Decay of helical and non-helical magnetic knots

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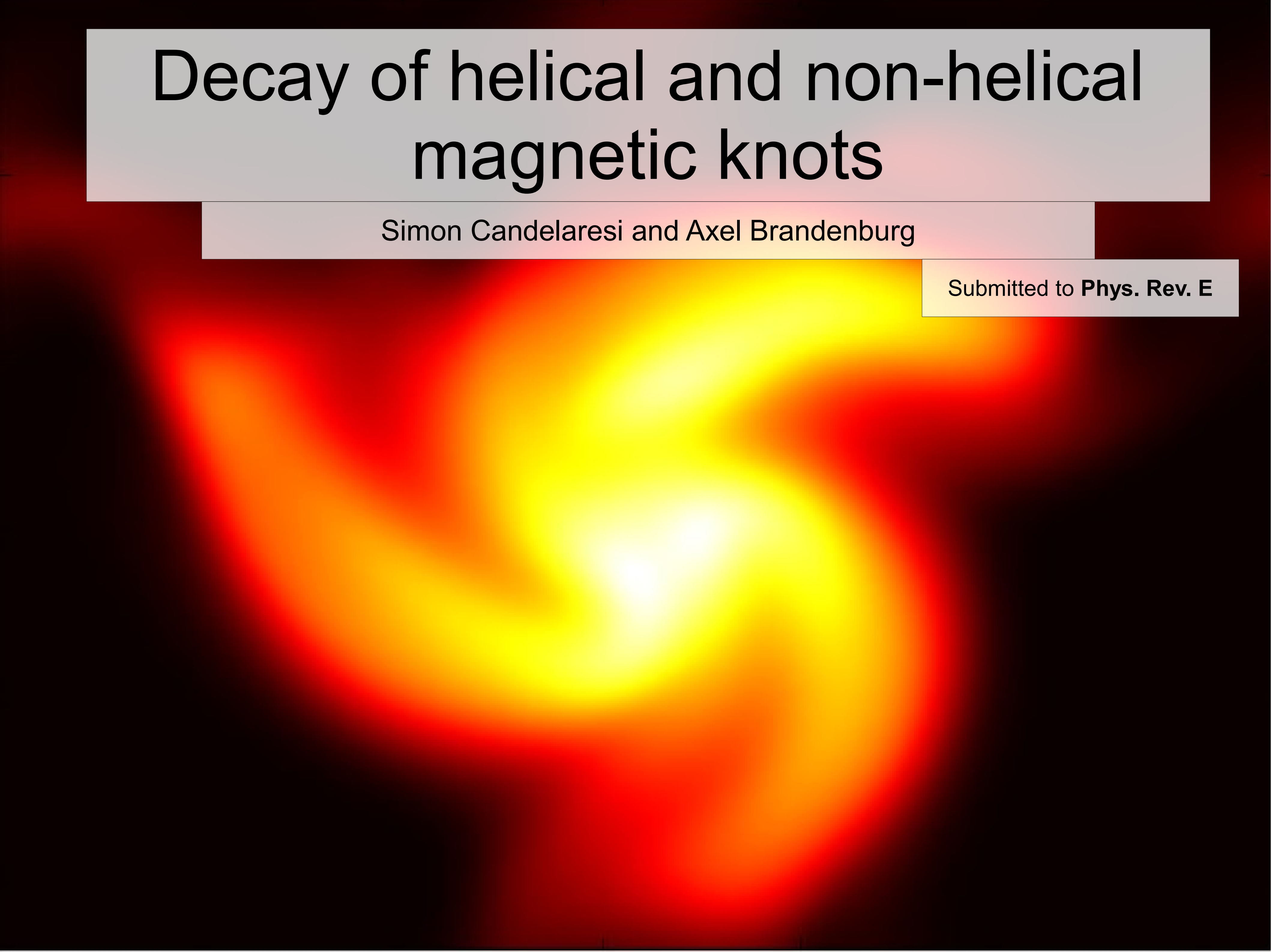
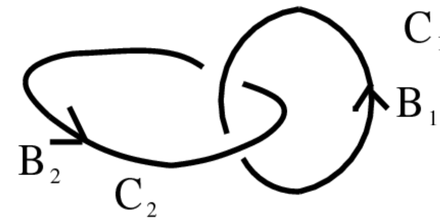


Magnetic Helicity

$$H_{\rm M} = \int_V \mathbf{A} \cdot \mathbf{B} \; {\rm d}V = 2n\phi_1\phi_2$$

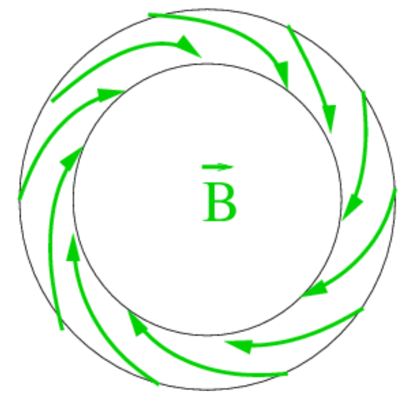
$$\phi_i = \int_{S_i} \mathbf{B} \cdot {\rm d}\mathbf{S}$$



Realizability condition:

$$E_{\rm m}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.



twisted field

magnetic helicity conservation

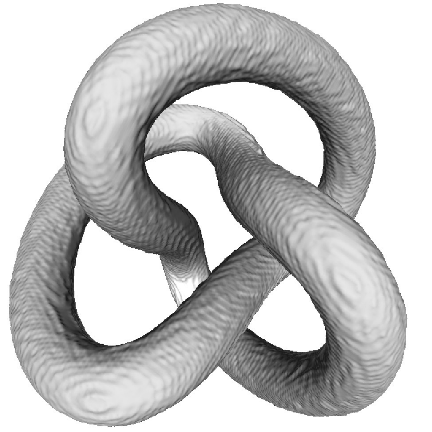
$${\rm Re}_{\rm M} \rightarrow \infty$$

$$\frac{dH_{\rm M}}{dt} = 0$$


trefoil knot

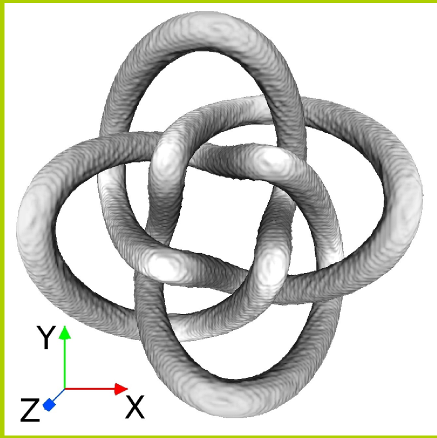
Helical and non-helical setups

$H_{\mathrm{M}} \neq 0$



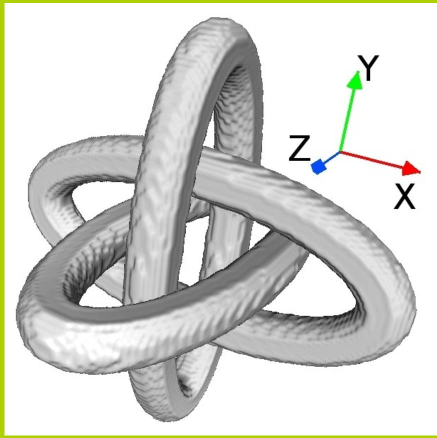
Trefoil knot

$H_{\mathrm{M}} = 0$



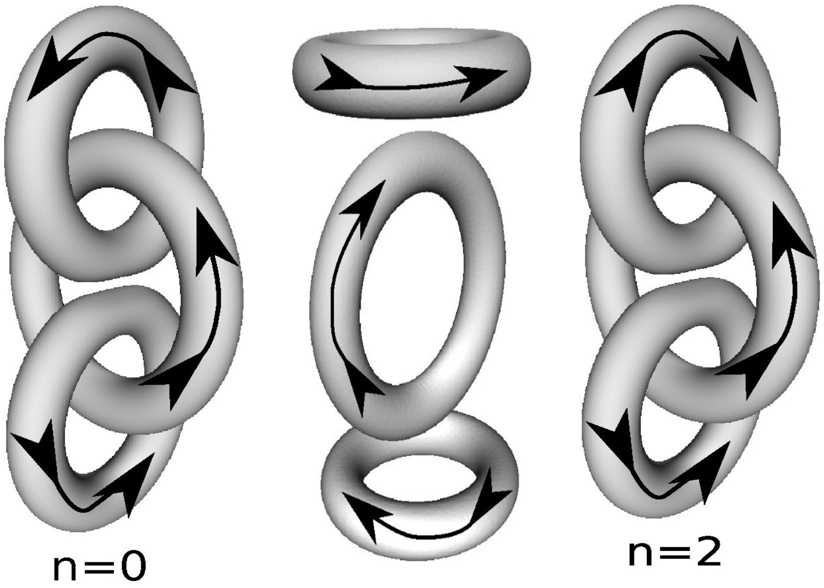
IUCAA knot

$H_{\mathrm{M}} = 0$



Borromean rings

Compare with
(Del Sordo et al. 2010):



Simulations

- 256^3 mesh point
- Isothermal compressible gas
- Viscous medium
- Periodic boundaries

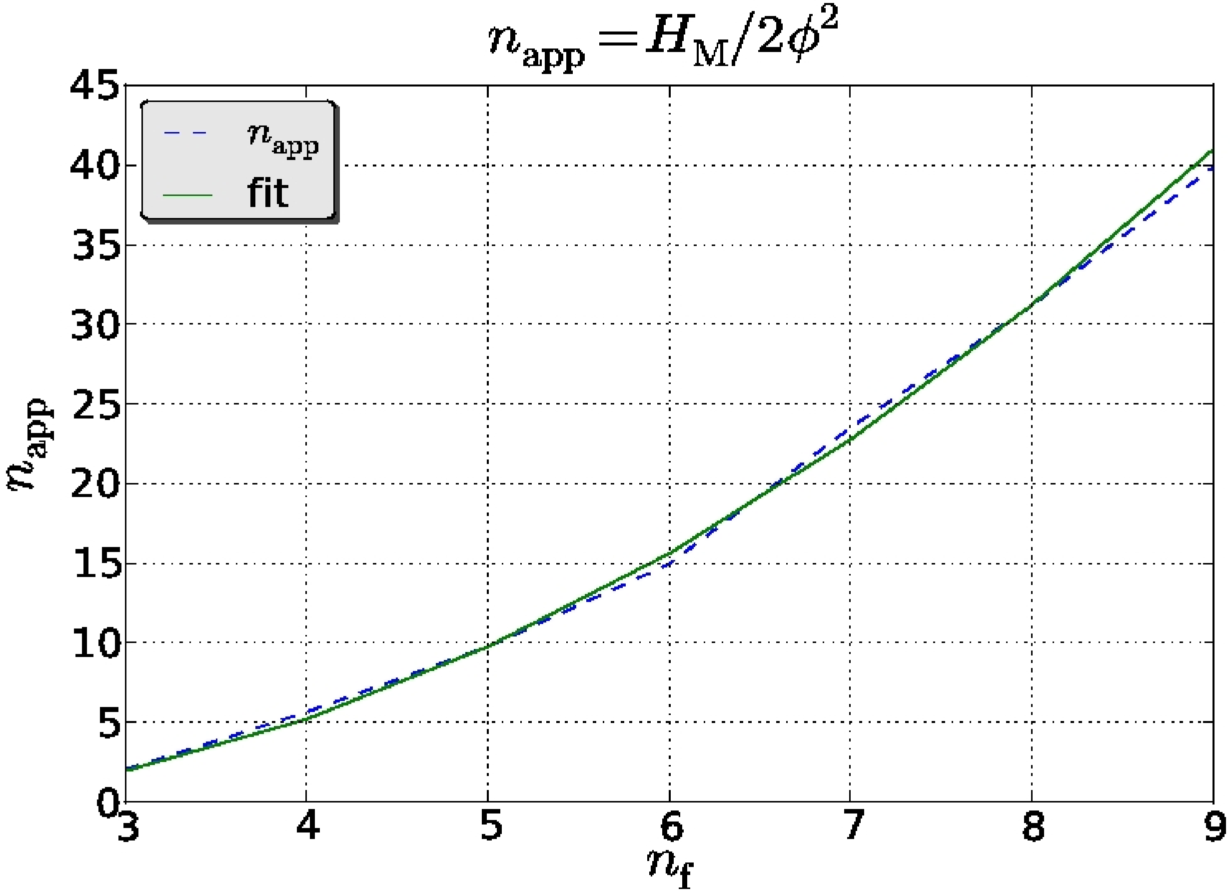
$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}$$

$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \mathbf{J} \times \mathbf{B}/\rho + \mathbf{F}_{\mathrm{visc}}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \mathbf{U}$$

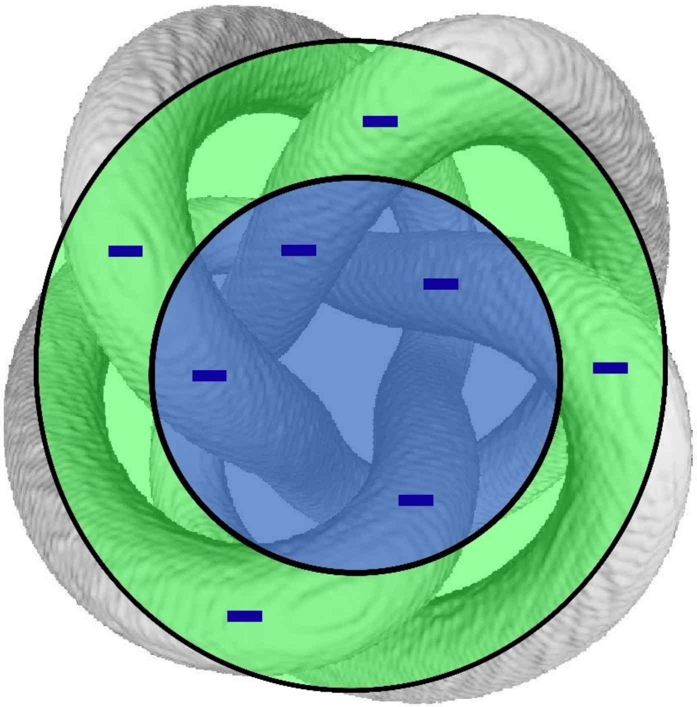
Helicity of n-foil knots

$$H_{\rm M} = 2 n_{\rm app} \phi^2$$

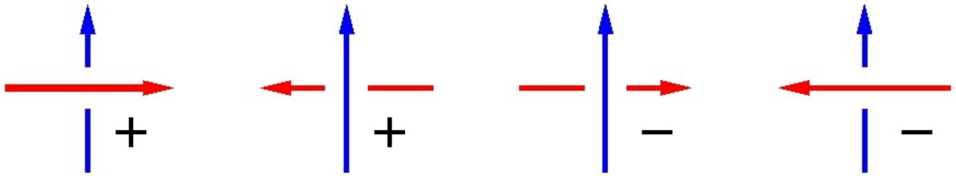


$$H_{\rm M} = (n_{\rm f} - 2) n_{\rm f} \phi^2 / 2$$

Linking number



Sign of the crossings
for the 4-foil knot



$$n_{\text{linking}} = (n_+ - n_-)/2$$

Number of crossings increases like n_{f}^2

$$H_{\text{M}} \propto n_{\text{linking}}$$

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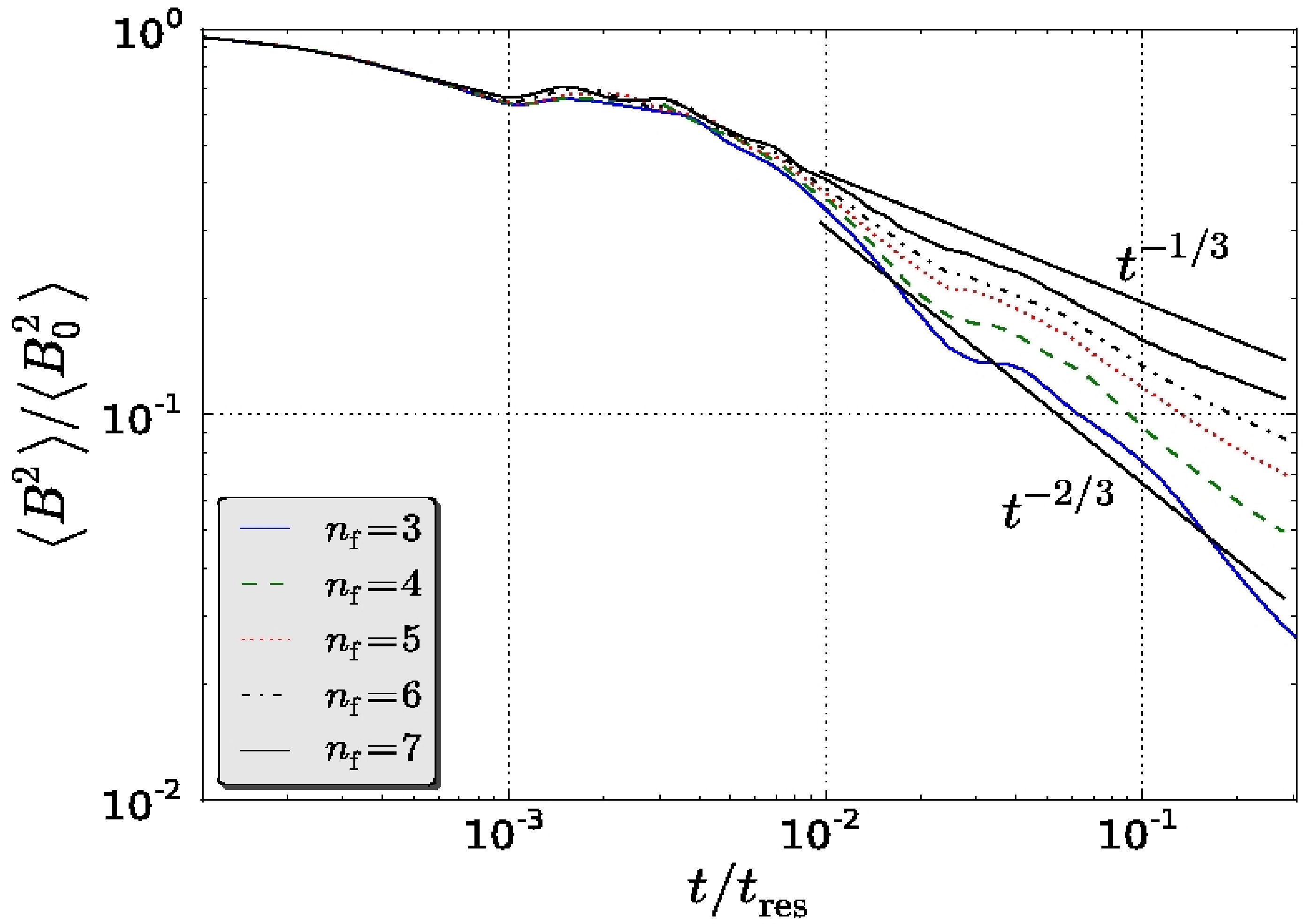
$$H_{\text{M}} \propto n_{\text{f}}^2$$

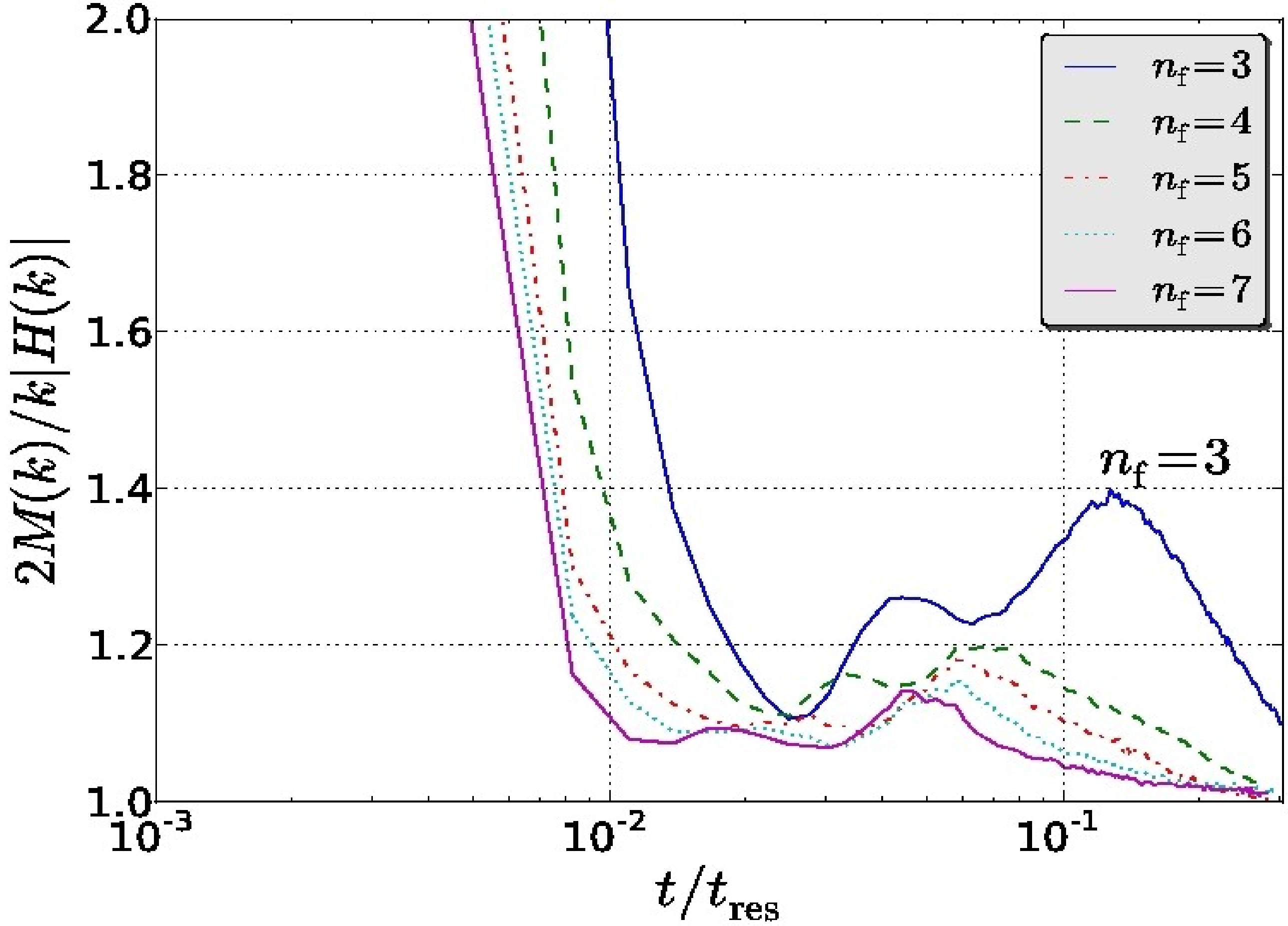
Magnetic energy decay



Slower decay for higher n_f .

Realizability condition

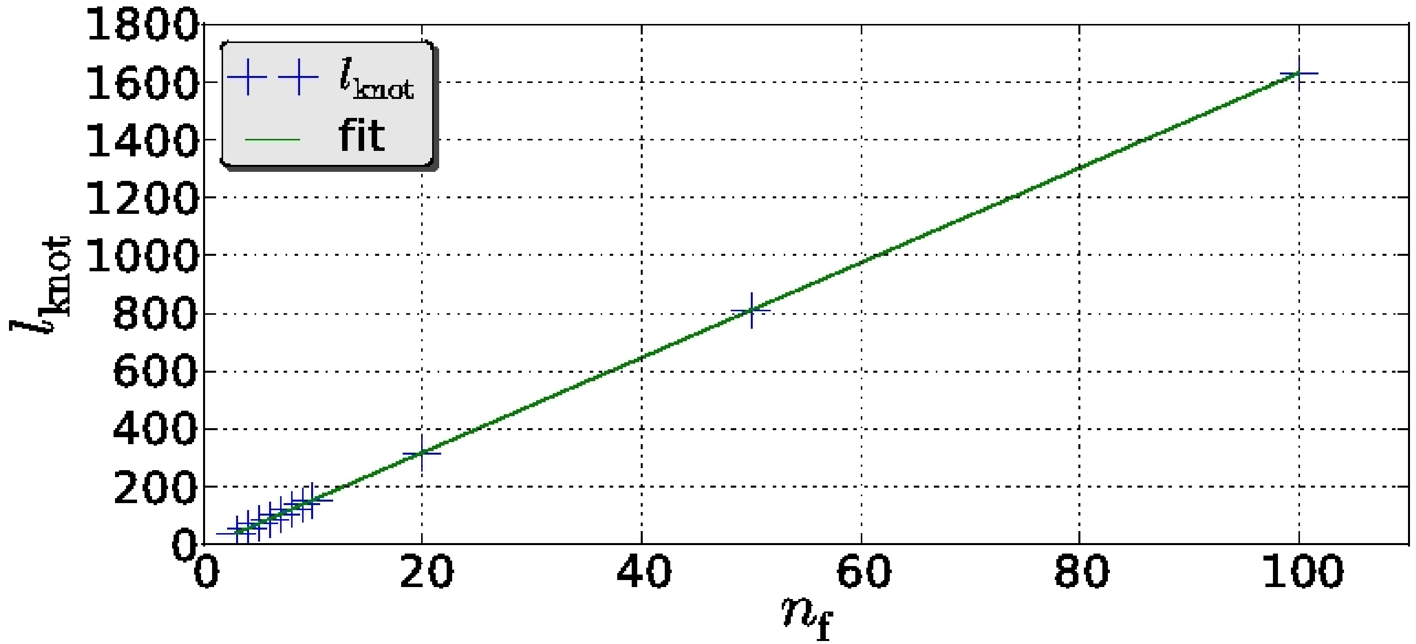
$2M(k)/|H(k)|k$



$k = 2$

Realizability condition more important for high n_f.

Helicity vs. energy



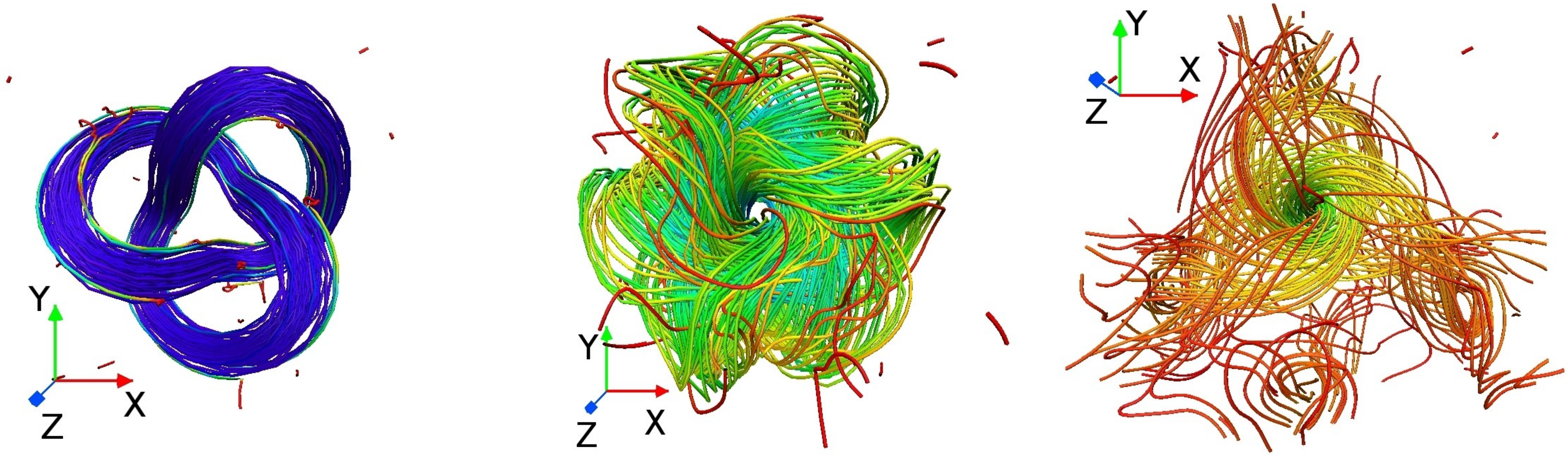
$$E_{\mathrm{M}} \propto l_{\mathrm{knot}} \propto n_{\mathrm{f}}$$

$$H_{\mathrm{M}} \propto n_{\mathrm{f}}^2$$

Knot is more strongly packed with increasing n_{f}.

Magnetic energy is closer to its lower limit for high n_{f}.

Magnetic helicity conservation

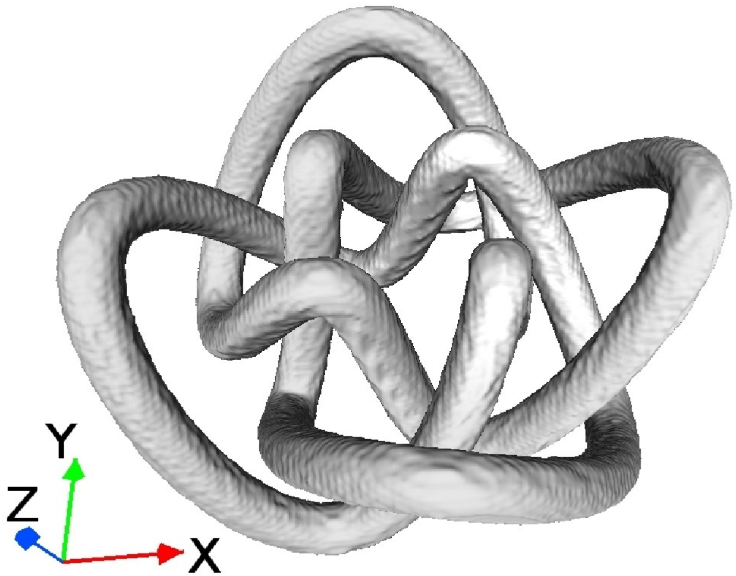


$t = 0$ $t = 6$ $t = 39$

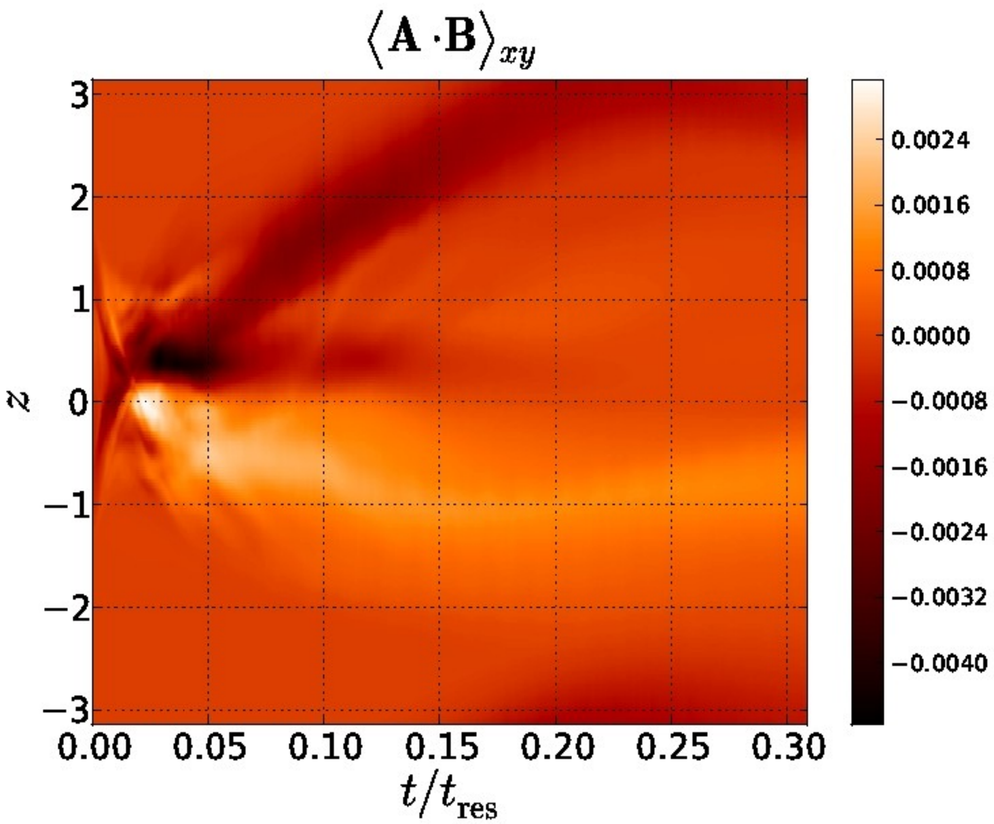
- Magnetic helicity is approximately conserved.
- Self-linking is transformed into twisting after reconnection.

IUCAA knot

Parametrization: $$\mathbf{x}(s) = \begin{pmatrix} (C + \sin 4s)\sin 3s \\ (C + \sin 4s)\cos 3s \\ D\cos(8s - \underline{\varphi}) \end{pmatrix}$$



Asymmetry due to parametrization

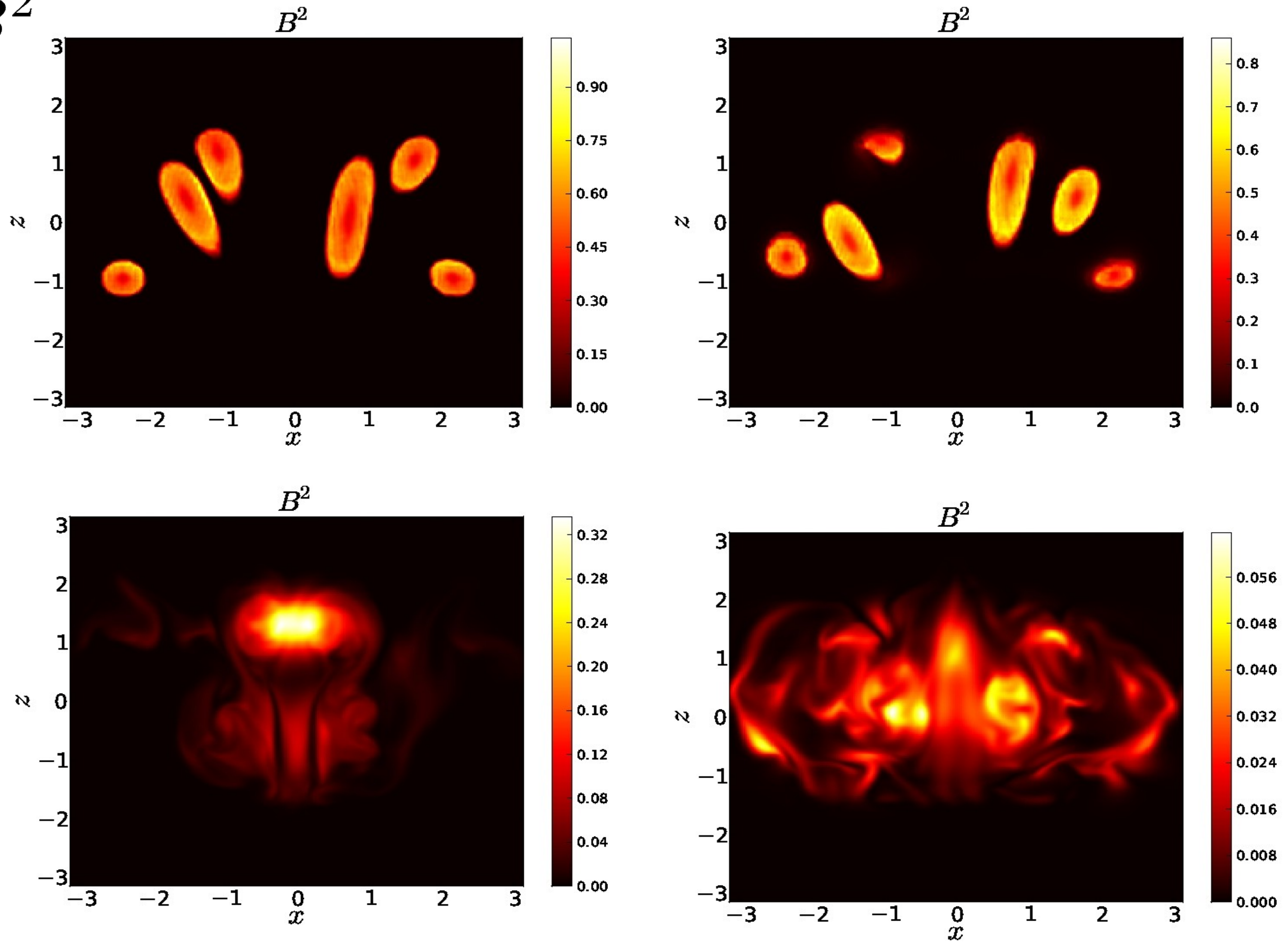


Field ejection

B^2

$t = 0$

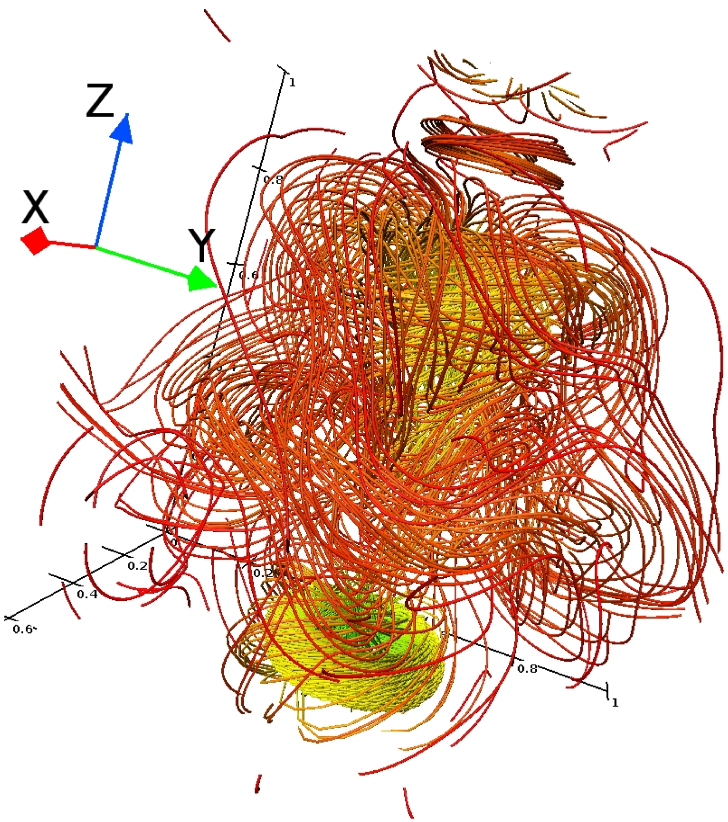
$t = 100$



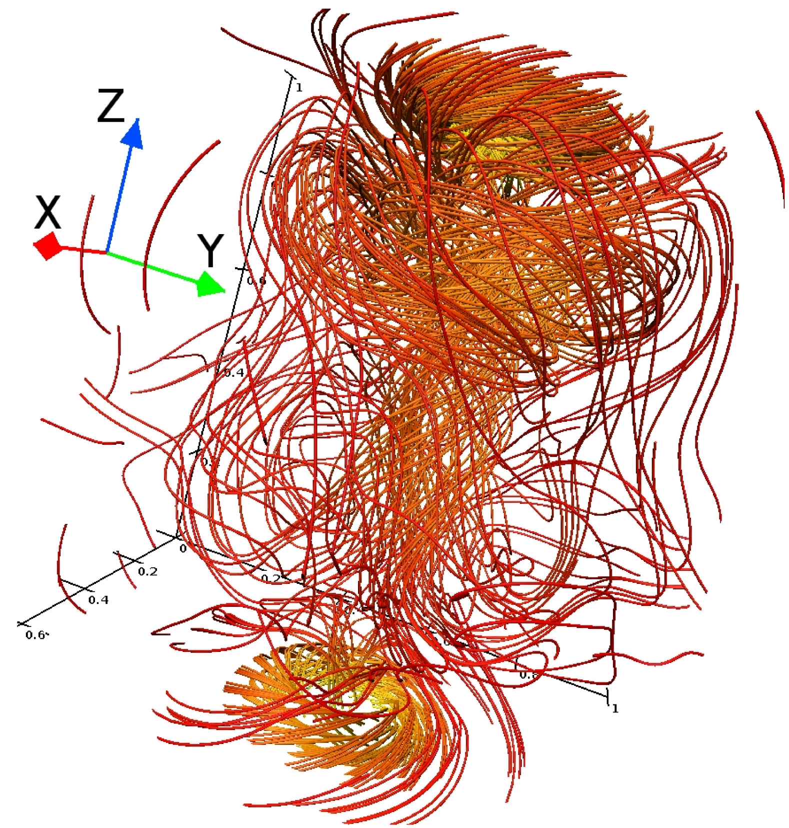
$\varphi = 4/3\pi$

$\varphi = (4/3 + 0.2)\pi$

Field ejection



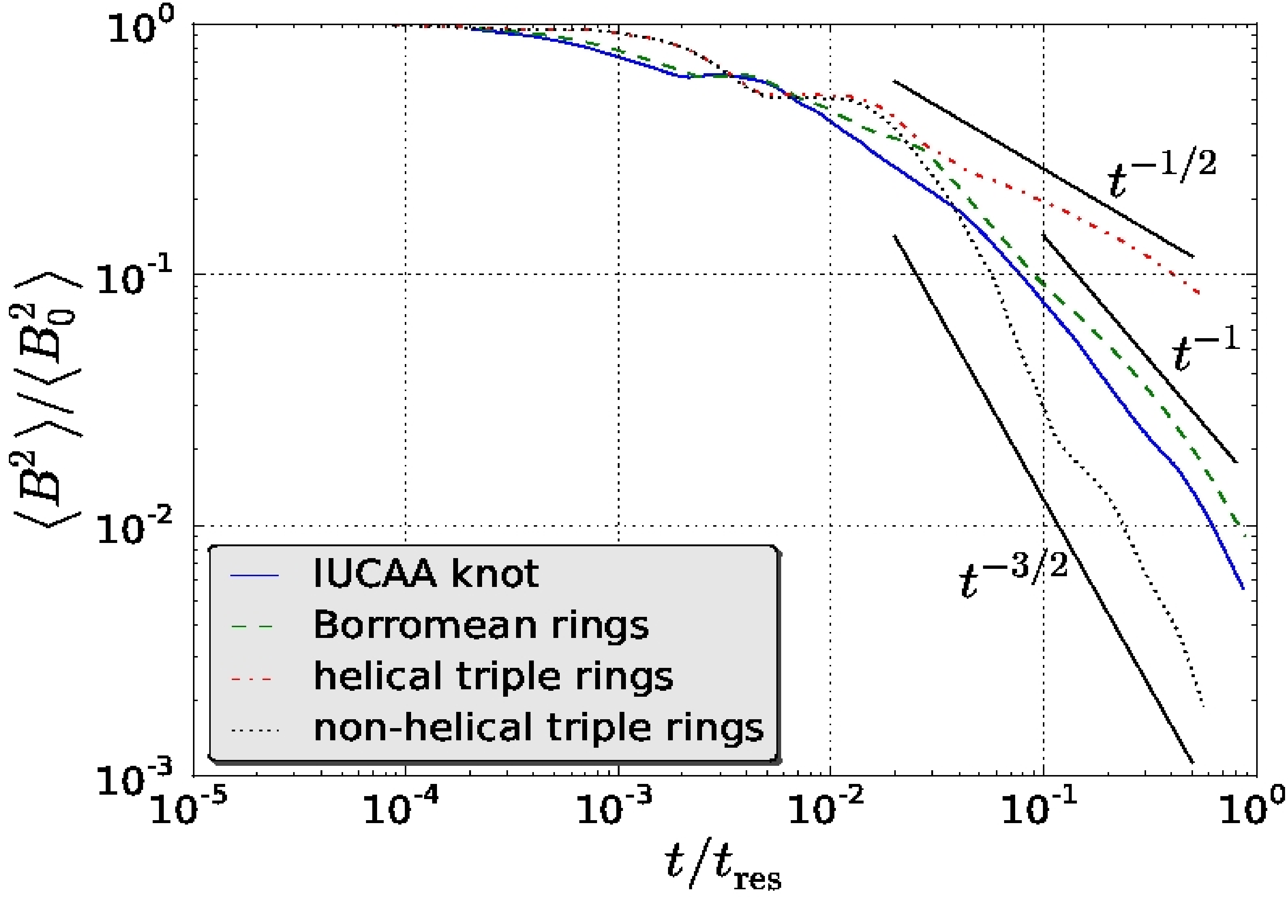
$t = 39$



$t = 78$

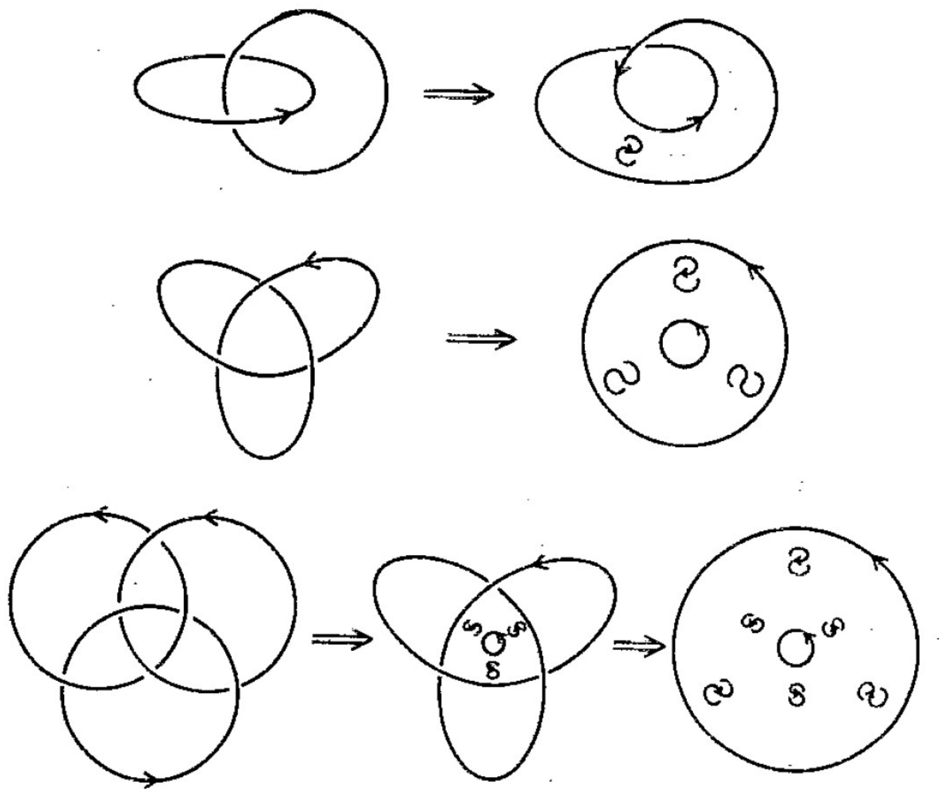
Magnetic field ejection → Isolated helical structures

Decay comparison



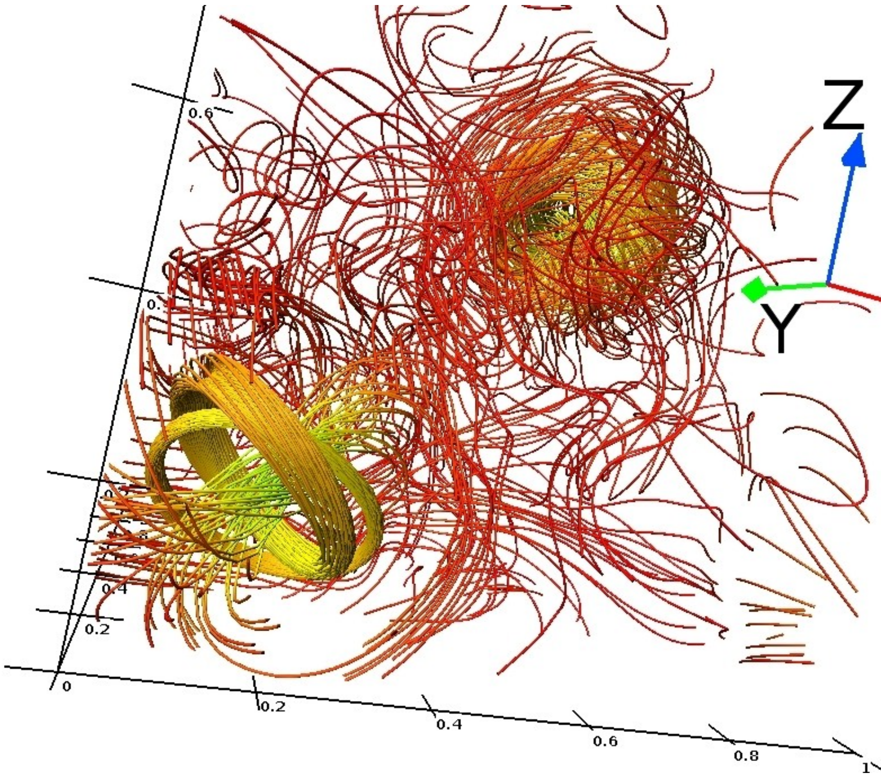
Reconnection characteristics

Conversion of linking into twisting

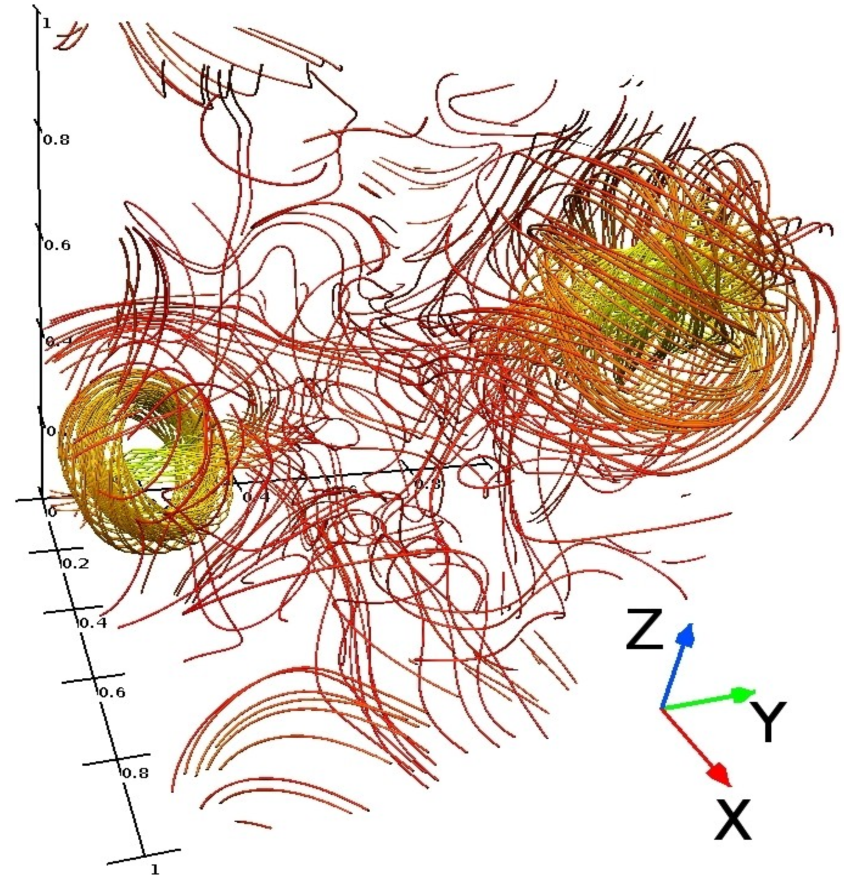


Ruzmaikin and Akhmetiev (1994)

Reconnection characteristics



$t = 70$



$t = 78$

3 rings → Twisted ring + interlocked rings → 2 twisted rings

Conclusions

- Stronger packing for high n_{f} leads to different decay slopes.
- Higher order invariants?

- Non-forced ejection of magnetic field
- Isolated helical structures inhibit energy decay
- Reconsider realizability condition

References

Del Sordo et al. 2010

Fabio Del Sordo, Simon Candelaresi, and Axel Brandenburg.
Magnetic-field decay of three interlocked flux rings with zero linking number.
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Ruzmaikin and Akhmetiev 1994

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Phys. Plasmas, vol. 1, pp. 331–336, 1994.