Coupled Multiphysics Simulations Using Trixi.jl

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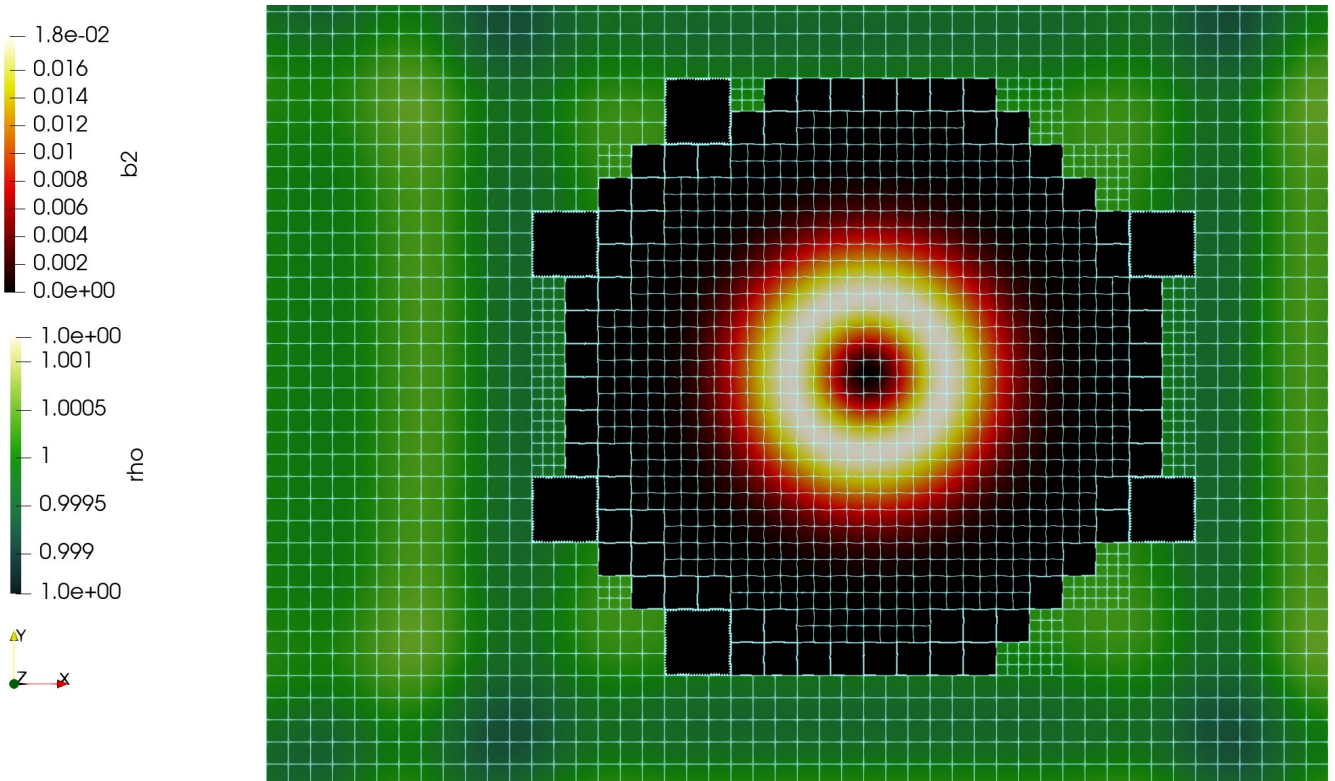
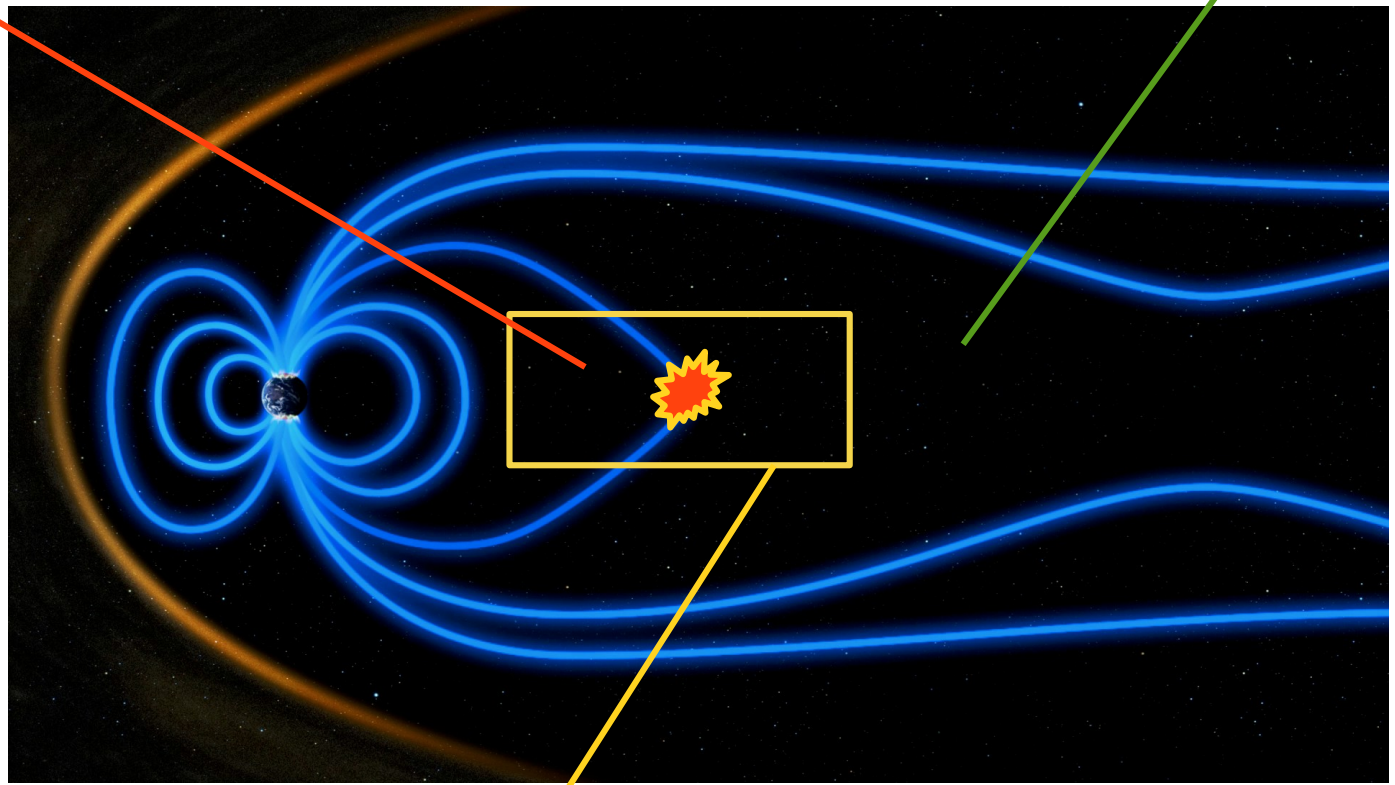


Universität Augsburg University

Modeling

kinetic model
(expensive)

magnetohydrodynamics (MHD)
(cheap)



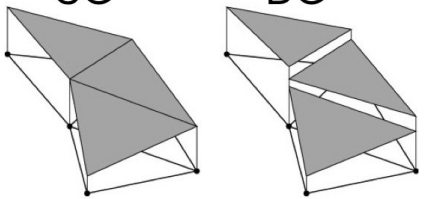
(ESA)

interface coupling

Trixi.jl

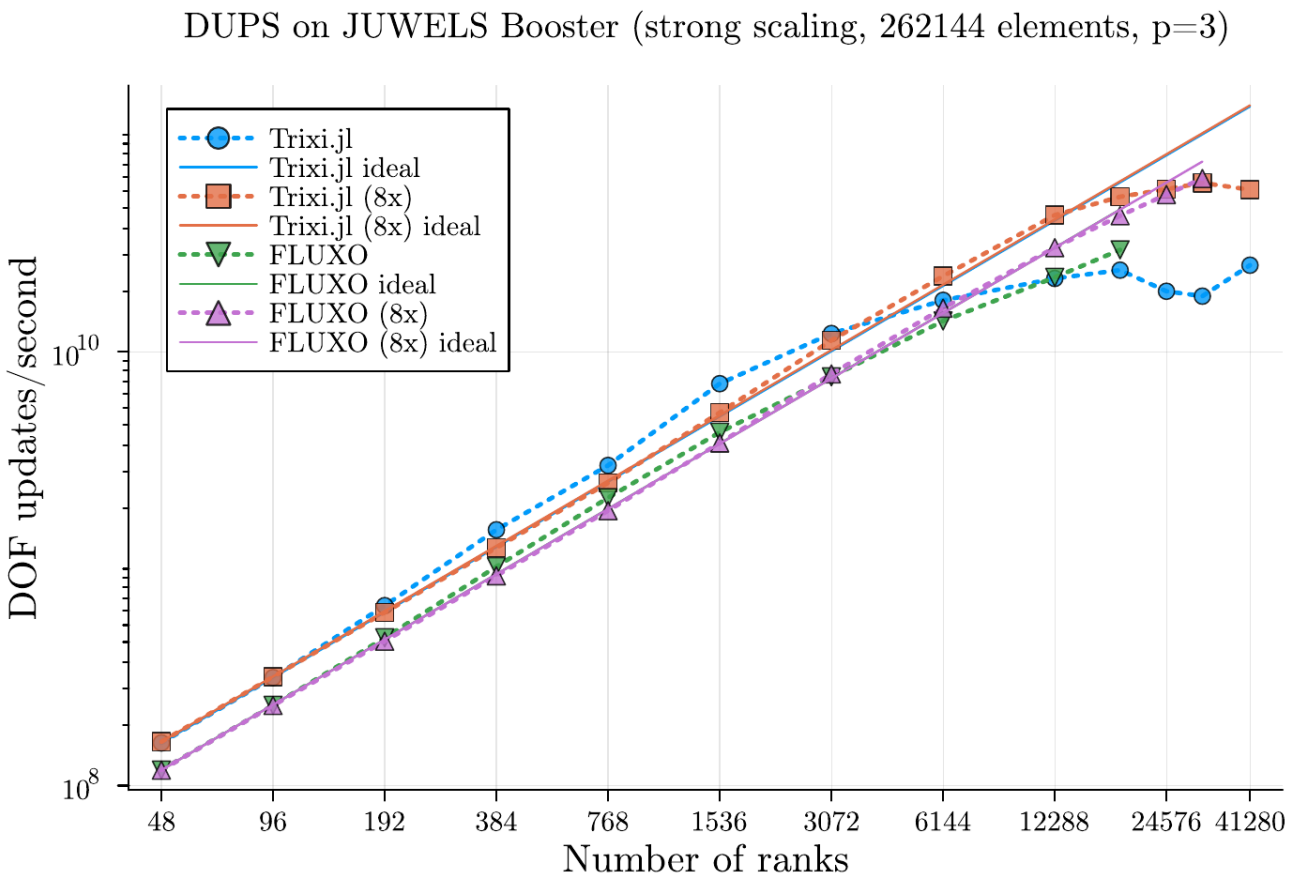
Discontinuous Galerkin

CG DG



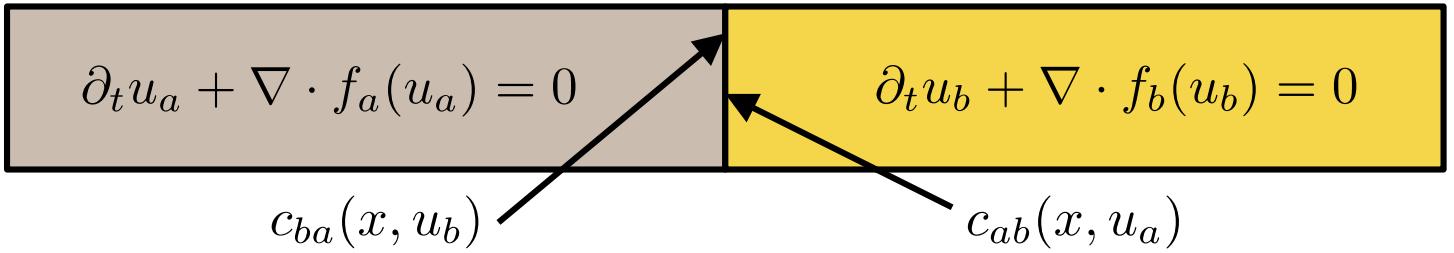
(Stack Overflow)

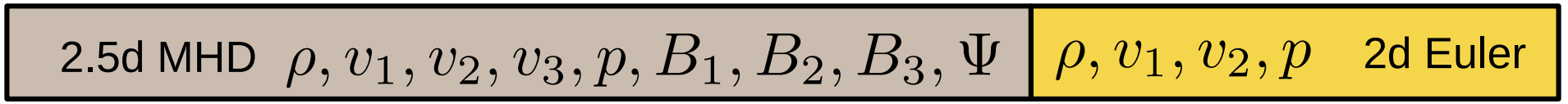
OrdinaryDiffEq (SciML)



Coupling via Converter Functions

Two system with any number of shared variables, including 0:

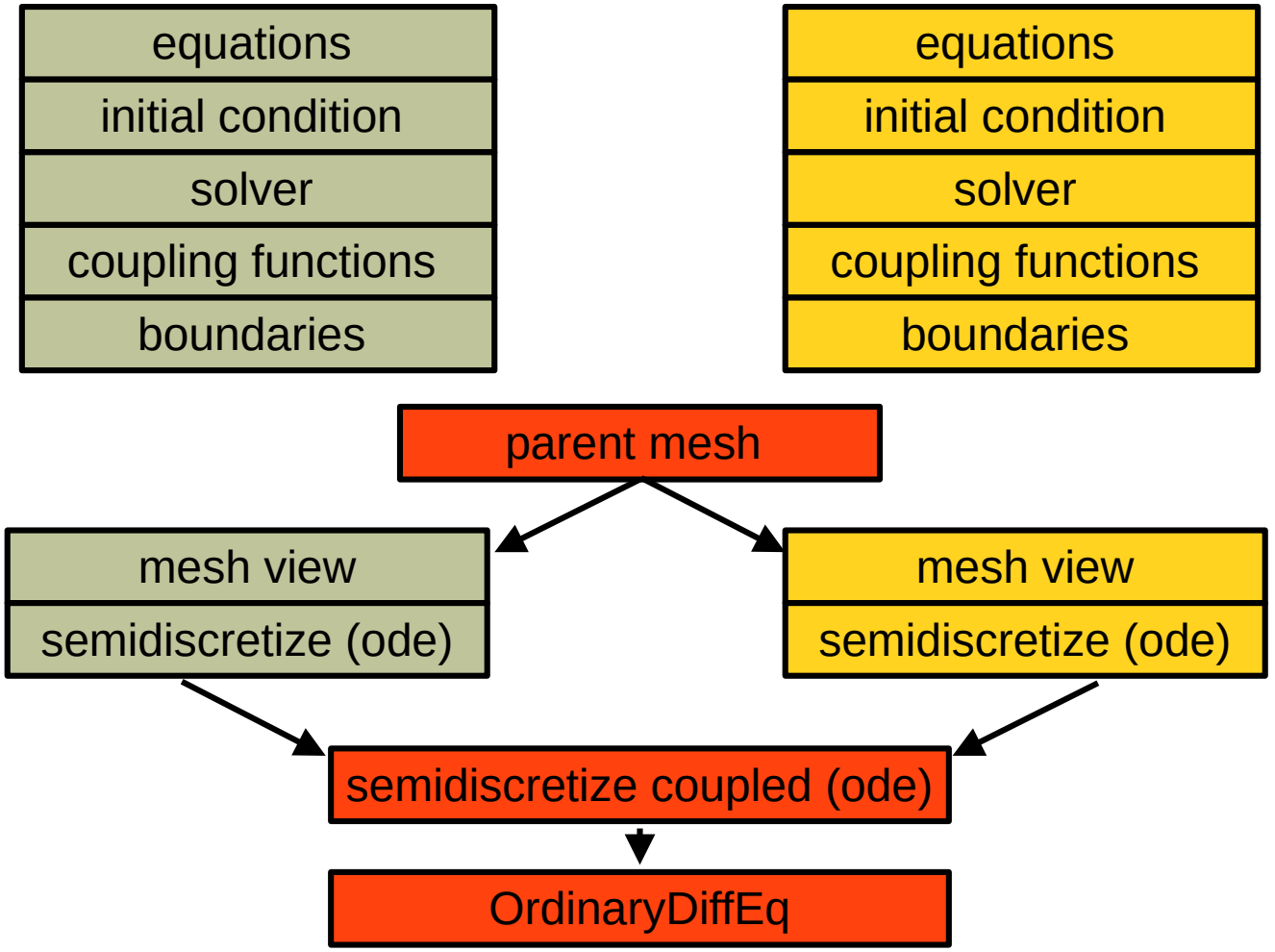




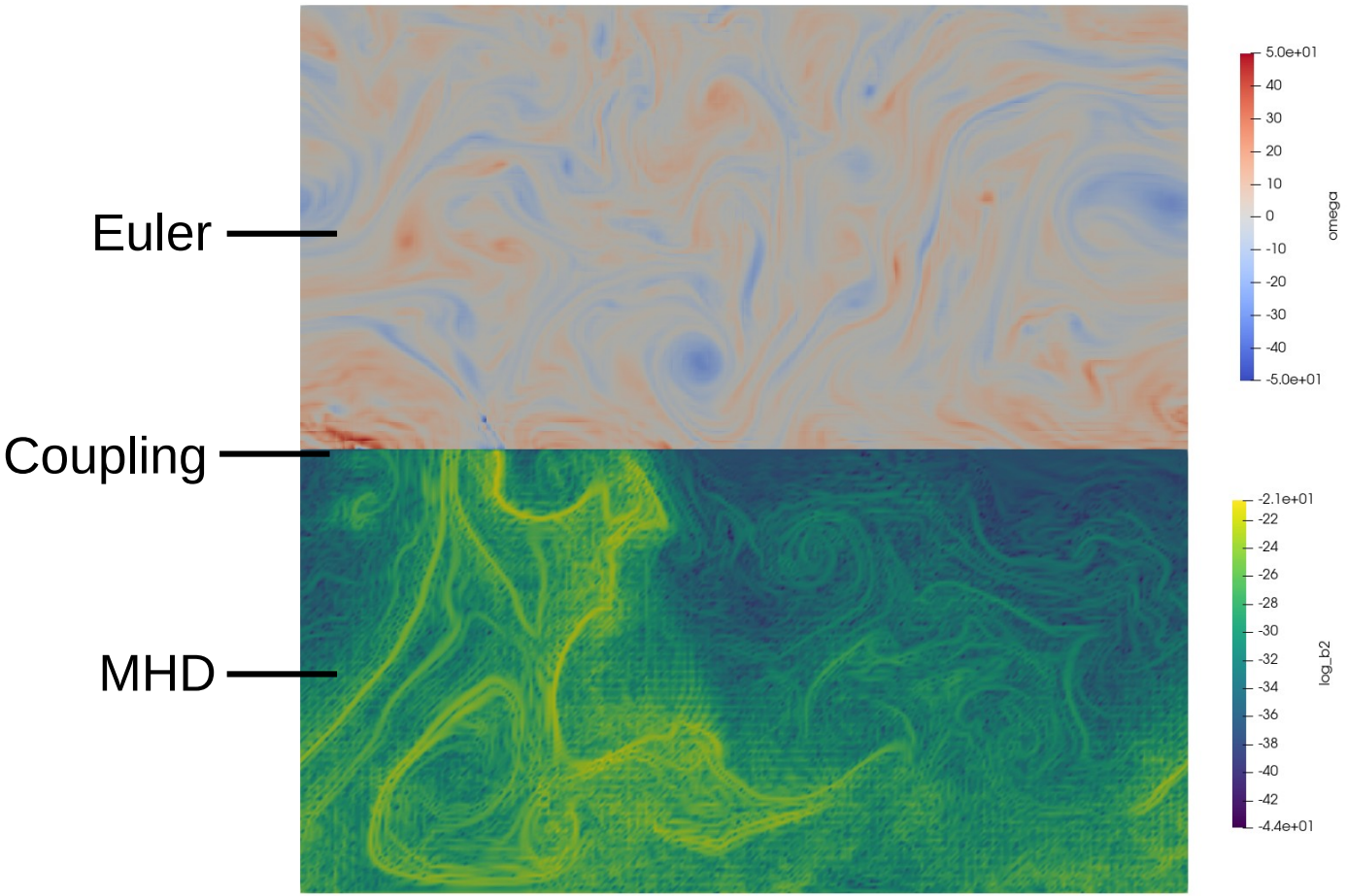

```
coupling_function12 = (x, u, equations_other, equations_own)
                      -> SVector(u[1], u[2], u[3], 0.0, u[4], 0.0, 0.0, 0.0, 0.0)
coupling_function21 = (x, u, equations_other, equations_own) -> SVector(u[1], u[2], u[3], u[5])
```

- User can define converter functions.
- Any pair of systems can be coupled.

Work Flow Coupling

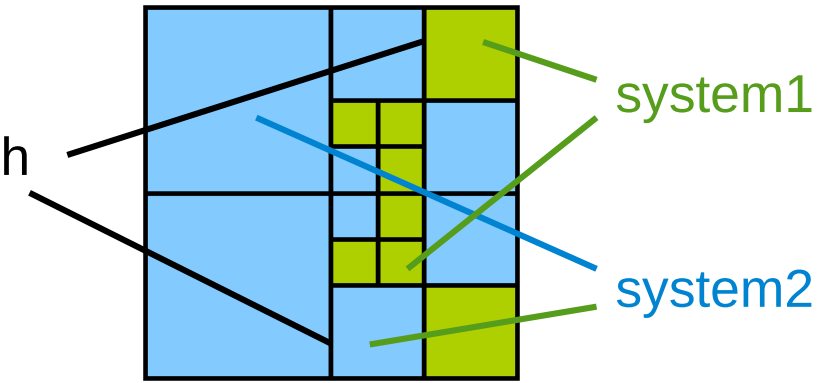


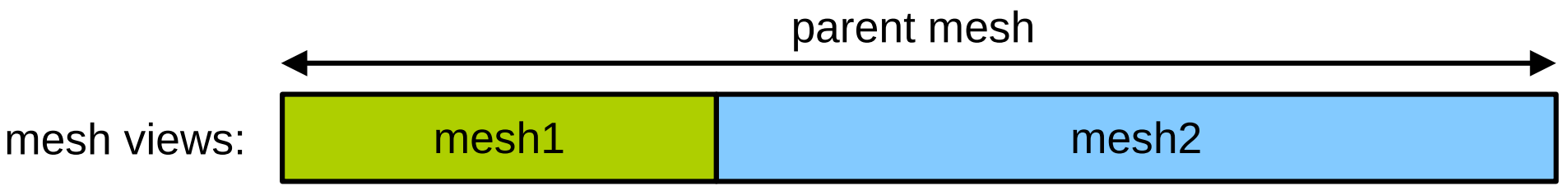
Coupled Dynamo



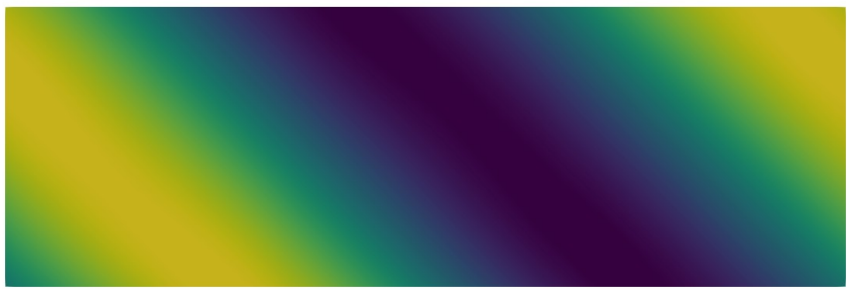
p4est Meshes

We can define the b.c. for each cell boundary.



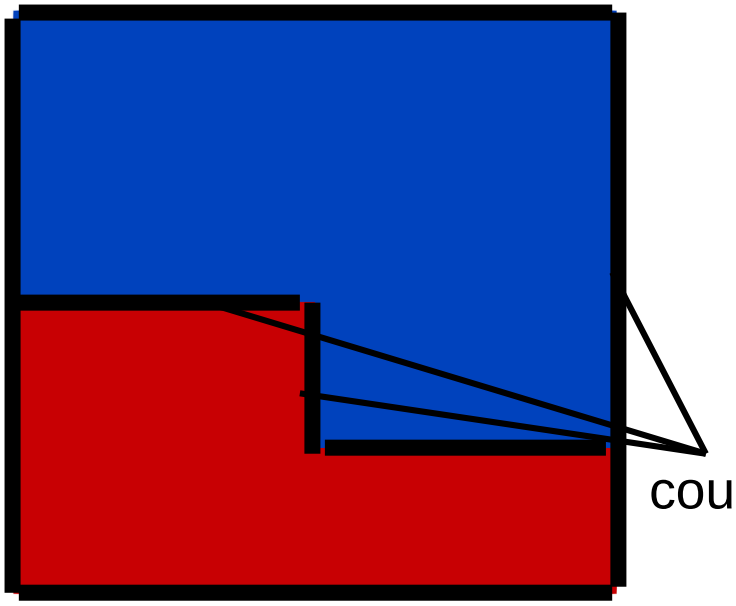


p4est Mesh Views



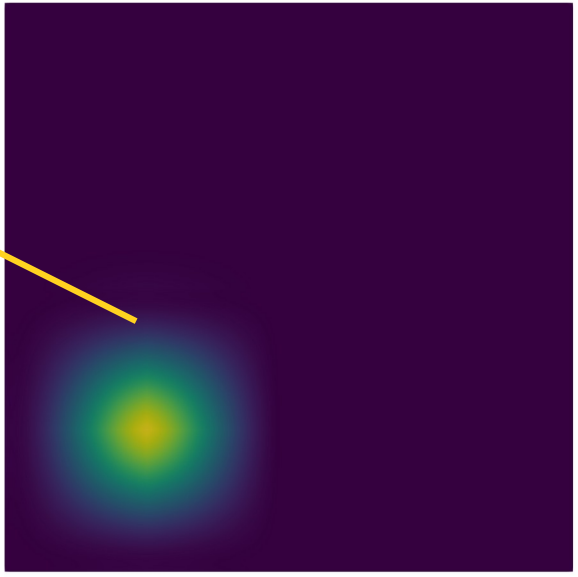
- p4est mesh views in Trixi.jl
- arbitrary topology
- views on smallest (cell) scale

Linear Advection



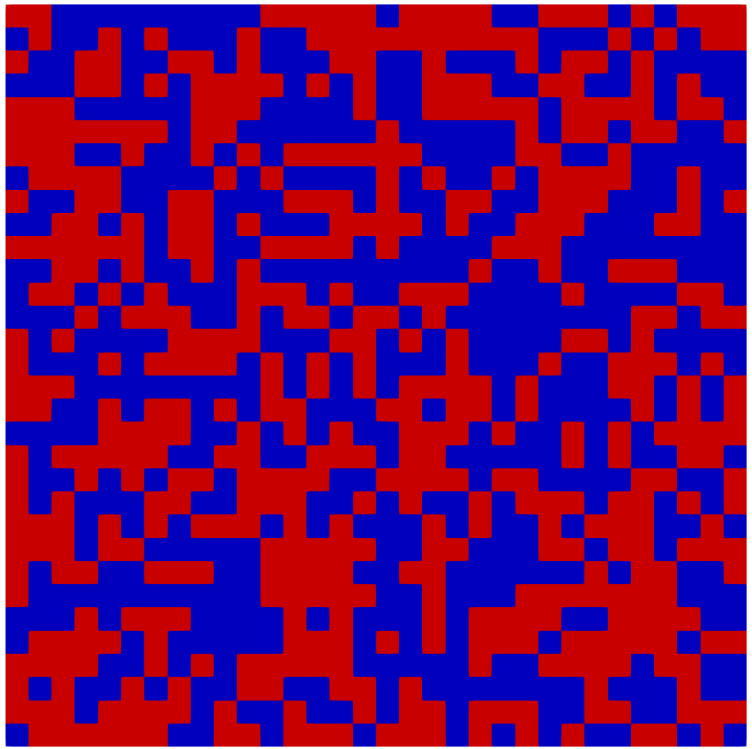
coupled interfaces

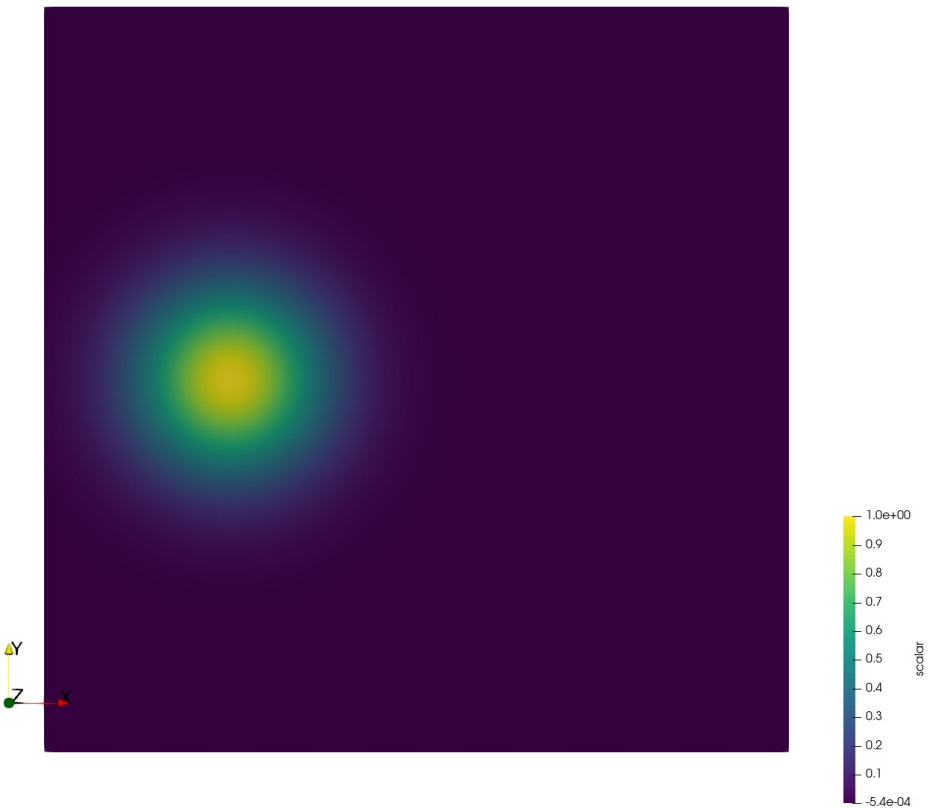
scalar advection



$$\partial_t u + a_x \partial_x u + a_y \partial_y u = 0 \qquad (a_x, a_y) = (0.05, 0.02)$$

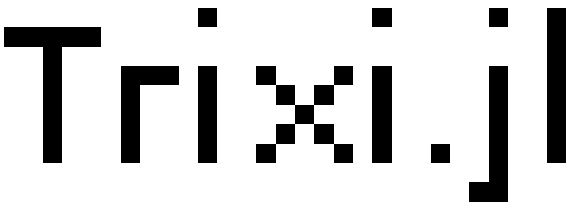
Linear Advection



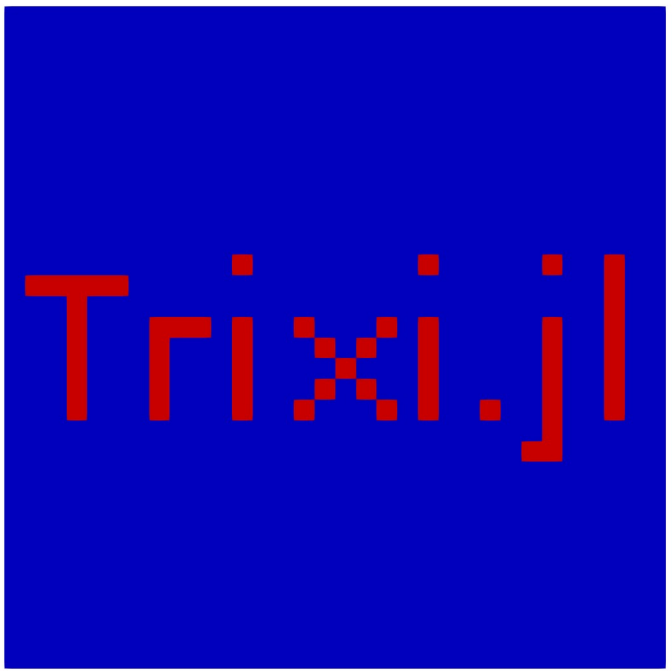


Linear Advection

BMP file:

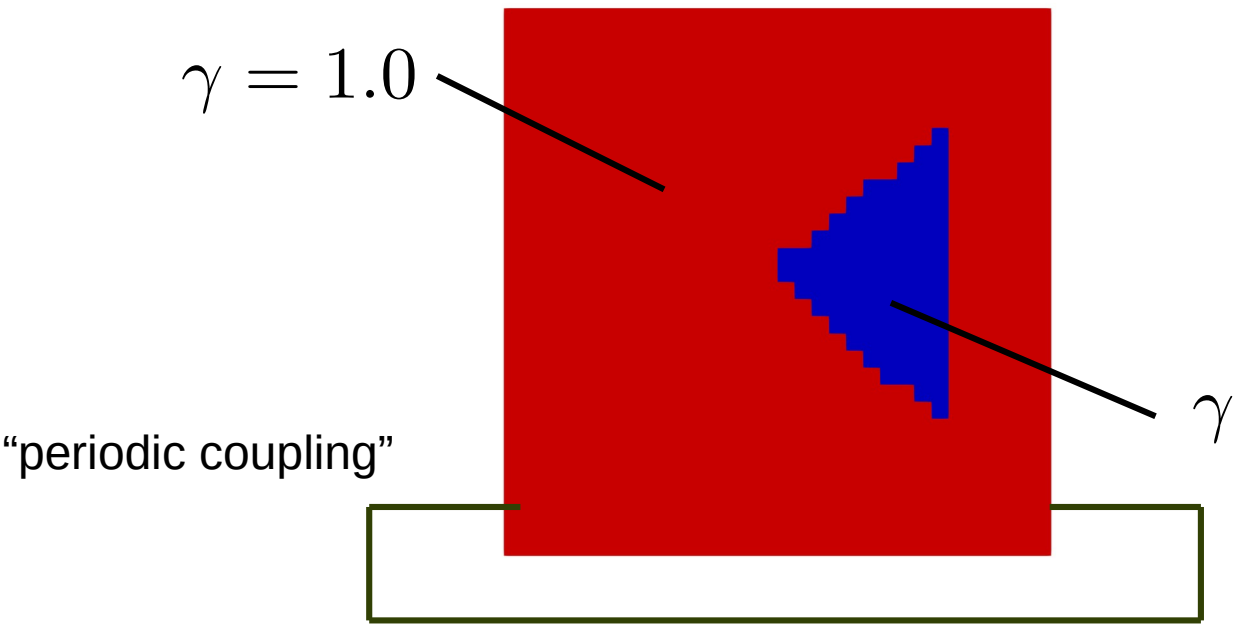


p4est domains:



Isothermal-Polytropic

$$\partial t \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \end{pmatrix} + \partial x \begin{pmatrix} \rho v_1 \\ \rho v_1^2 + \rho^\gamma \\ \rho v_1 v_2 \end{pmatrix} + \partial y \begin{pmatrix} \rho v_2 \\ \rho v_1 v_2 \\ \rho v_2^2 + \rho^\gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

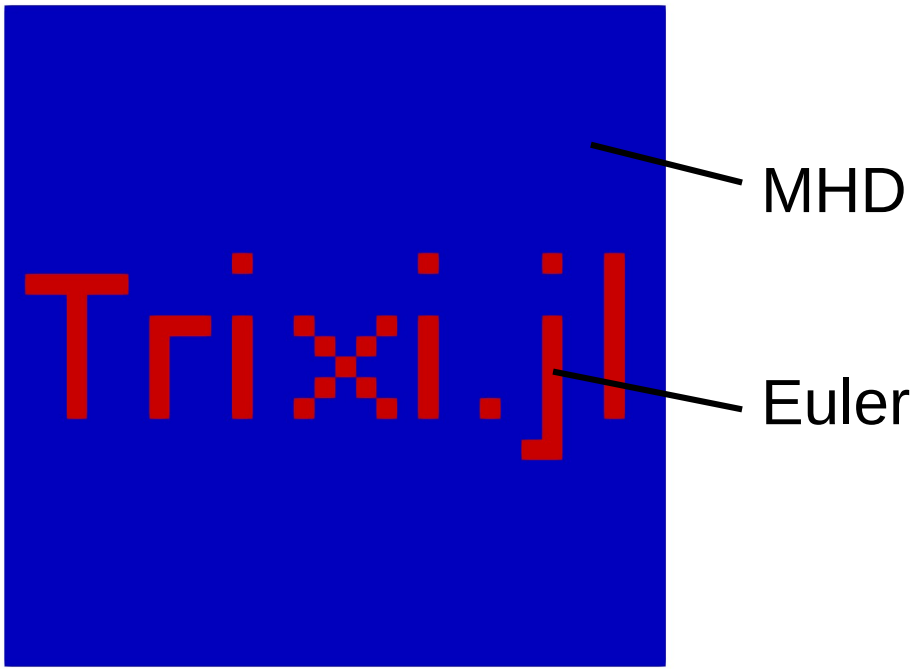


coupling functions:

$$c_{ab} = u^b$$

$$c_{ba} = u^a$$

MHD-Euler

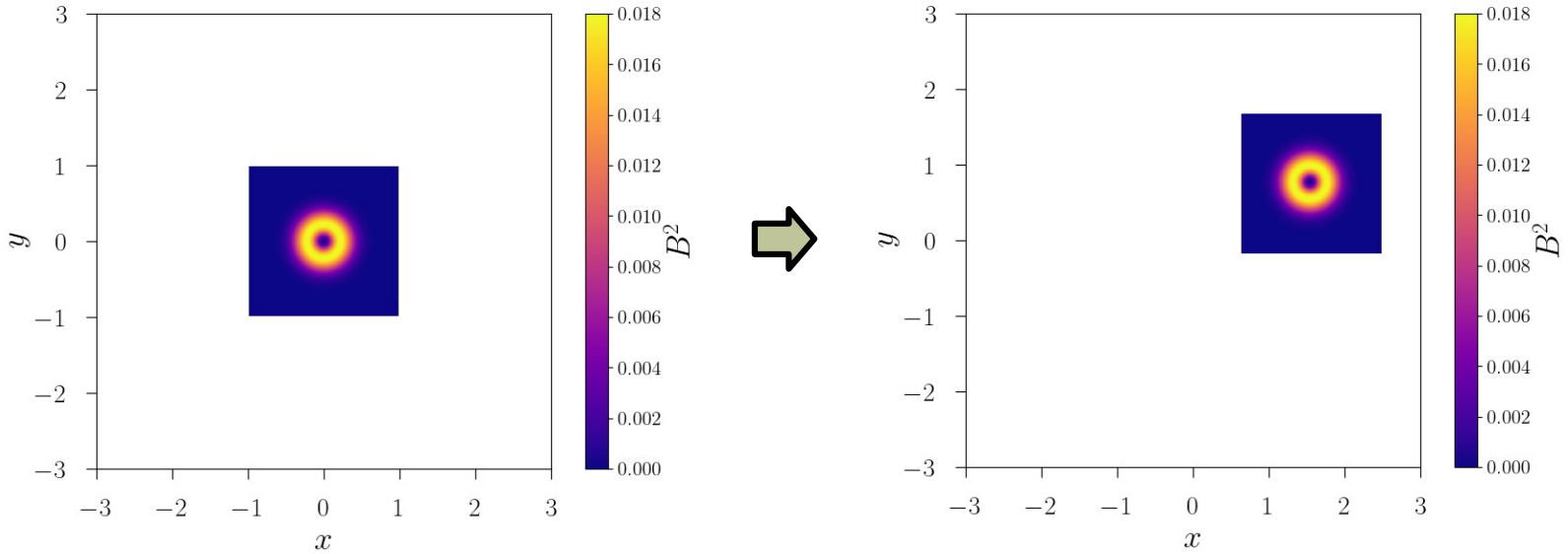

Trixi.jl
MHD
Euler


1.0e+00
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0.992
0.99
9.9e-01
rho

Adaptive Model Selection

1. Generate the new grid (mesh views).
2. Write new u-solution vectors using converter functions.
3. Generate new ODE for OrdinaryDiffEq (integrator).
4. Reinitialize ODE integrator with new problem and new solution vector.

Euler and MHD



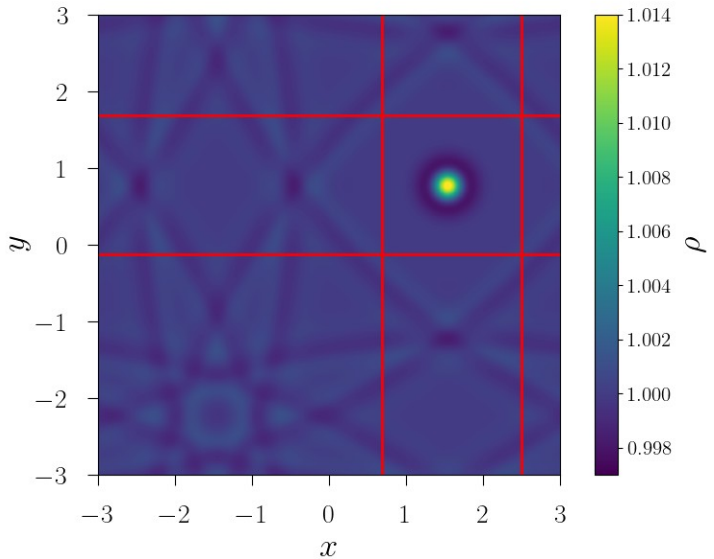
Time spent on coupled boundaries: 0.8%.

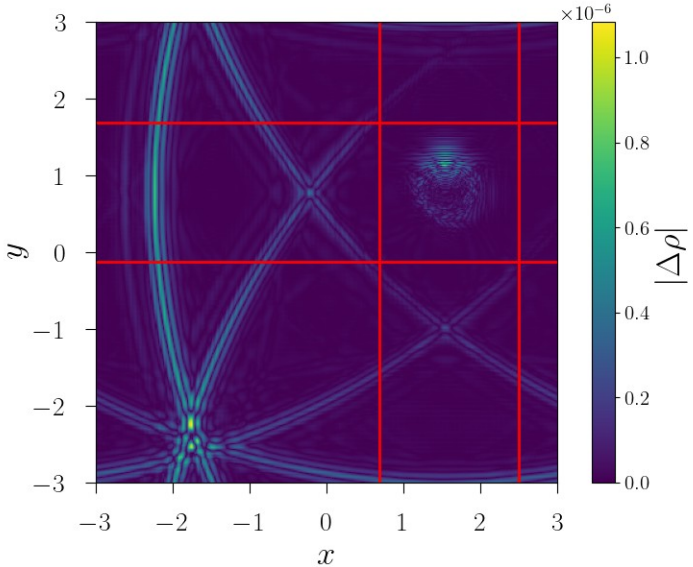
Time spent on model selection: 1.08s per model selection

Timings: coupled → 765.6s
full MHD → 1905.3s

Estimated maximum speed up: 3.31x; here: 2.49x

Density Evolution



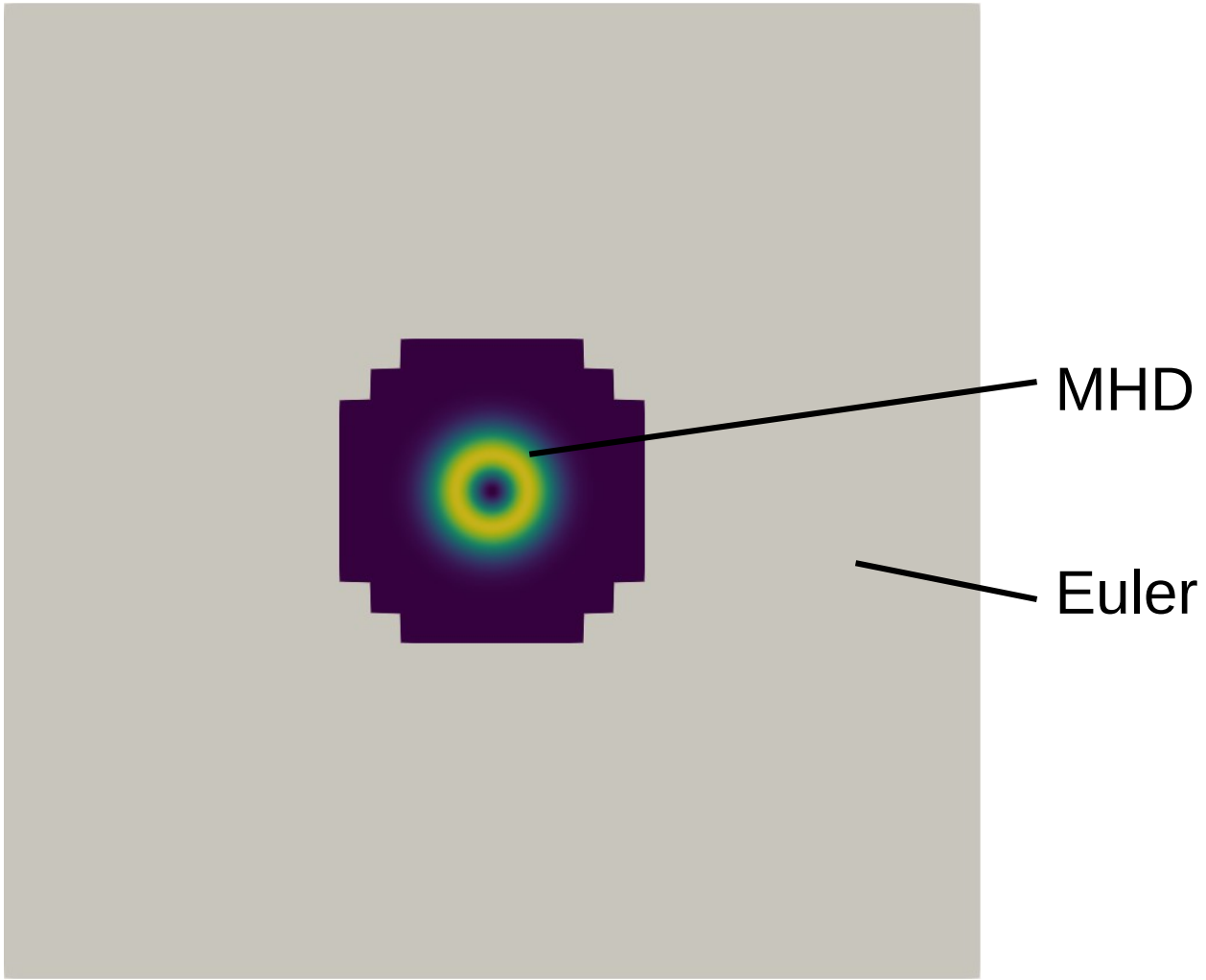


full MHD vs. coupled

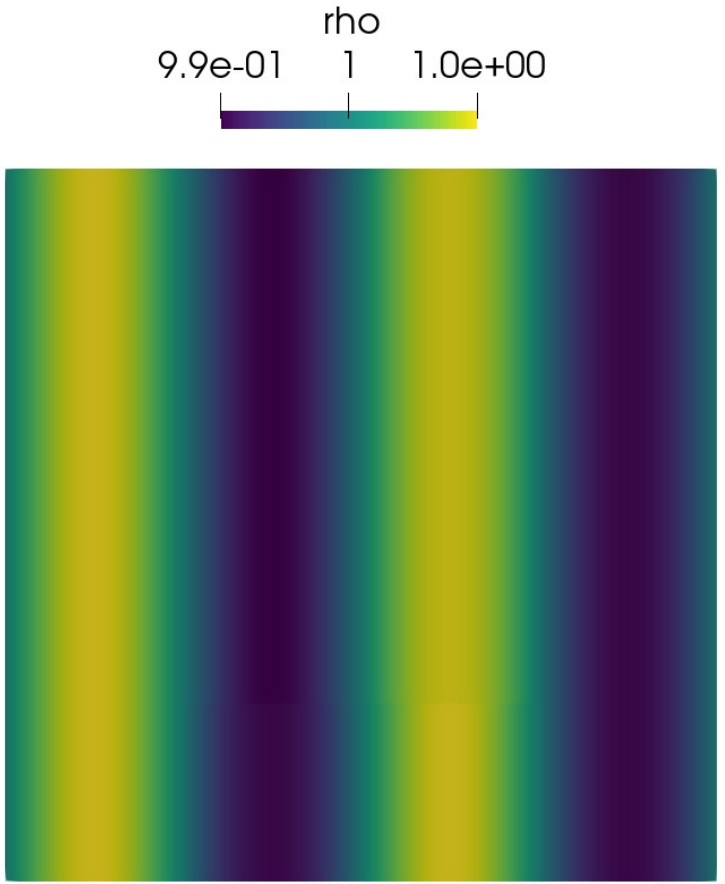
➡ No artificial discontinuities.

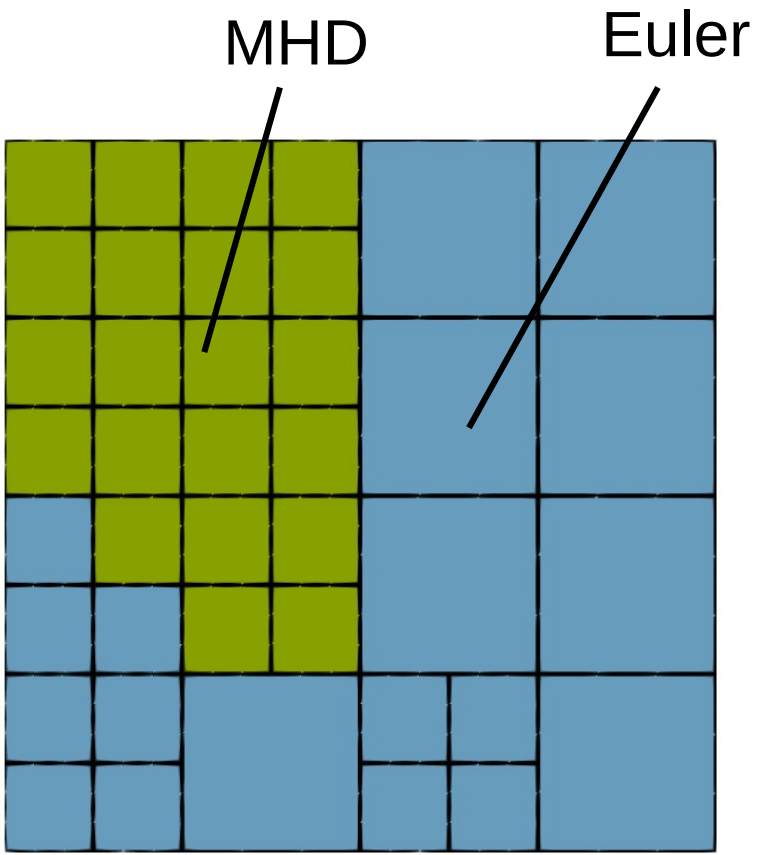
➡ Small differences to full MHD simulation.

MHD-Euler AMS

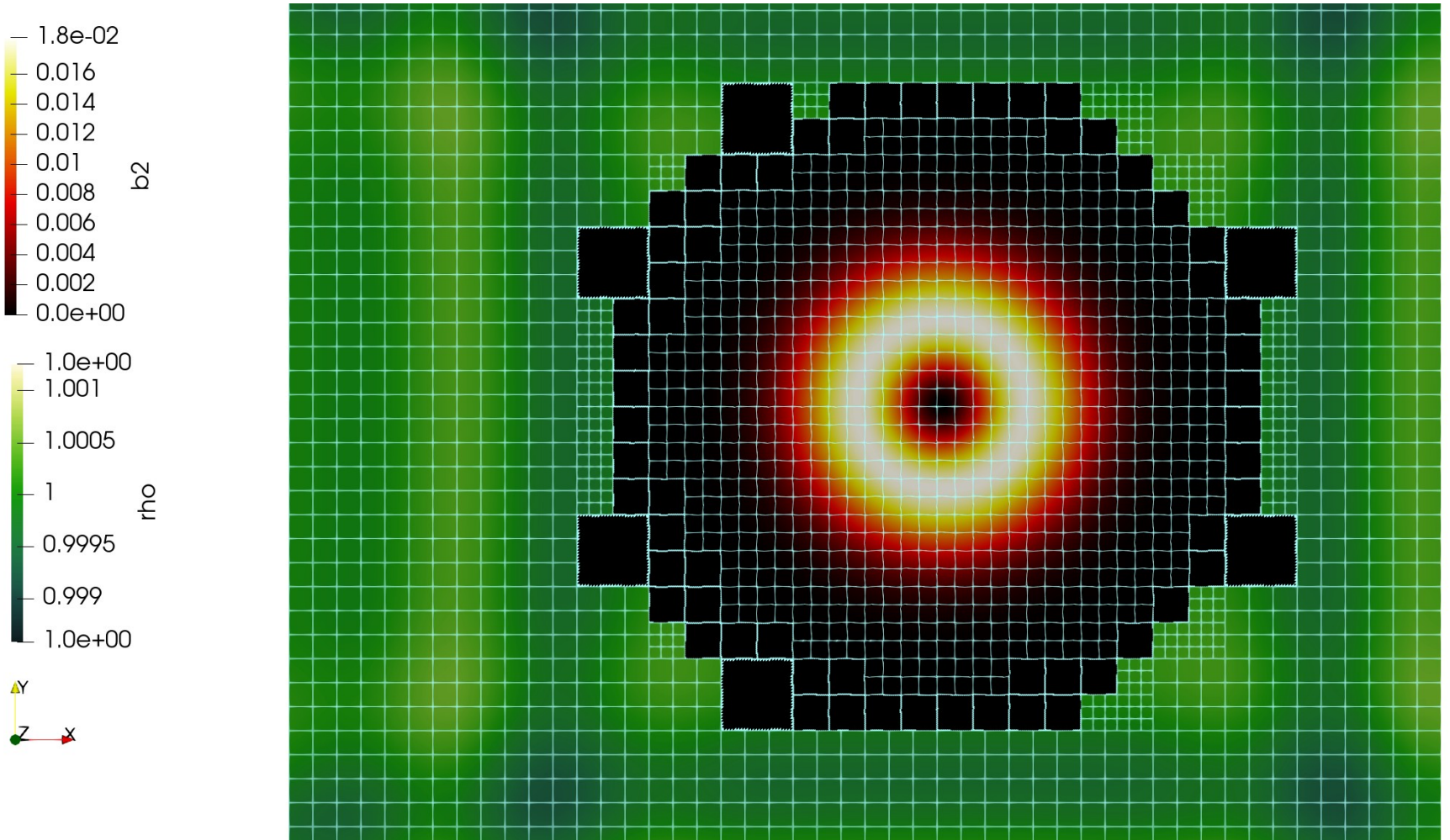


Hanging Nodes





AMS + AMR


1.8e-02
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0.0e+00
b2
1.0e+00
1.001
1.0005
1
0.9995
0.999
1.0e+00
rho
Y
Z
X

Convergence

l2

scalar (semi 1)		scalar (semi 2)	
error	EOC	error	EOC
1.33e-04	-	1.33e-04	-
7.82e-06	4.09	7.82e-06	4.09
4.93e-07	3.99	4.93e-07	3.99
mean	4.04	mean	4.04

Conclusion

- Free domain definitions.
- Import meshes from an image.
- Coupled p4est simulations.
- Non-simply connected meshes in p4est.
- Adaptive model selection with arbitrary criteria.
- Adaptive model selection with AMR

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