Magnetic helicity transport in the advective gauge family

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- Advecto-resistive gauge
- Gauge transformation (Λ method)
- Instability and its nature
- Helicity transport

Magnetic helicity fluxes

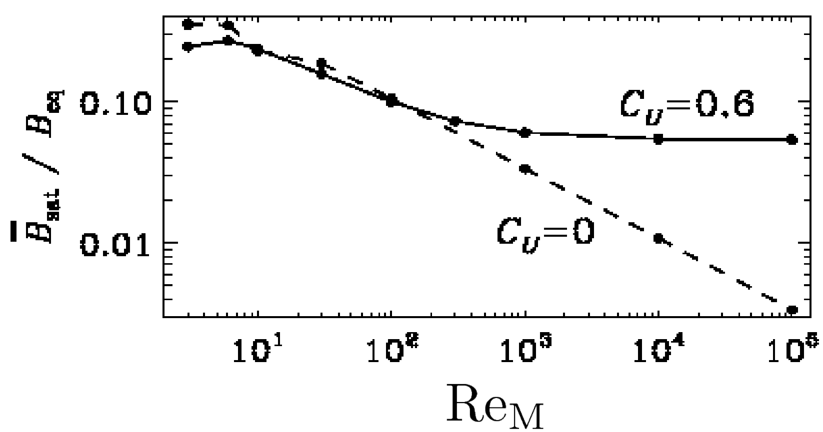
magnetic helicity density: $h_{\rm M} = \mathbf{A}\cdot\mathbf{B}$

dynamo process: $\overline{h}_{\rm K,f} = \overline{\boldsymbol{\omega}\cdot\mathbf{u}}$ → $\overline{h}_{\rm C,f} = \overline{\mathbf{j}\cdot\mathbf{b}}$ → $\overline{h}_{\rm M,f} = \overline{\mathbf{a}\cdot\mathbf{b}}$

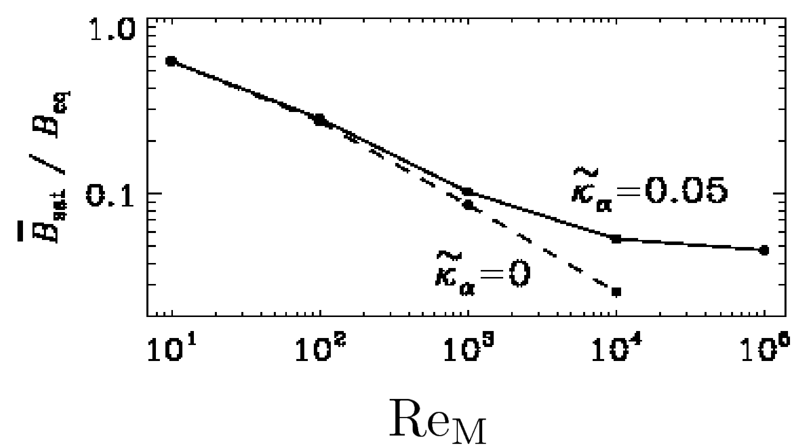
$\overline{\mathbf{a}\cdot\mathbf{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^{9}$ galaxies: ${\rm Re}_{\rm M} = 10^{29}$

advective fluxes



diffusive fluxes



Brandenburg, A. et al., Mon. Not. Roy. Astron. Soc., (2009)

Advective gauge

induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J})$$

uncurl

resistive gauge

$$\frac{\partial \mathbf{A}^{\mathrm{r}}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}^{\mathrm{r}}$$

advecto-resistive gauge

$$\frac{\partial \mathbf{A}^{\mathrm{ar}}}{\partial t} = \mathbf{U} \times \mathbf{B} - \eta \mathbf{J} - \boldsymbol{\nabla}(\mathbf{U} \cdot \mathbf{A}^{\mathrm{ar}} - \eta \boldsymbol{\nabla} \cdot \mathbf{A}^{\mathrm{ar}})$$

- measure helicity transport
- spatial distribution of the magnetic helicity

Instability

MHD equations

$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot\mathbf{U}$$

$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2\boldsymbol{\nabla}\ln\rho + \frac{c_{\mathrm{L}}}{\rho}\mathbf{J}\times\mathbf{B} + \mathbf{F}_{\mathrm{visc}} + \mathbf{f}$$

advective derivative: $\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{U}\cdot\boldsymbol{\nabla}$

But: Advecto-resistive gauge is numerically unstable.

Λ method

- Work in the resistive gauge
- Make a gauge transformation
- Evolve also the gauge field

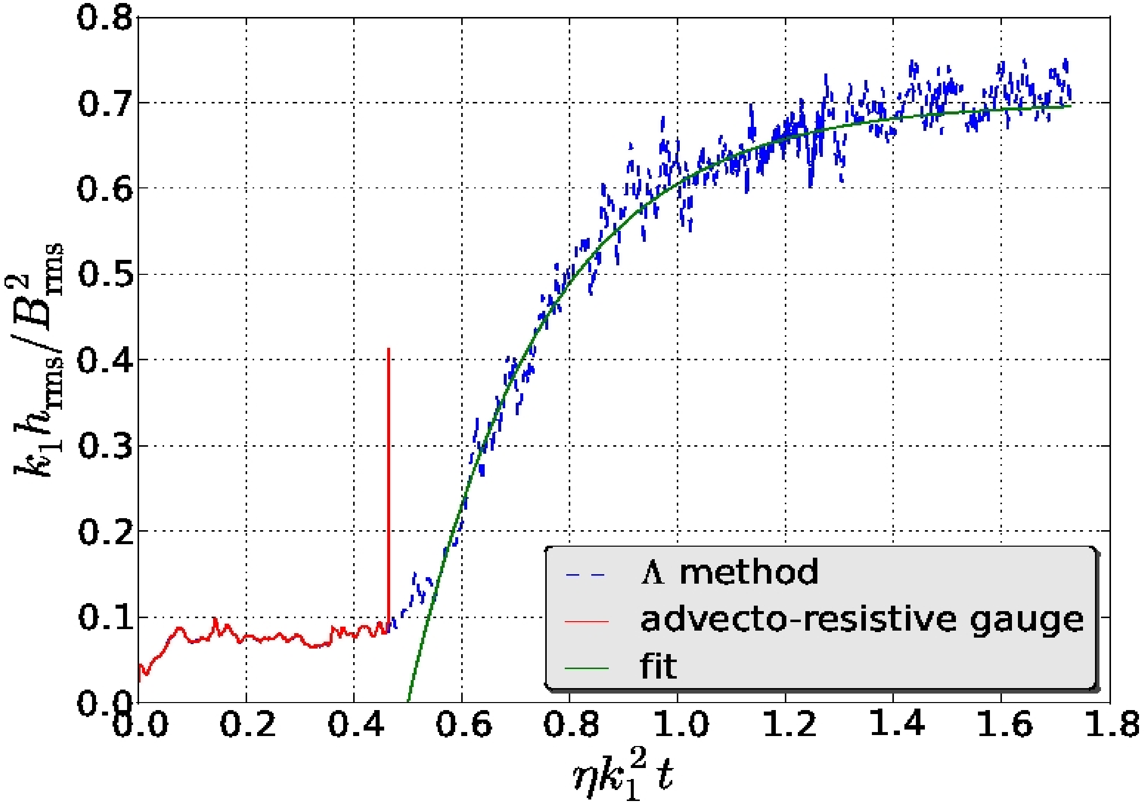
$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$

resistive gauge $$\frac{\partial \mathbf{A}^{\mathrm{r}}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta\nabla^2 \mathbf{A}^{\mathrm{r}}$$

gauge transformation $$\mathbf{A}^{\mathrm{ar}} = \mathbf{A}^{\mathrm{r}} + \boldsymbol{\nabla}\Lambda^{\mathrm{r:ar}}$$

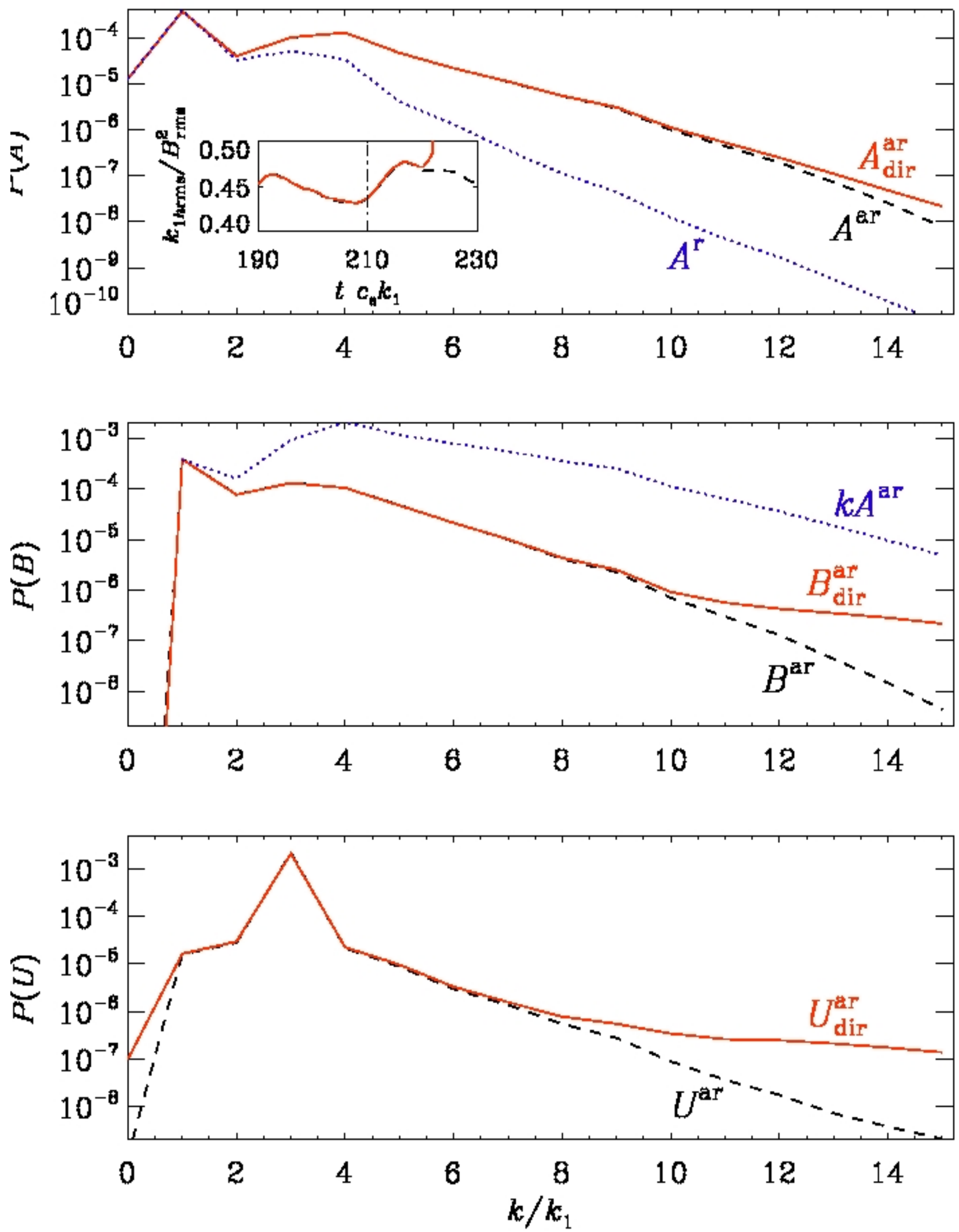
evolve Λ $$\frac{\mathrm{D}\Lambda^{\mathrm{r:ar}}}{\mathrm{D}t} = -\mathbf{U}\cdot\mathbf{A}^{\mathrm{r}} + \eta\nabla^2\Lambda^{\mathrm{r:ar}}$$

Λ method vs. direct gauge



Normalized magnetic helicity versus time. The direct method becomes unstable already in the kinematic regime while the Λ method is inherently stable.

Nature of the instability



$$\frac{\mathrm{D}A_i^{\mathrm{ar}}}{\mathrm{D}t} = -U_{j,i}A_j^{\mathrm{ar}} + \eta\nabla^2 A_i^{\mathrm{ar}}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla}\Lambda)$$

irrotational contributions to B and J

Lorentz force increases

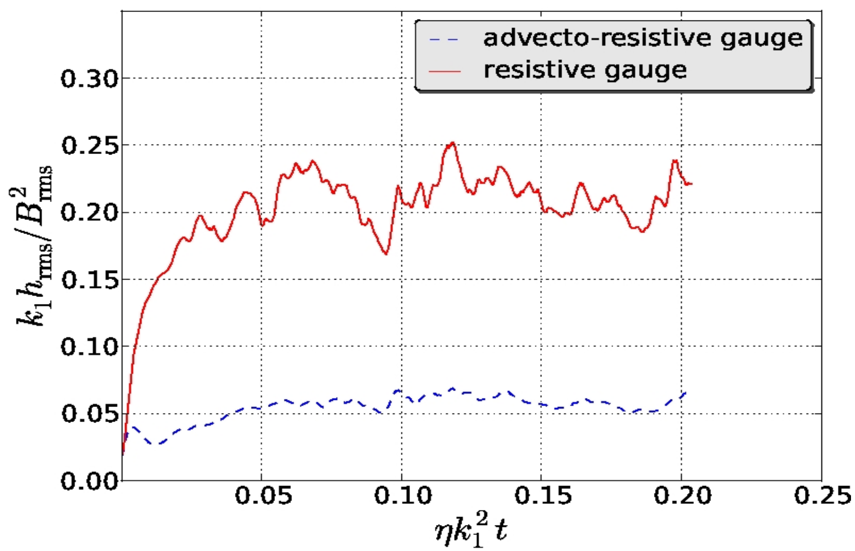
velocity increases

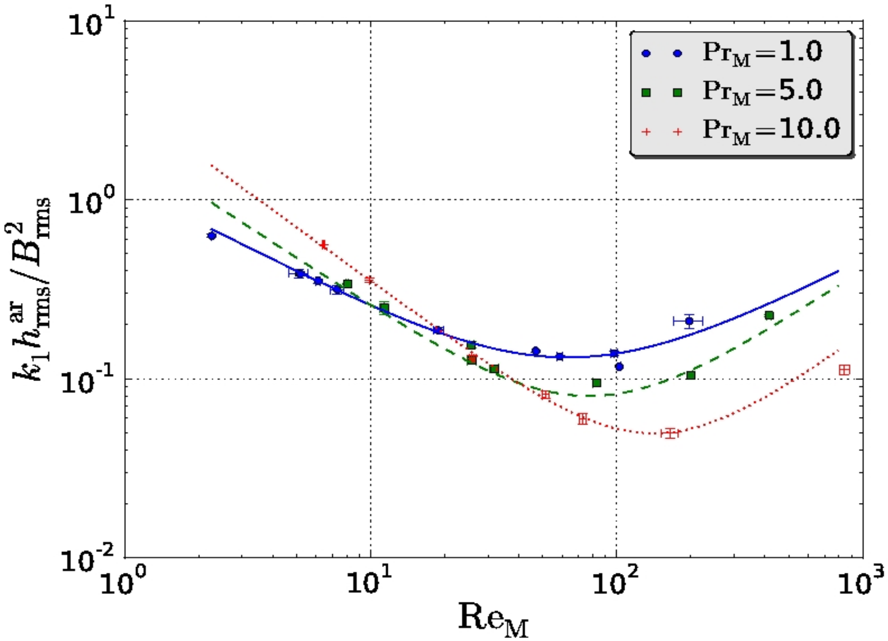
crash

Kinematic regime

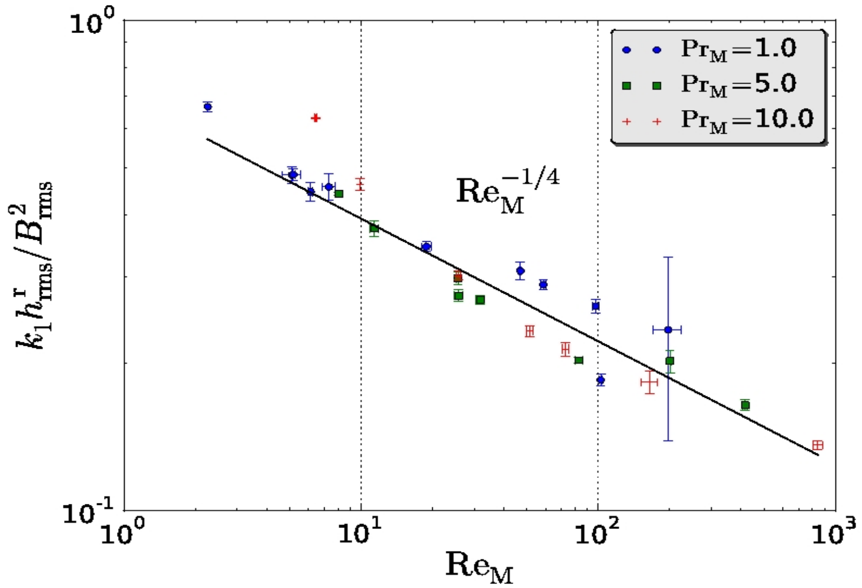
Different spatial fluctuations for h^{r} and h^{ar}

In the advecto-resistive gauge helicity transport becomes important for high Re

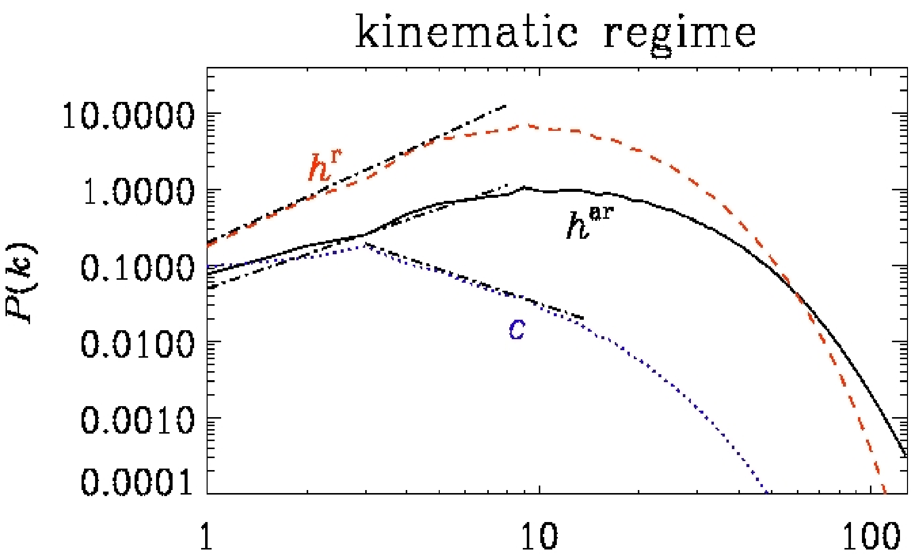


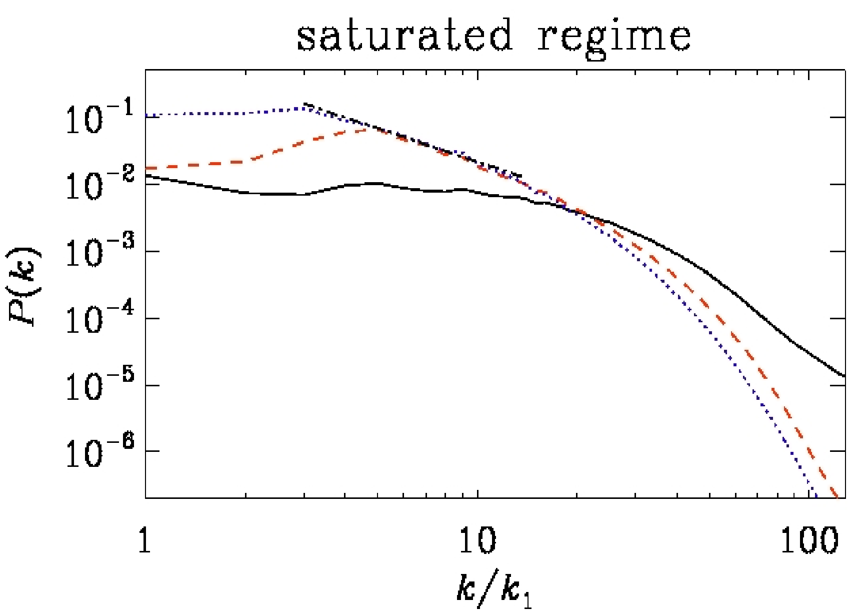


$$\frac{k_1 h^{\mathrm{ar}}}{B_{\mathrm{rms}}^2} = c\mathrm{Re_M}^{-a}(1 + b\mathrm{Re_M}^{2a})$$



Comparison with passive scalar





$$\frac{\partial h^{\mathrm{ar}}}{\partial t} = -2\eta \mathbf{J} \cdot \mathbf{B} - \boldsymbol{\nabla} \cdot \mathbf{F}^{\mathrm{ar}}$$

$$\mathbf{F}^{\mathrm{ar}} = h^{\mathrm{ar}}\mathbf{U} - \eta(\boldsymbol{\nabla} \cdot \mathbf{A}^{\mathrm{ar}})\mathbf{B} + \eta \mathbf{J} \times \mathbf{A}^{\mathrm{ar}}$$

passive scalar: $\frac{\mathrm{D}C}{\mathrm{D}t} = \kappa \nabla^2 C$

In the kinematic regime h behaves like a passive scalar.

h^{ar} has strong high-k tail

efficient turbulent cascade in the advecto-resistive gauge

Conclusions and Outlook

- Advecto-resistive gauge is unstable.
- Λ method can be used universally.
- The advecto-resistive gauge efficiently makes magnetic helicity cascade to higher wave numbers.
- In the ar gauge magnetic helicity behaves like a passive scalar in the kinematic regime and for high Rm.