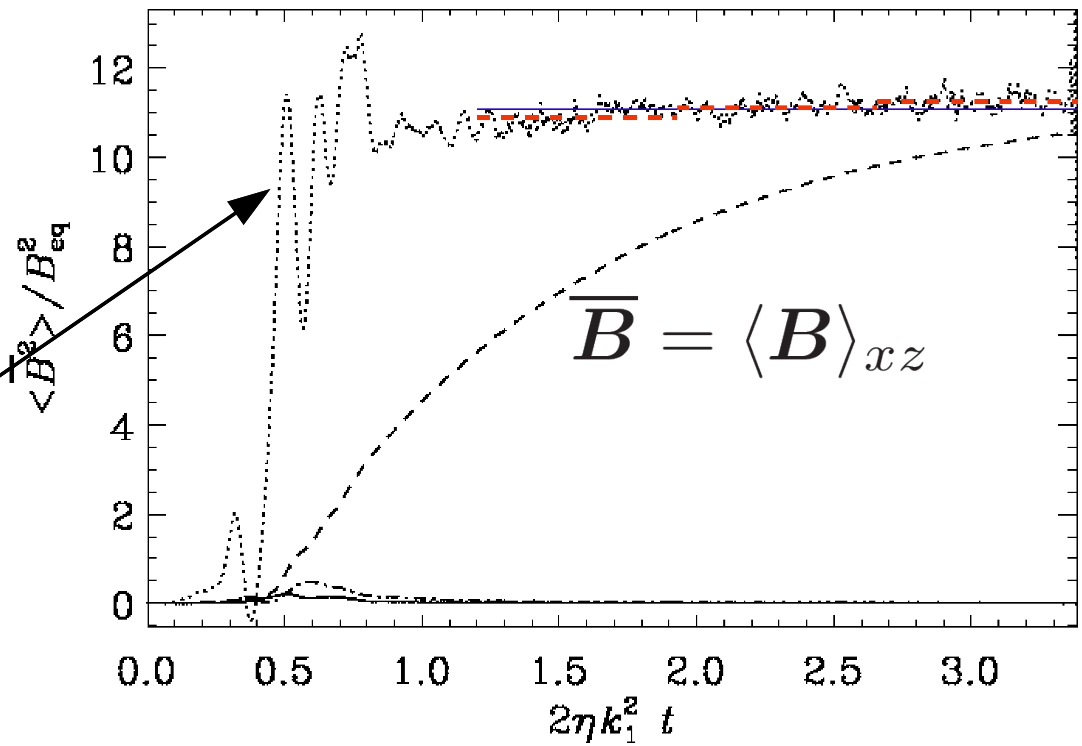
Saturation Magnetic Field

resistive growth:

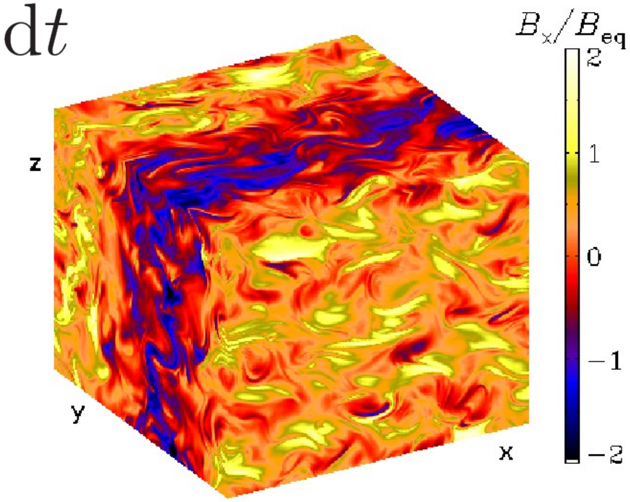
$$M(t) = M_0 - M_1 e^{-t/\tau}$$

$$\tau = (2\eta\epsilon_{\rm m}^2 k_{\rm m}^2)^{-1}$$

$$M_0 = M(t) + \tau \frac{\rm d}{{\rm d}t} M(t)$$



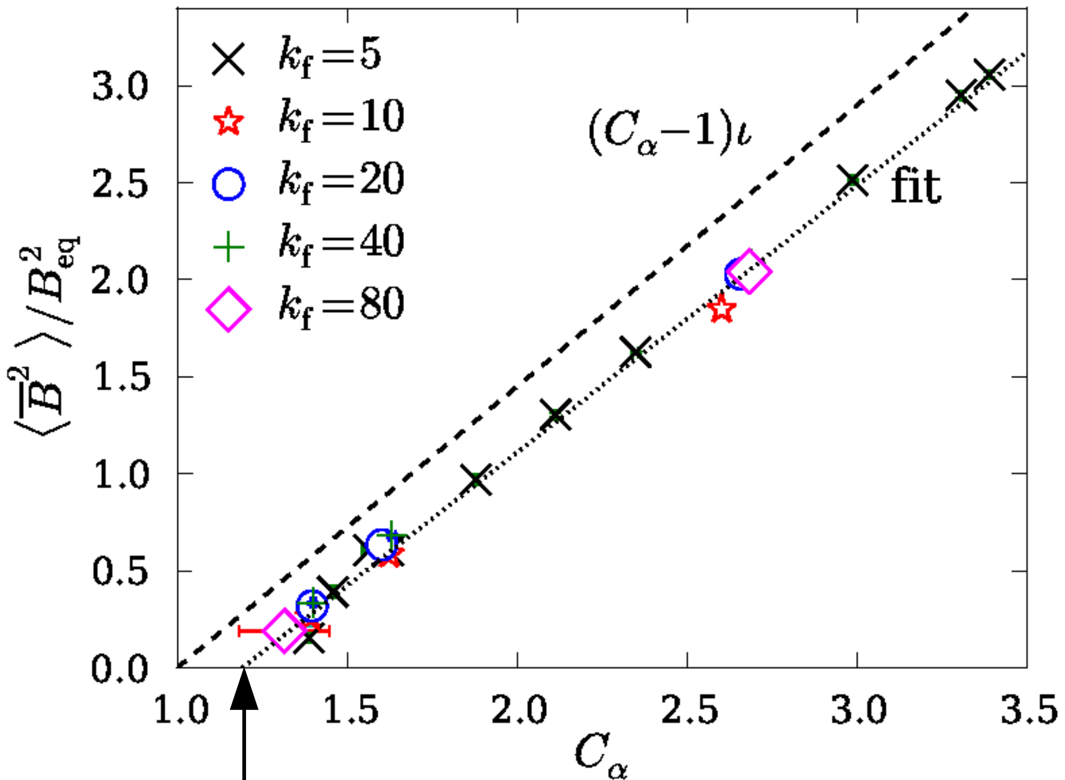
$$B_{\rm sat}^2 = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} M(t) + \tau \frac{\rm d}{{\rm d}t} M(t) \; {\rm d}t$$



(Candelaresi, Brandenburg 2012)

Saturation Magnetic Field

Prediction: $\overline{B}^2_{\rm sat}/B^2_{\rm eq} = (C_\alpha - 1)\iota$ *(Blackman, Brandenburg 2002)*



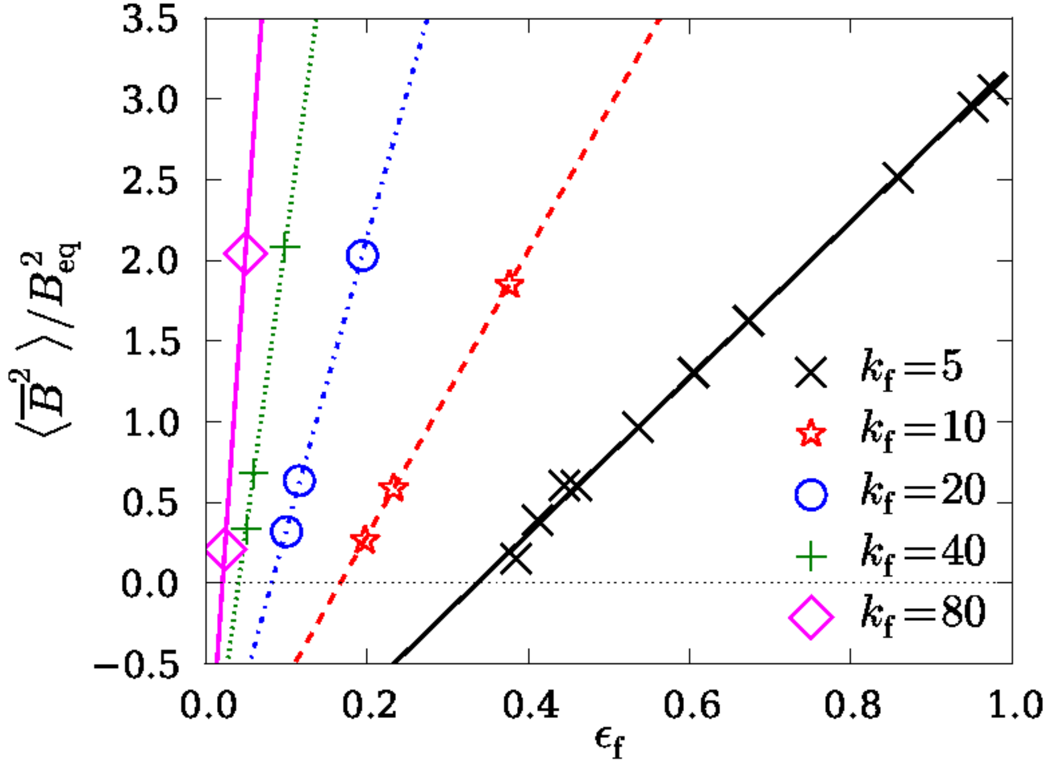
$\mathrm{Re_M} \approx 6$

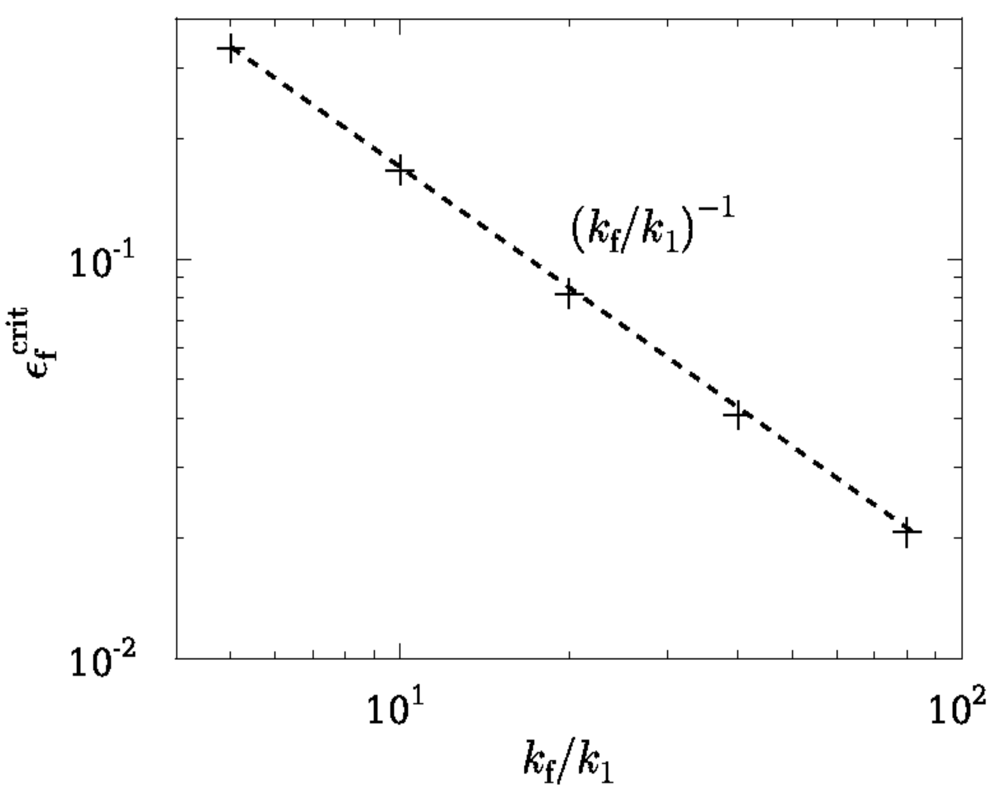
$\mathrm{Pr_M} = \nu/\mu = 1$

$C_\alpha^{\rm crit} \approx 1.2$

(Candelaresi, Brandenburg 2012)

Saturation Magnetic Field





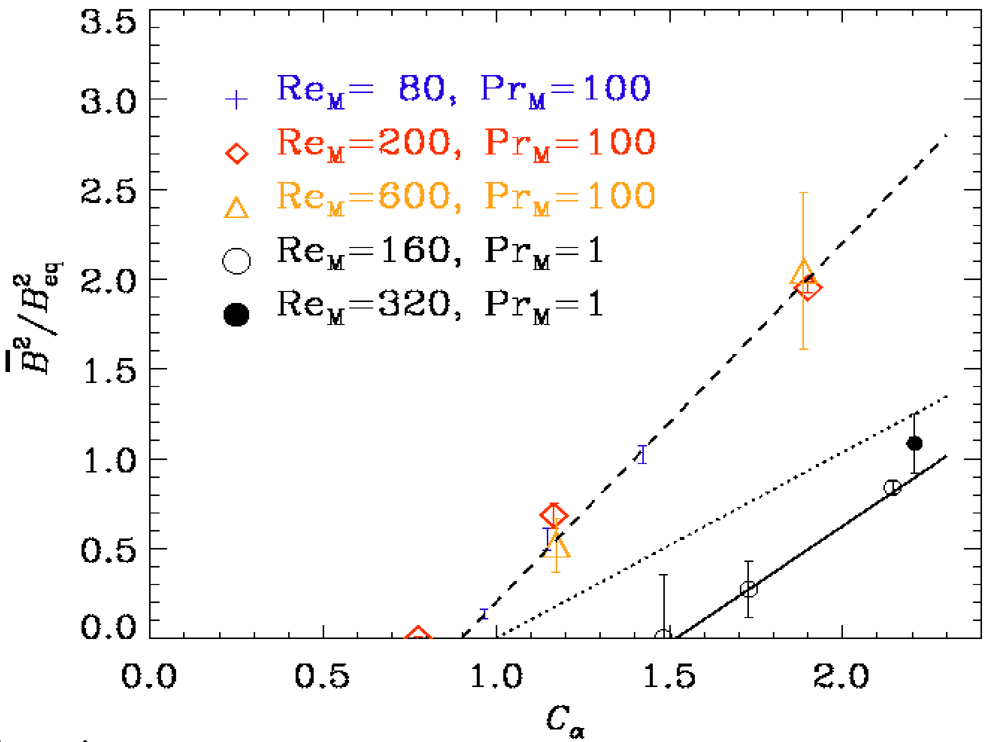
Predictions: $\overline{B}^2_{sat}/B^2_{eq} = \epsilon_f \left(\frac{k_f}{k_m}\right) - \iota$ $\qquad \epsilon_f^{crit} = \iota \epsilon_m \left(\frac{k_f}{k_m}\right)^{-1}$

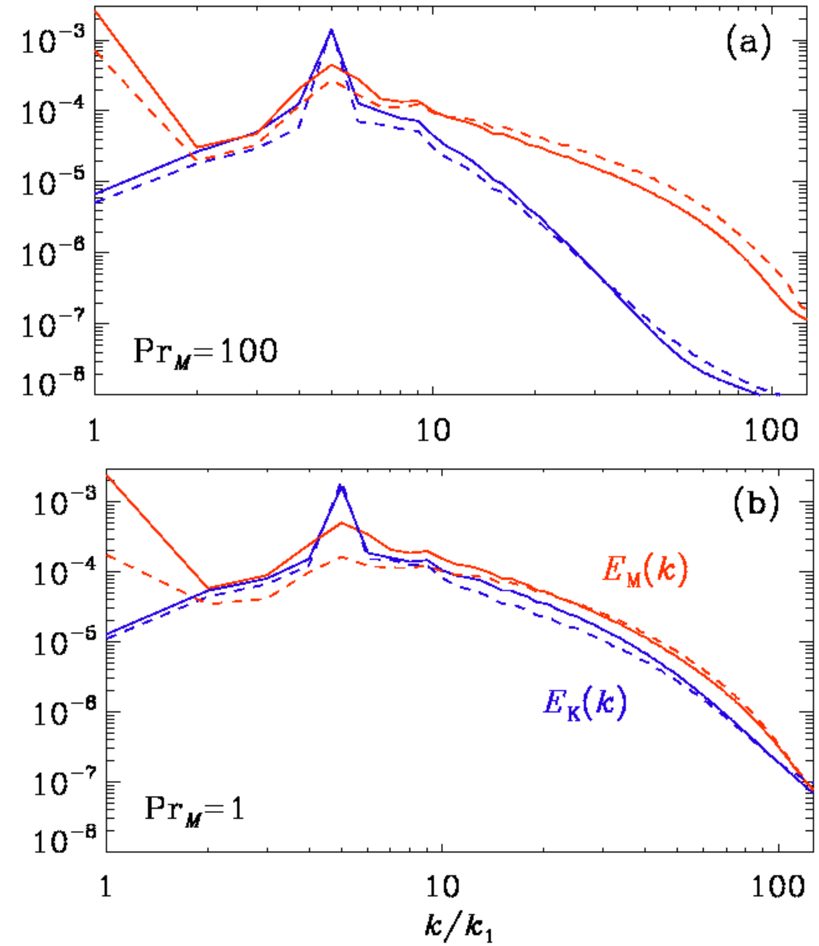
NB: $k_1 = k_m$

High $\mathrm{Re_M}$

$\mathrm{Re_M} \to 2000$ *(Pietarila Graham, et. al. 2012)*

Match their parameters.





No change in C_α^{crit} for high $\mathrm{Re_M}$.

(Candelaresi, Brandenburg 2012)

B_{eq} is underestimated for high $\mathrm{Pr_M}$ due to viscous losses.

Magnetic Helicity Fluxes

mean-field considerations

Induction equation:

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Mean-field induction equation:

$$\partial_t \overline{\boldsymbol{B}} = \eta \nabla^2 \overline{\boldsymbol{B}} + \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}}) \qquad \boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$$

Electromotive force (EMF): $\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \boldsymbol{b}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \nabla \times \overline{\boldsymbol{B}}$

$$\alpha = \alpha_{\rm K} + \alpha_{\rm M}$$

$$\alpha_{\rm K} = -\tau \overline{\boldsymbol{\omega} \cdot \boldsymbol{u}}/3$$

$$\alpha_{\rm M} = \tau \overline{\boldsymbol{j} \cdot \boldsymbol{b}}/(3\overline{\rho}) \approx \tau k^2 \overline{\boldsymbol{a} \cdot \boldsymbol{b}}/(3\overline{\rho}) = \overline{h}_{\rm f}$$ *(Pouquet et al. 1976)*

$$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \alpha \overline{\boldsymbol{B}} + \eta_{\rm T} \boldsymbol{\nabla}^2 \overline{\boldsymbol{B}}$$

Moffatt, H. K., 1978, Krause and Raedler, 1980

Magnetic Helicity Fluxes

I) $$\frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}}}{B_{eq}^2} + \frac{\alpha_M}{R_m} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

II) $$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \alpha \overline{\boldsymbol{B}} + \eta_T \boldsymbol{\nabla}^2 \overline{\boldsymbol{B}}$$

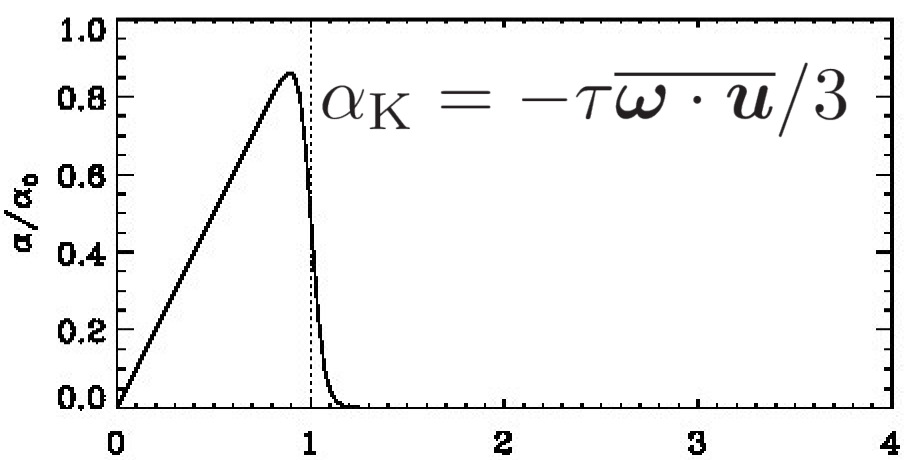
III) $$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_t \nabla \times \overline{\boldsymbol{B}}$$

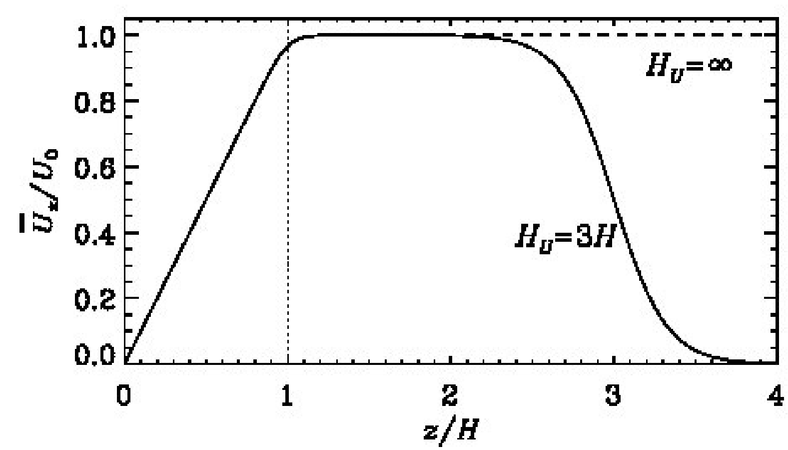
1D mean-field in z

α diffusion
$$\kappa_\alpha \frac{\partial \alpha_M}{\partial z}$$

advective:
$$\alpha_M \overline{\boldsymbol{U}}$$

Helical forcing profile:

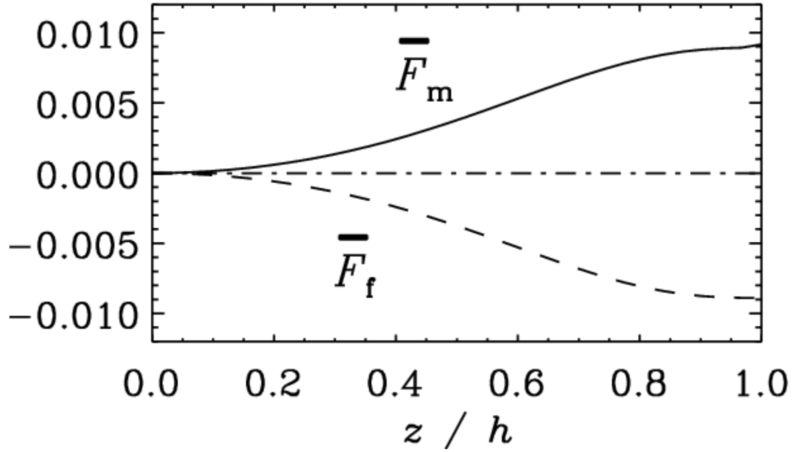


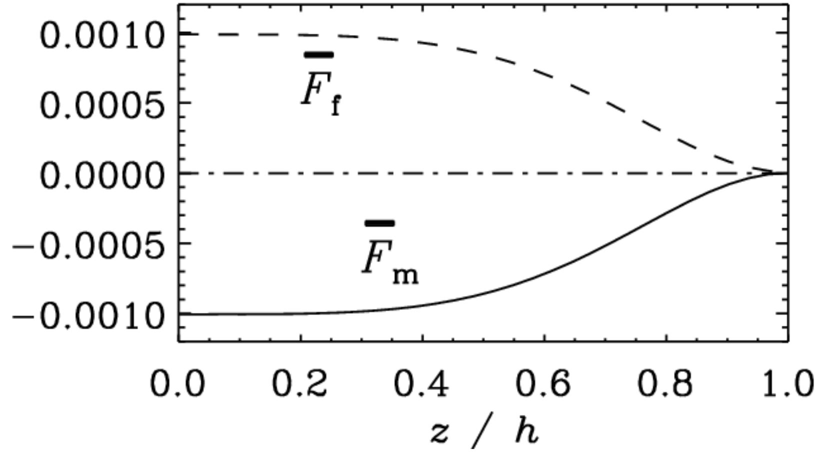


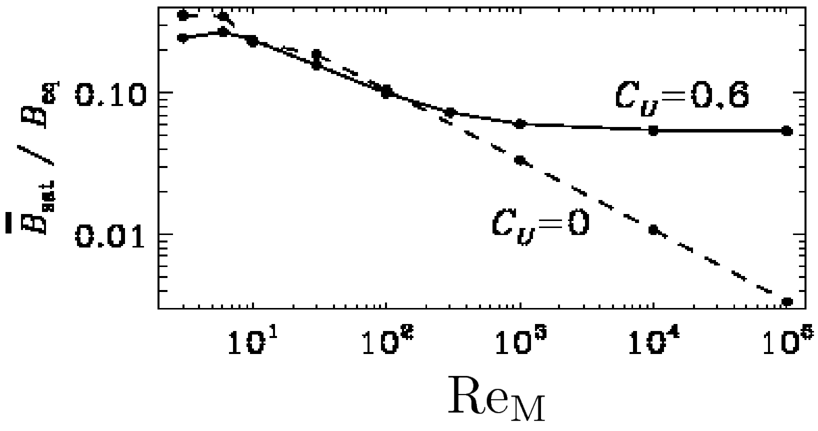
(Brandenburg, et al., 2009)

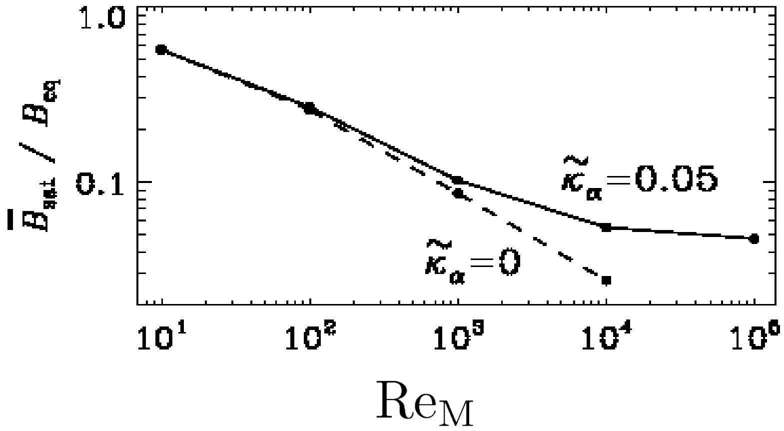
Magnetic Helicity Fluxes

open boundary	vs.	closed boundary
symmetric		antisymmetric
wind		κ_α









Summary

- MF prediction reproduced in DNS.
- Discrepancy of (Graham) due to SSD contamination.
- Magnetic helicity fluxes alleviate catastrophic alpha quenching.

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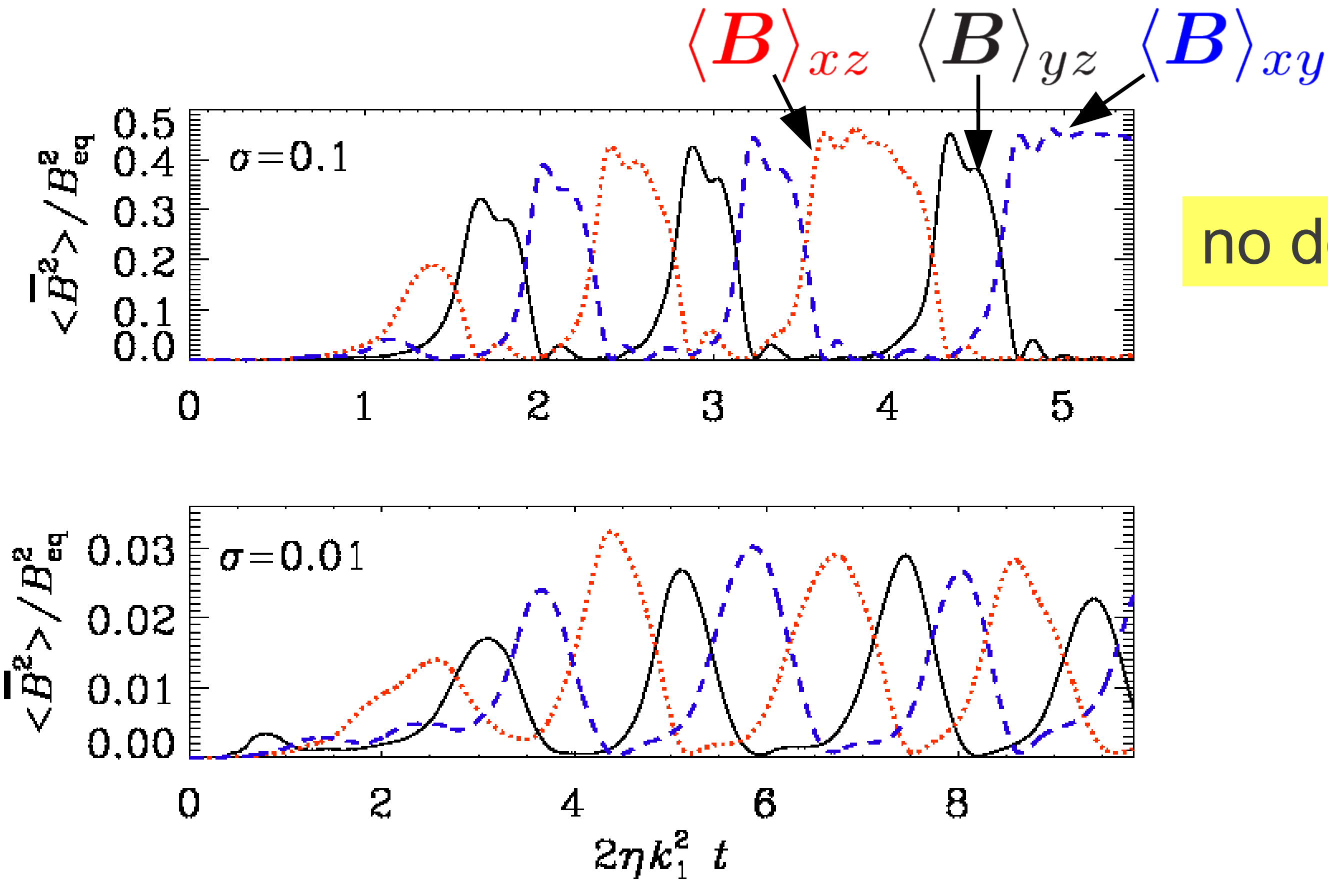
ABC-Flow Forcing

Forcing: $$\boldsymbol{f}(\boldsymbol{x},t)=\frac{f_0}{\sqrt{\frac{3}{2}(1+\sigma^2)}}\begin{pmatrix}\sin(X_3)+\sigma\cos(X_2)\\ \sin(X_1)+\sigma\cos(X_3)\\ \sin(X_2)+\sigma\cos(X_1)\end{pmatrix}$$

$$X_i=k_\mathrm{f}x_i+\theta_0\cos(\omega_i t)$$



no dominant mode