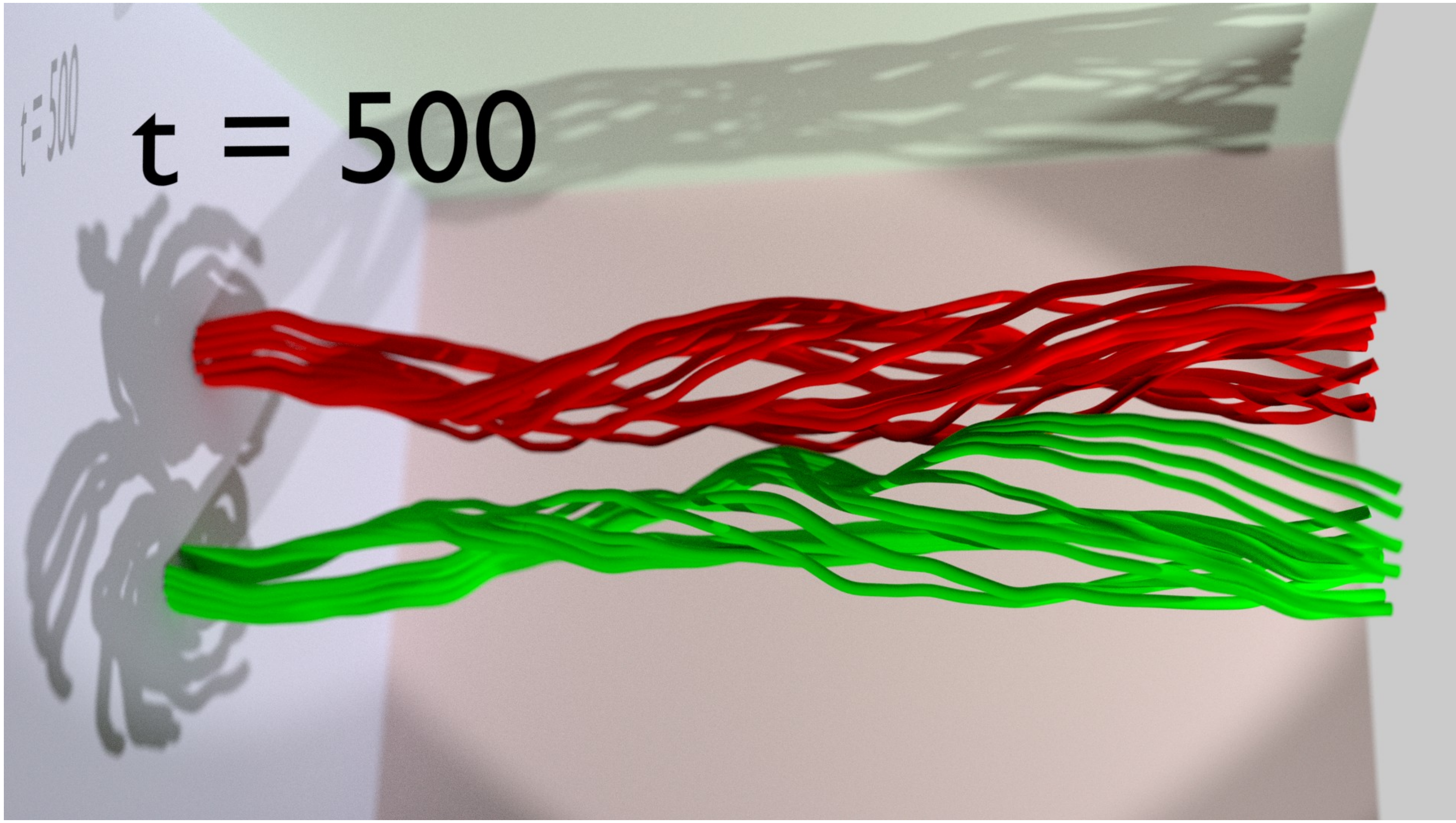
Relaxation of Vortex Braids

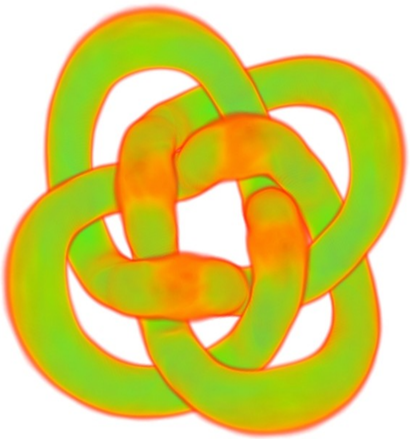
Simon Candelaresi, Gunnar Hornig,
Benjamin Podger, David I. Pontin

UNIVERSITY OF DUNDEE



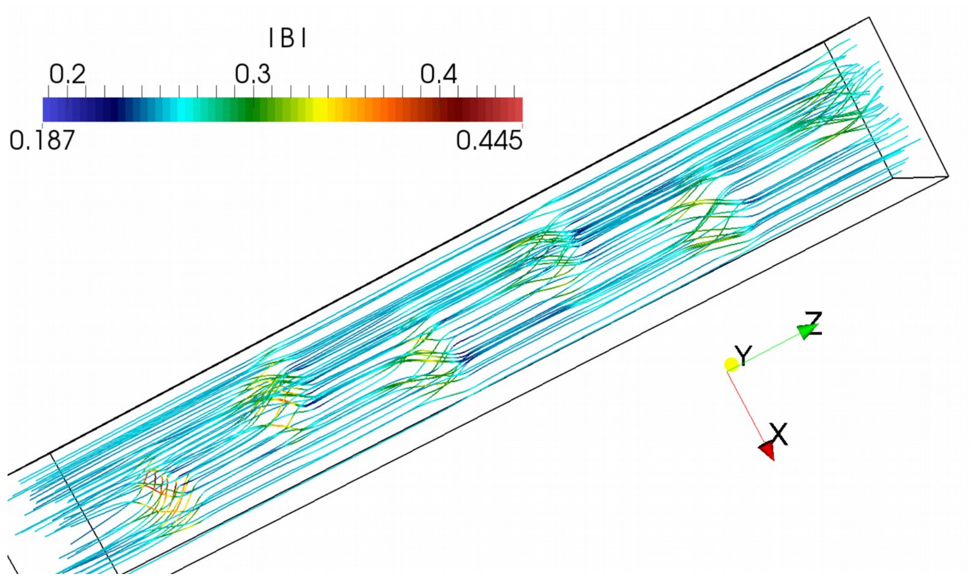
Vortex Field Lines

vorticity: $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{u}$



Similar evolution equation for vorticity and magnetic field in magnetohydrodynamics.

- How far do the similarities go?
- How does the field line topology affect the dynamics?

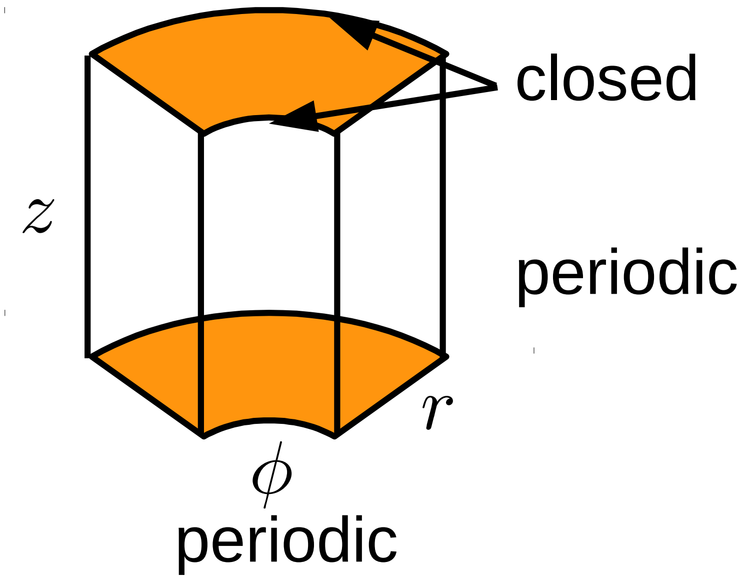


Vortex Braid Experiments

Full viscous simulations with the PencilCode.

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -c_\mathrm{s}^2 \boldsymbol{\nabla} \ln \rho + 2\mathbf{u} \times \boldsymbol{\Omega} + \mathbf{F}_\mathrm{visc}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\nabla \cdot \mathbf{u}$$

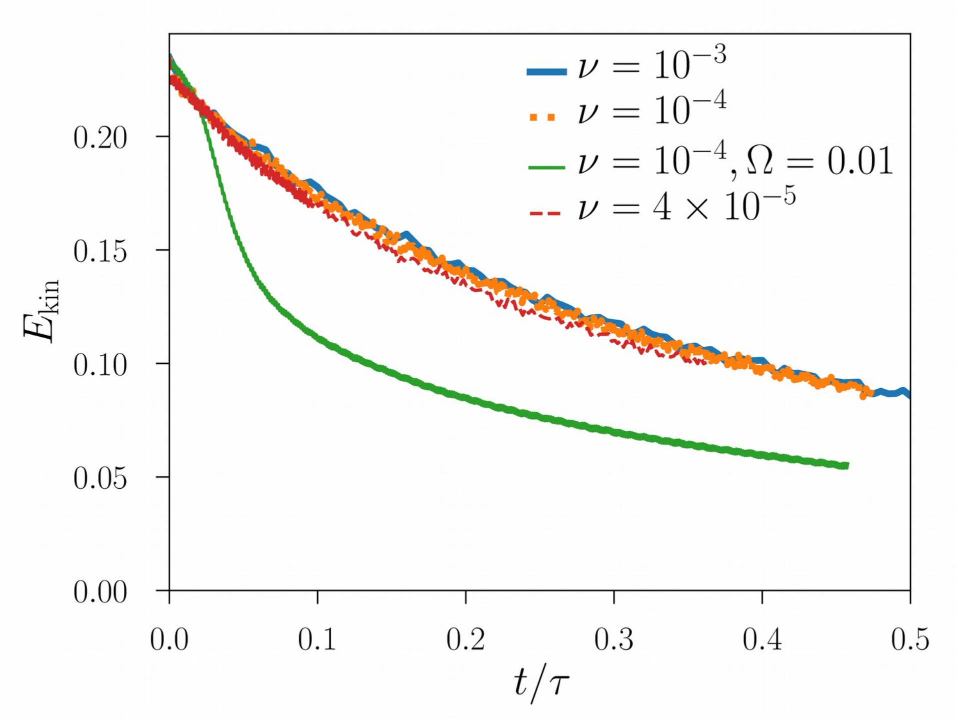


Initially braided vortex field.

kinetic energy: $E_\mathrm{kin} = \dfrac{1}{2} \int \rho \mathbf{u}^2 \, \mathrm{d}V$

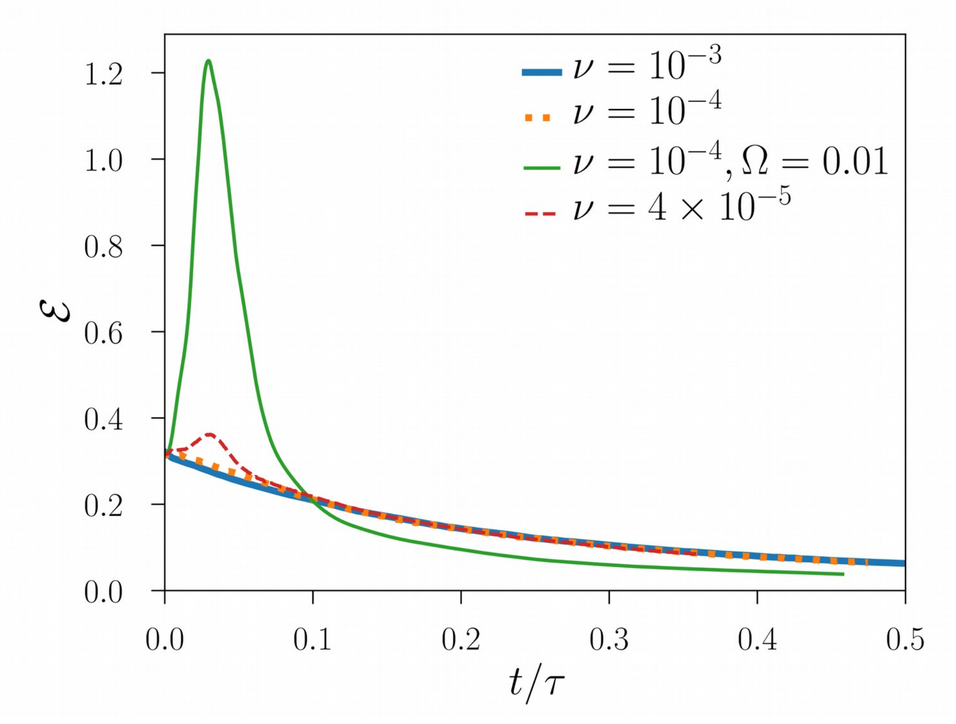
enstrophy: $\mathcal{E} = \int \boldsymbol{\omega}^2 \, \mathrm{d}V$

Vortex Braid Kinetic Energy



- Viscous relaxation strongly affected by the background vorticity.
- Field topology affects relaxation.

Vortex Braid Enstrophy



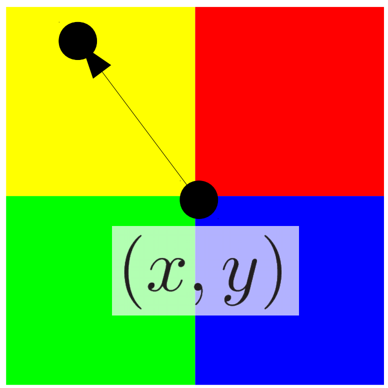
- Enstrophy not a conserved quantity (unlike magnetic energy).
- Enstrophy generation through non-viscosus effects.

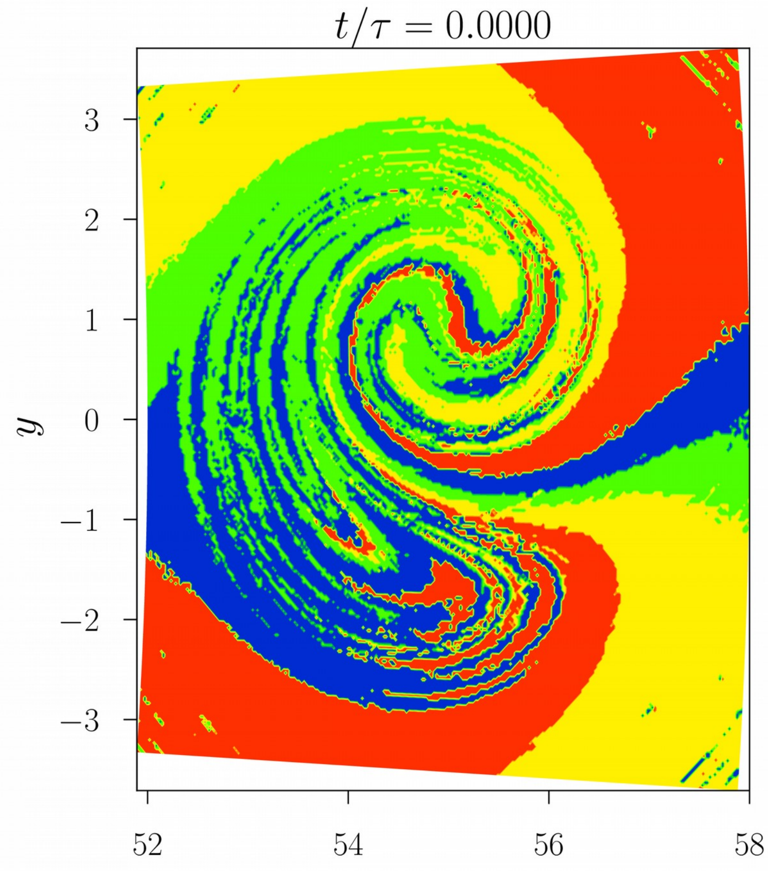
Field Line Mapping

mapping: $(x, y) \to \mathbf{F}_z(x, y)$

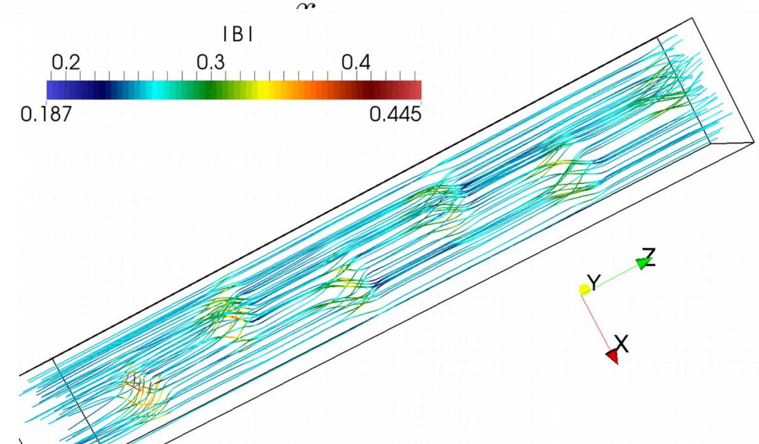
fixed points: $\mathbf{F}_1(x, y) = \begin{pmatrix} x \\ y \end{pmatrix}$

color coding:





fixed point index: $T = \sum_i t_i \quad t_i = \pm 1$



Fixed point index is conserved.

Conclusions

- Topology preserving relaxation of vortex fields.
- Dynamical generation of enstrophy.
- Unbraiding into two twisted vortex flux tubes.

simon.candelaresi@gmail.com