Magnetic helicity transport in the advective gauge family

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- Advecto-resistive gauge
- Gauge transformation (Λ method)
- Instability and its nature
- Helicity transport

Advective gauge

induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

uncurl

resistive gauge

$$\frac{\partial \boldsymbol{A}^{\mathrm{r}}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A}^{\mathrm{r}}$$

advecto-resistive gauge

$$\frac{\partial \boldsymbol{A}^{\mathrm{ar}}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J} - \boldsymbol{\nabla}(\boldsymbol{U} \cdot \boldsymbol{A}^{\mathrm{ar}} - \eta \boldsymbol{\nabla} \cdot \boldsymbol{A}^{\mathrm{ar}})$$

Instability

MHD equations

$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot\boldsymbol{U}$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{s}}^2\boldsymbol{\nabla}\ln\rho + \frac{c_{\mathrm{L}}}{\rho}\boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{F}_{\mathrm{visc}} + \boldsymbol{f}$$

advective derivative: $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U}\cdot\boldsymbol{\nabla}$

But: Advecto-resistive gauge is numerically unstable.

Λ method

- Work in the resistive gauge
- Make a gauge transformation
- Evolve also the gauge field

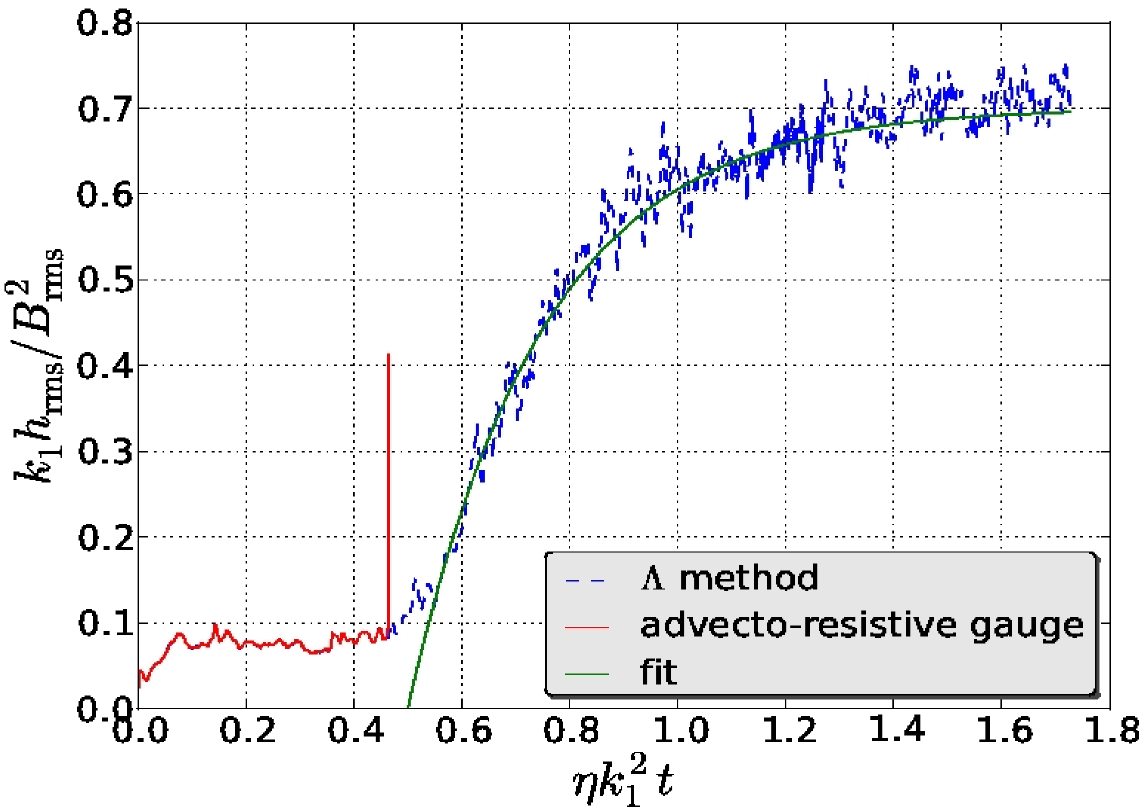
~~$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$~~

resistive gauge $$\frac{\partial \boldsymbol{A}^{\mathrm{r}}}{\partial t} = \boldsymbol{U}\times\boldsymbol{B} + \eta\nabla^2\boldsymbol{A}^{\mathrm{r}}$$

gauge transformation $$\boldsymbol{A}^{\mathrm{ar}} = \boldsymbol{A}^{\mathrm{r}} + \boldsymbol{\nabla}\Lambda^{\mathrm{r:ar}}$$

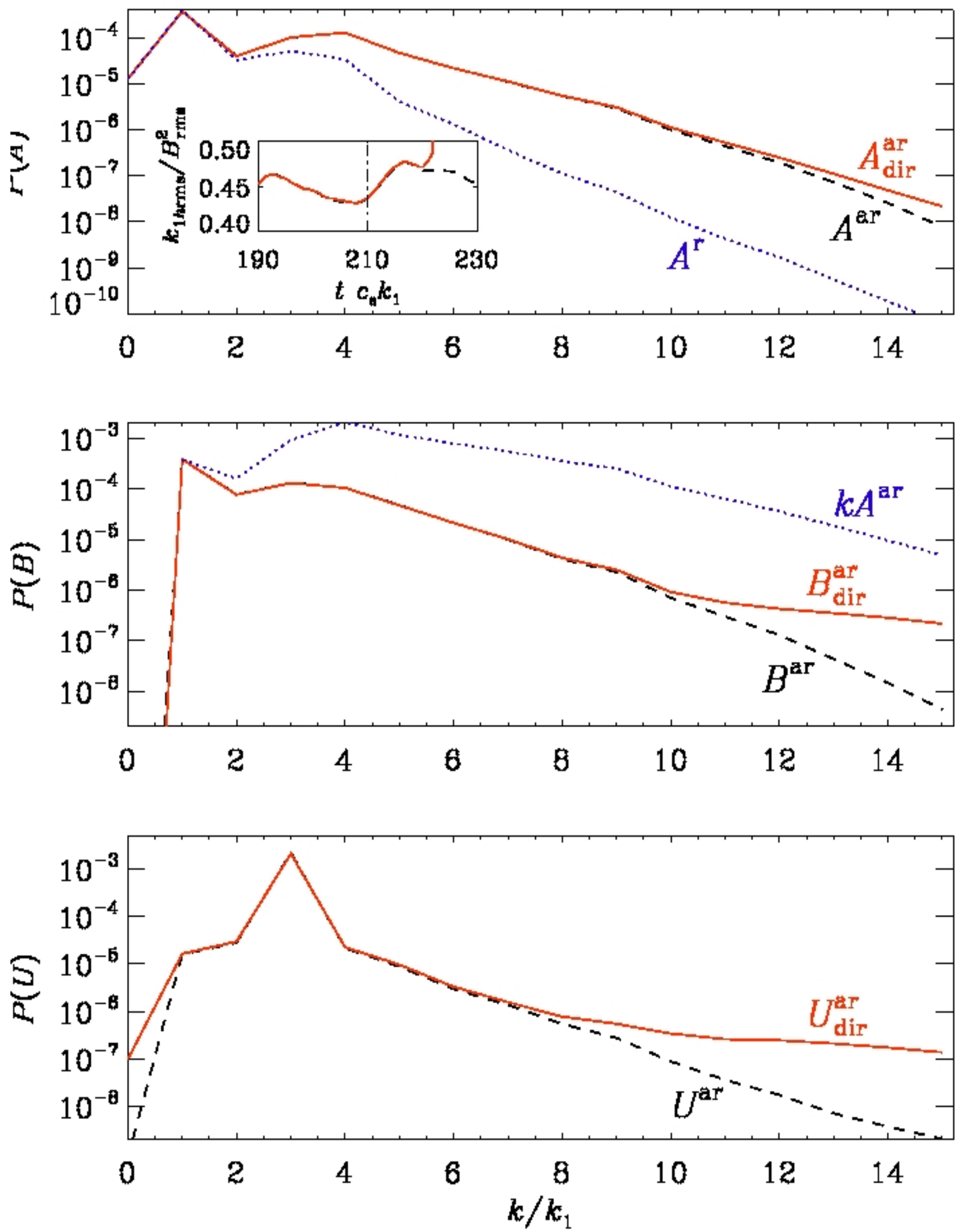
evolve Λ $$\frac{\mathrm{D}\Lambda^{\mathrm{r:ar}}}{\mathrm{D}t} = -\boldsymbol{U}\cdot\boldsymbol{A}^{\mathrm{r}} + \eta\nabla^2\Lambda^{\mathrm{r:ar}}$$

Λ method vs. direct gauge



Normalized magnetic helicity versus time. The direct method becomes unstable already in the kinematic regime while the Λ method is inherently stable.

Nature of the instability



$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\Lambda)$$

irrotational contributions to B and J

Lorentz force increases

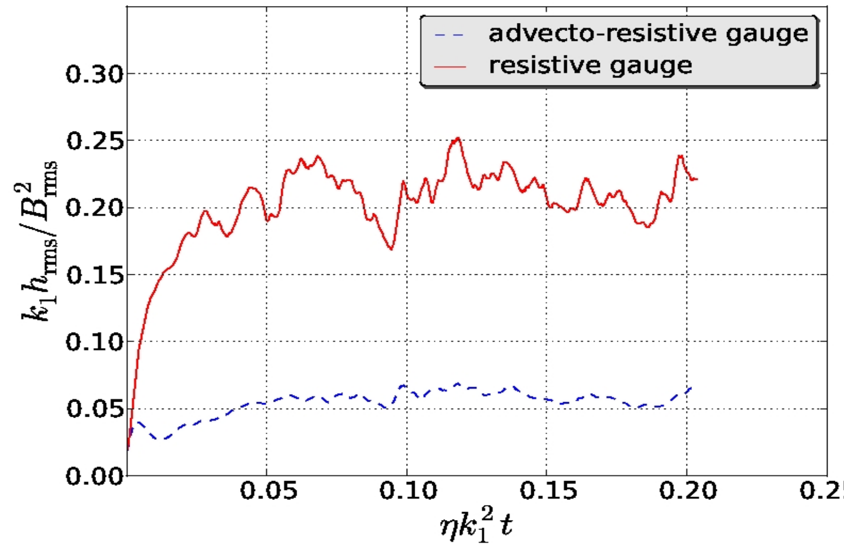
velocity increases

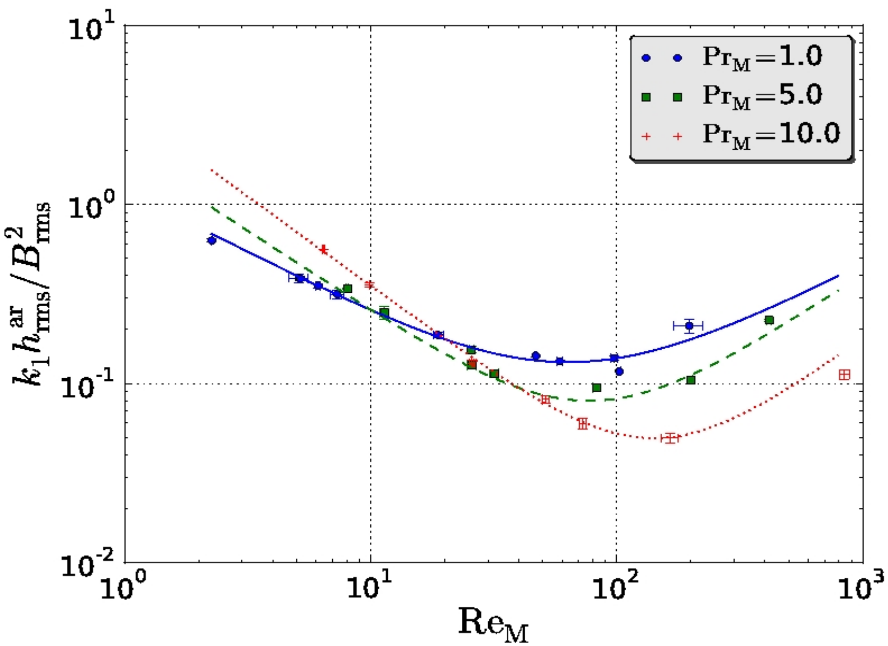
crash

Kinematic regime

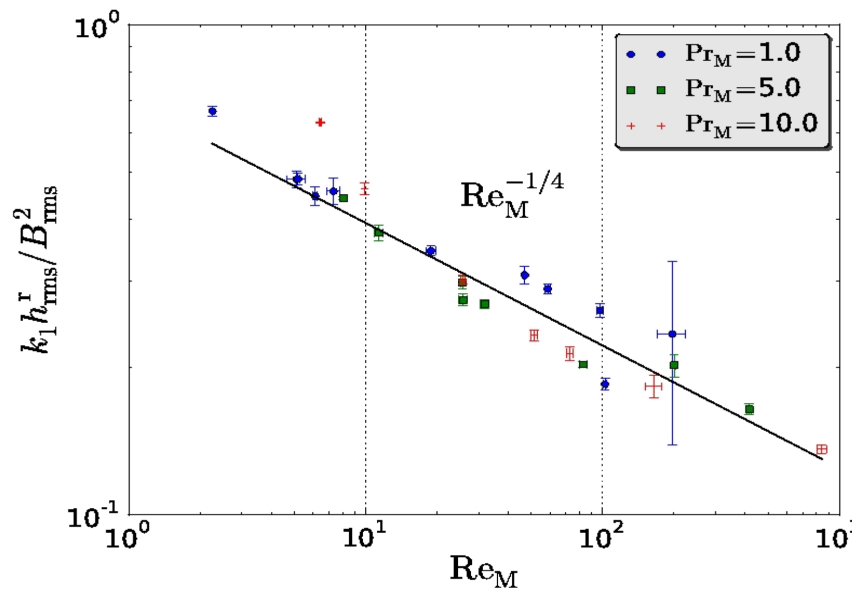
Different spatial fluctuations for h^r and h^ar

In the advecto-resistive gauge helicity transport becomes important for high Re

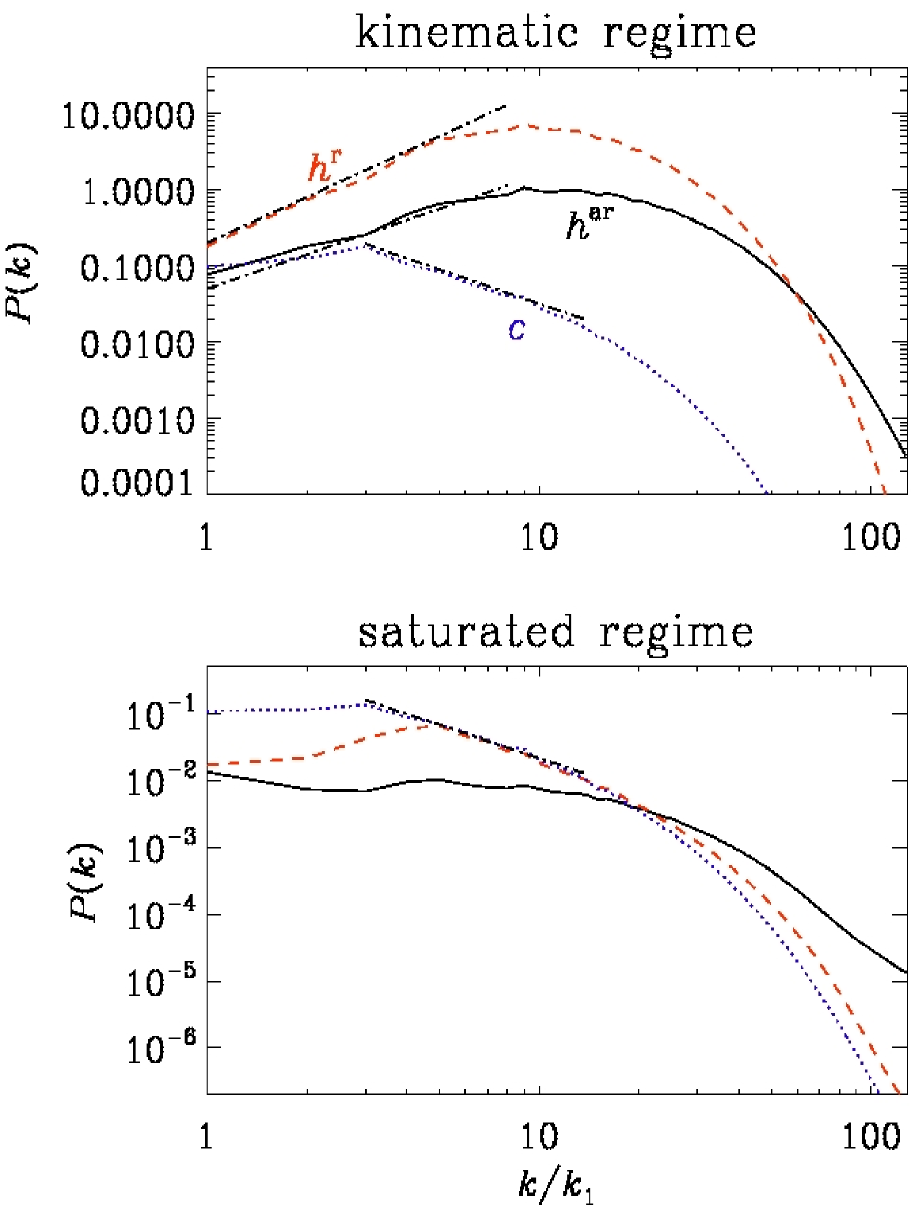




$$\frac{k_1 h^{\rm ar}}{B^2_{\rm rms}} = c\mathrm{Re}_{\rm M}^{-a}\left(1 + b\mathrm{Re}_{\rm M}^{2a}\right)$$



Comparison with passive scalar



$$\frac{\partial h^{\mathrm{ar}}}{\partial t} = -2\eta \boldsymbol{J}\cdot\boldsymbol{B} - \boldsymbol{\nabla}\cdot\boldsymbol{F}^{\mathrm{ar}}$$

$$\boldsymbol{F}^{\mathrm{ar}} = h^{\mathrm{ar}}\boldsymbol{U} - \eta(\boldsymbol{\nabla}\cdot\boldsymbol{A}^{\mathrm{ar}})\boldsymbol{B} + \eta\boldsymbol{J}\times\boldsymbol{A}^{\mathrm{ar}}$$

passive scalar: $\frac{\mathrm{D}C}{\mathrm{D}t} = \kappa\nabla^2 C$

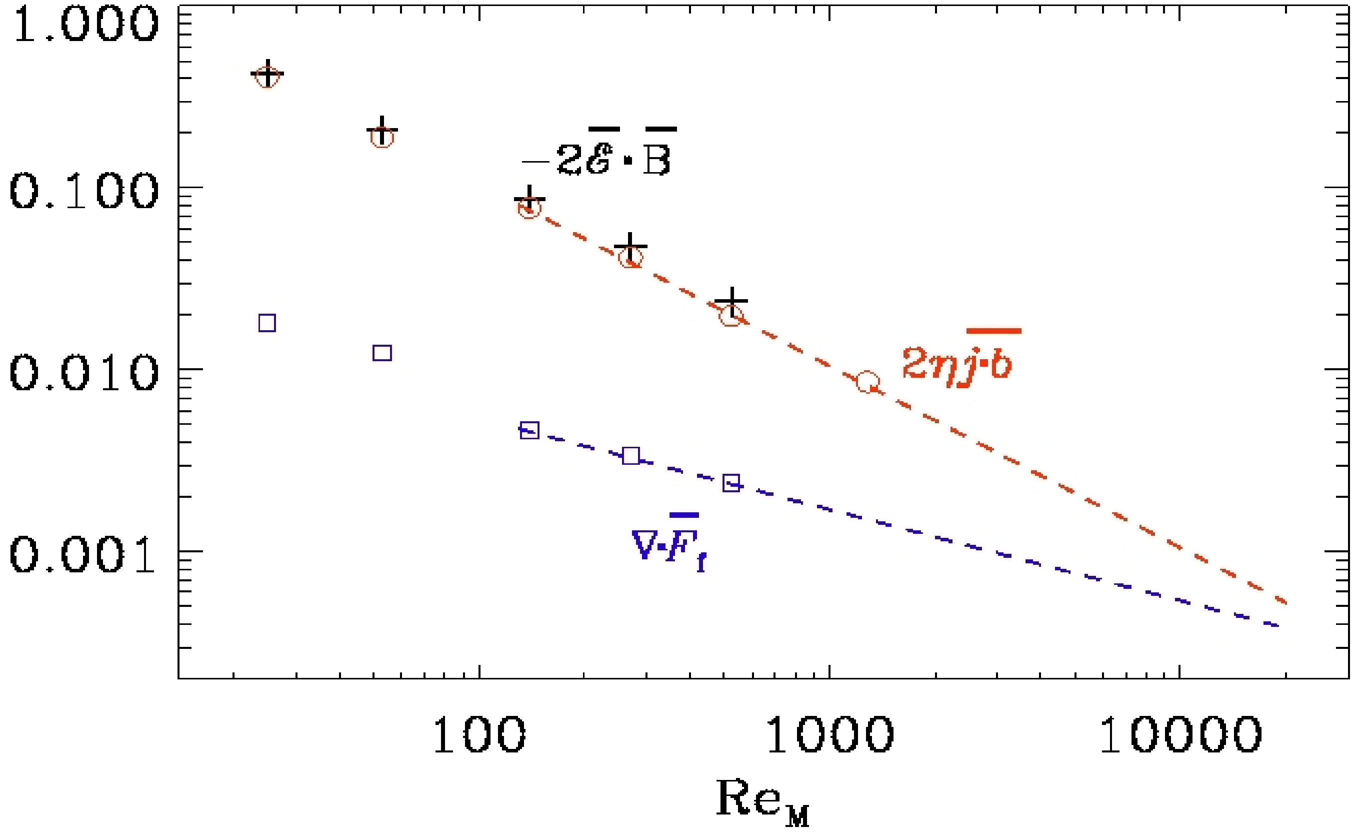
In the kinematic regime h behaves like a passive scalar.

h^ar has strong high-k tail

↓

efficient turbulent cascade in the advecto-resistive gauge

Revisiting earlier works



Conclusions and Outlook

- Advecto-resistive gauge is unstable.
- Λ method can be used universally.
- The advecto-resistive gauge efficiently makes magnetic helicity cascade to higher wave numbers.
- In the ar gauge magnetic helicity behaves like a passive scalar in the kinematic regime and for high Rm.

- Understand the high Rm hook for h^ar better.