Magnetic helicity fluxes in an α^2 dynamo

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Motivation

α quenching

- total $\alpha = \alpha_K + \alpha_M$
- $\alpha_M = \frac{1}{3}\tau\overline{\mathbf{j}\cdot\mathbf{b}} \sim$ current magnetic helicity density
- related to $\overline{h}_\mathrm{f} \equiv \overline{\mathbf{a}\cdot\mathbf{b}}$
- is produced as $\overline{h}_\mathrm{m} \equiv \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}$ builds up

alleviate α quenching by removing $\overline{h}_\mathrm{f}$

- either fluxes from outflows $\overline{\mathbf{F}}_\mathrm{f} = \overline{h}_\mathrm{f}\overline{\mathbf{U}}$
- or by diffusive fluxes $\overline{\mathbf{F}}_\mathrm{f} = -\kappa_\mathrm{h}\boldsymbol{\nabla}\overline{h}_\mathrm{f}$

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Linear Model

Starting point

- α^2-dynamo (is simplest)
- 1d model $\sim$ to galactic fountain flow model of Shukurov et al. (2006)
- $\alpha_K = \alpha_0 \, z/H$

turbulence is helical $\Rightarrow$ kinetic helicity

$$\alpha_K = -\frac{1}{3}\tau\overline{\omega \cdot \mathbf{u}}$$

with $\alpha = \alpha_K + \alpha_M$ the electromotive force becomes

$$\overline{\mathcal{E}} = \alpha\overline{\mathbf{B}} - \eta_t\mu_0\overline{\mathbf{J}}$$

with the magnetic diffusivity

$$\eta_t = \frac{1}{3}\tau\overline{\mathbf{u^2}}$$

Dynamo equations

Dynamo equations

$$\frac{\partial \overline{B}_x}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z \overline{B}_x + \overline{\mathcal{E}}_y\right) + \eta \frac{\partial^2 \overline{B}_x}{\partial z^2} \quad (1)$$

$$\frac{\partial \overline{B}_y}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z \overline{B}_y - \overline{\mathcal{E}}_x\right) + \eta \frac{\partial^2 \overline{B}_y}{\partial z^2} \quad (2)$$

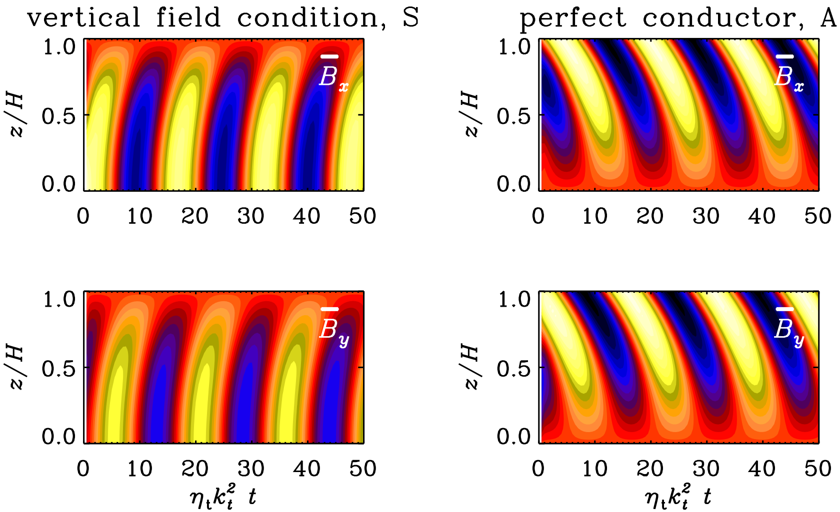
Marginal solutions

Excitation conditions, define $C_\alpha = \alpha_0/\eta_t k_1$, where $k_1 = \pi/H$

$C_\alpha^{\rm crit}$	S mode	A mode
vertical field at $z = H$	**2.56**	3.56
perfect conductor at $z = H$	3.57	**2.56**

all modes are oscillatory $\omega/\eta_t k_1 = 0.4$–0.6

Marginal solutions



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magnetic helicity evolution equations

- evolution equation for the mean magnetic helicity:

$$\frac{\partial \overline{h}_m}{\partial t} = 2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \nabla \cdot \overline{\mathbf{F}}_m$$

  with $\overline{\mathbf{F}}_m = \overline{\mathbf{E}} x \overline{\mathbf{A}}$ the flux of the magnetic helicity of the mean magnetic field

- evolution equation for the small scale magnetic helicity:

$$\frac{\partial \overline{h}_f}{\partial t} = -2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{j} \cdot \mathbf{b}} - \nabla \cdot \overline{\mathbf{F}}_f$$

- combined gives:

$$\frac{\partial h}{\partial t} = -2\eta\mu_0 \overline{\mathbf{J} \cdot \mathbf{B}} - \nabla \cdot \overline{\mathbf{F}}$$

Dynamical quenching formalism

equipartition magnetic field:

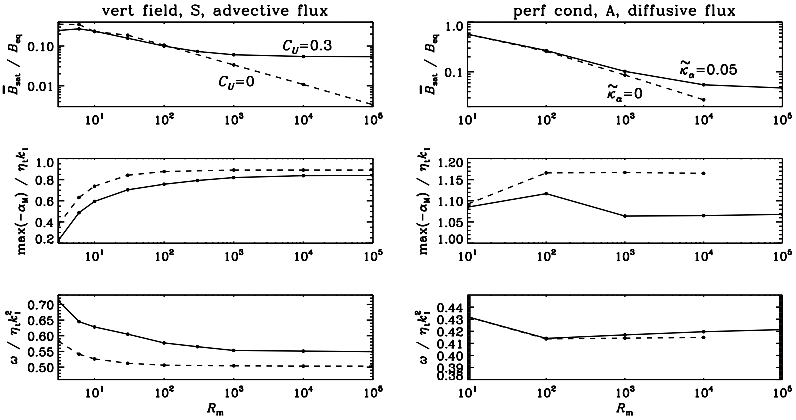
$$B_{eq} = (\mu_0 \rho \overline{\mathbf{u^2}})^{1/2}$$

$$\frac{\partial \alpha_m}{\partial t} = -2\eta_t k_f^2 \left(\frac{\overline{\mathcal{E}} \cdot \overline{\mathbf{B}}}{B_{eq}^2} + \frac{\alpha_M}{R_M} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha \qquad (3)$$

introducing Fickian diffusion term:

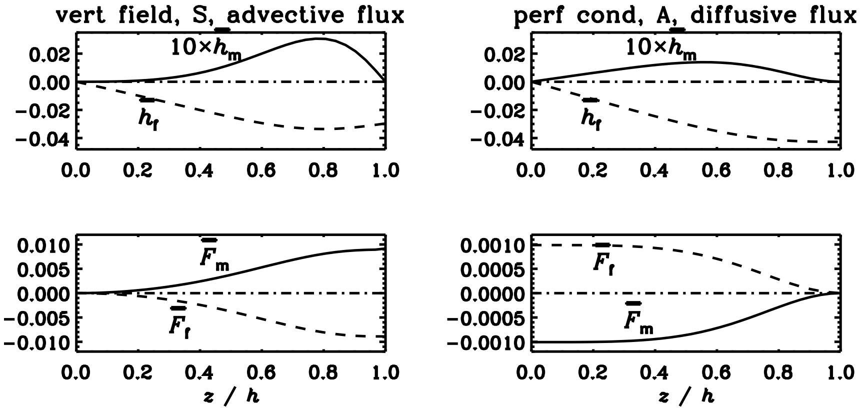
$$\overline{\mathcal{F}}_\alpha = \alpha_M \overline{U} - \kappa_\alpha \frac{\partial \alpha_M}{\partial z}$$

Nonlinear solutions



- for A mode flux through the equator is possible ⇒ diffusive flux $\nabla\alpha_M$
- catastrophic α quenching is alleviated by flux through equator
- B finite with flux
- finite B with flux for large R_m in either case

Magnetic helicity densities and their fluxes



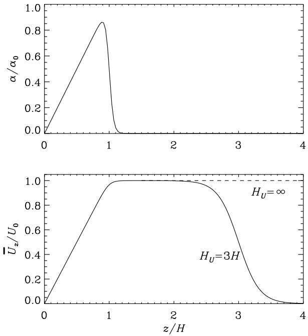
negative $\overline{F}_{\rm f}$ to the right

positive $\overline{F}_{\rm f}$ to the left

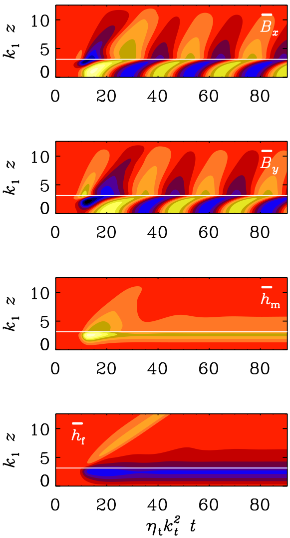
Model profiles and boundary conditions

$$\alpha = \alpha_0 \frac{z}{H} \cdot \frac{1}{2}\left(1 - \tanh\left(\frac{z-H}{w}\right)\right) \quad (4)$$

$$\overline{U}_z = U_0 \frac{z}{H}\left(1 + (z/H)^n\right)^{-1/n}\left(1 - \tanh\left(\frac{z-H_U}{w_U}\right)\right) \quad (5)$$



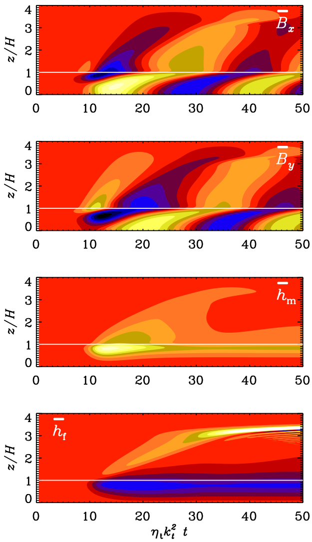
Results for different model profiles



$L = 4H$, $C_\alpha = 2$, $C_U = 0.3$ and $H_U \to \infty$ (flux always permitted)

- Small scale helicity escapes
- At first positive $\overline{h}_\mathrm{f}$ is shed
- is produced by $\eta_\mathrm{t}\overline{\mathbf{J}} \cdot \overline{\mathbf{B}}$ term
- Later also negative $\overline{h}_\mathrm{f}$ is shed

Results for different model profiles



$L = 3H,\ C_\alpha = 2,\ C_U = 0.3,\ R_M = 10^5$ and $H_U = H$

- Small scale helicity is confined
- shocks are created which cannot be resolved
- such models are unphysical
- helicity has to get out e.g. through diffusive flux
- $\rightarrow$ unphysical, it must get out

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Summary and Conclusions

- appropriate velocity field → ss helicity flux out of the domain
- Fickian diffusion term → helicity flux through the equator

Outlook and future work

- investigate magnetic helicity fluxes through the equator and out of the domain
- different wind profiles $u(z)$
- switch to pencil code
- extend to 2 and 3 dimensions
- migrate algorithms to GPUs (CUDA)
- 3 years 10.75 months left