Twist and Current Alignment within non-trivial Magnetic Field Topologies

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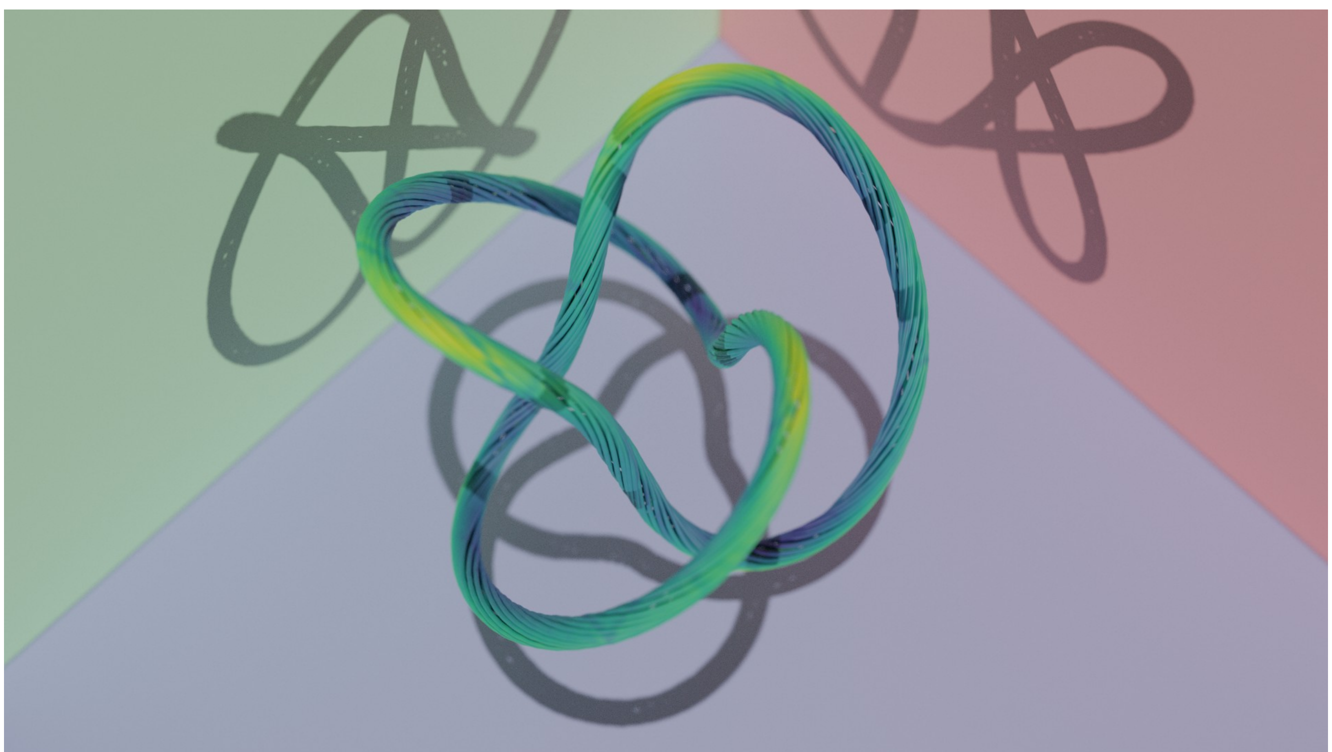
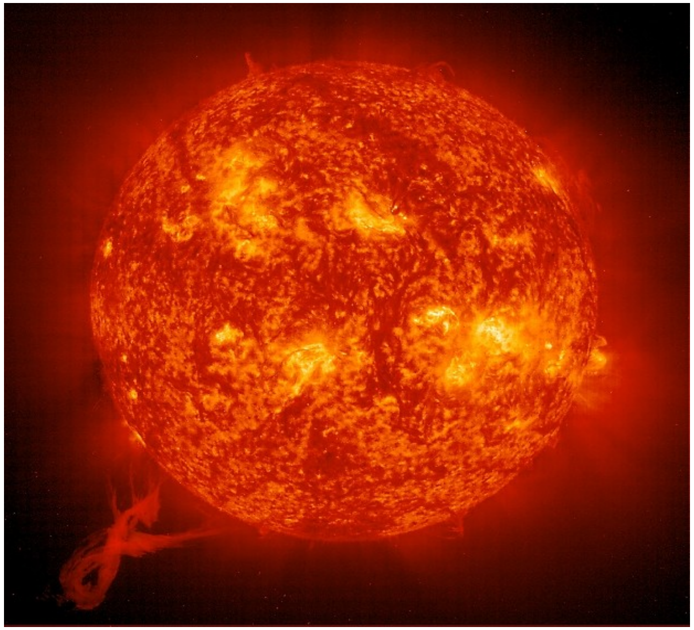
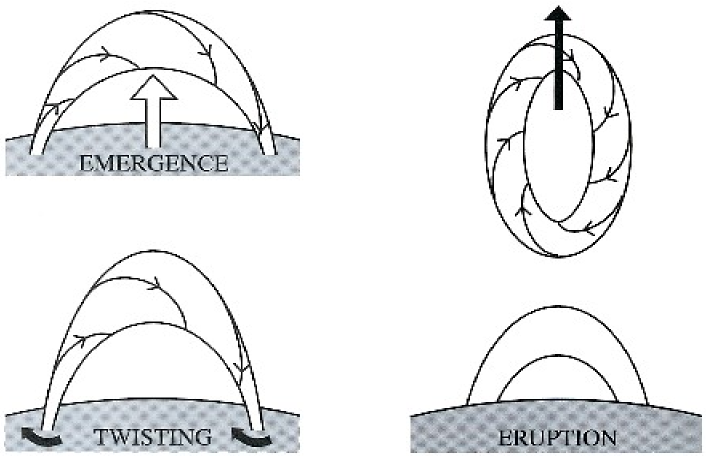
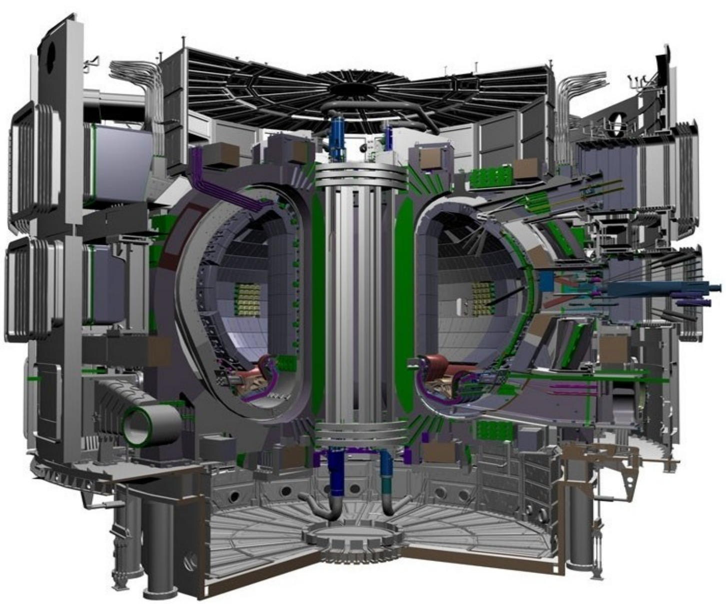


Twisted Magnetic Fields

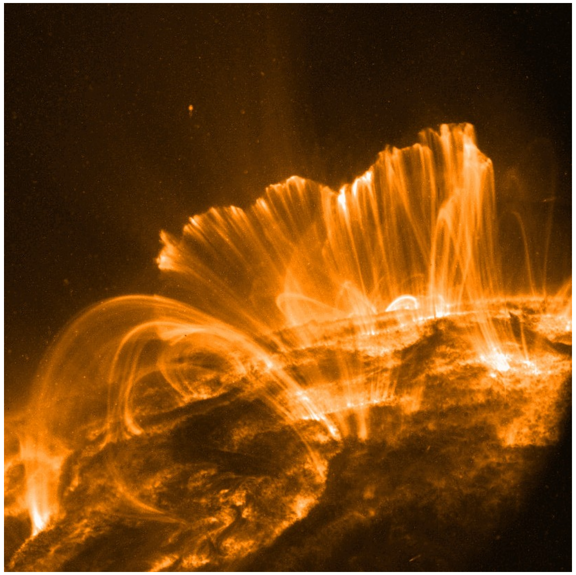




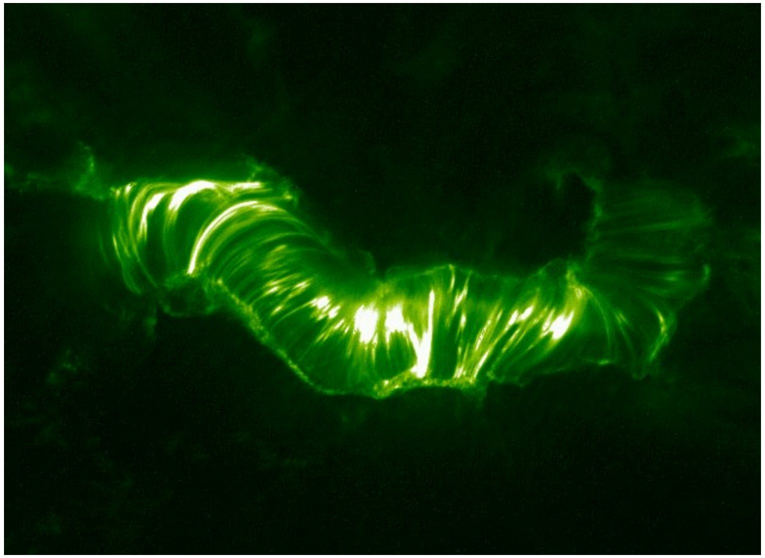


- Twisted fields are more likely to erupt *(Canfield et al. 1999)*.
- Twist increases the stability of magnetic fields in tokamaks.

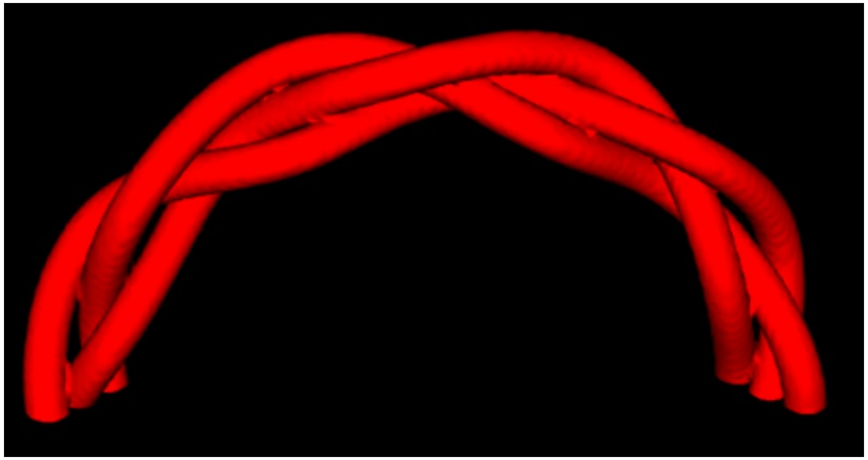
Solar Magnetic Field



(Trace)



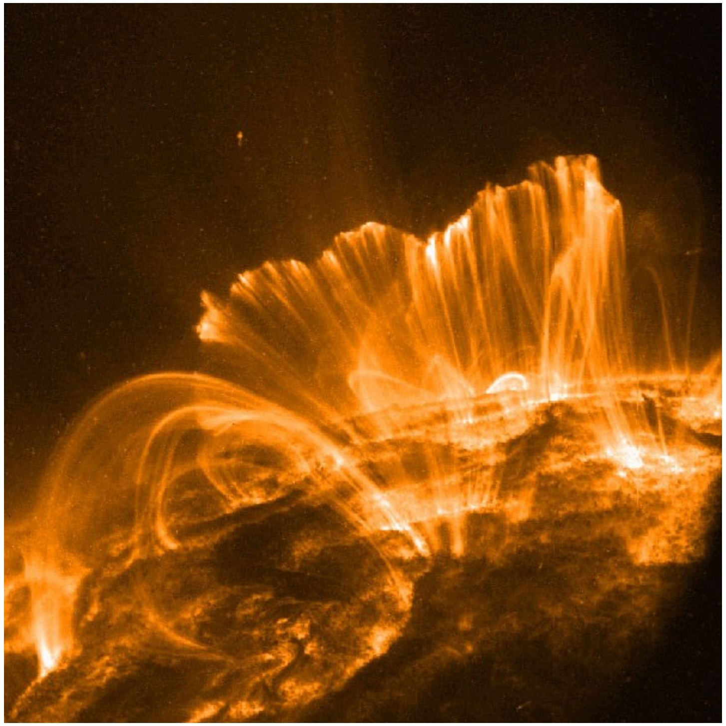
(Trace)

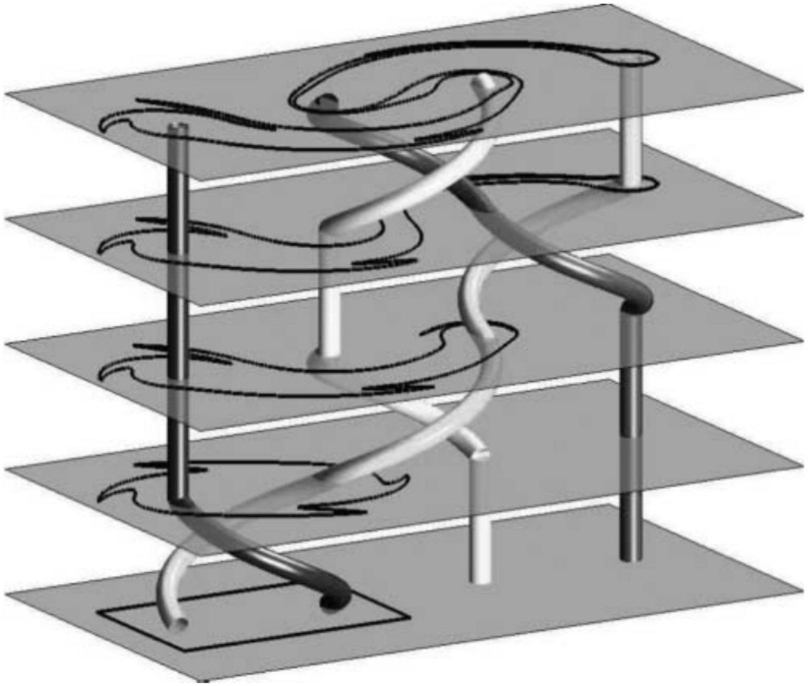


- Twisted flux tubes may rise to the corona. *(Prior and MacTaggart 2016)*.

Coronal Magnetic Fields

NASA

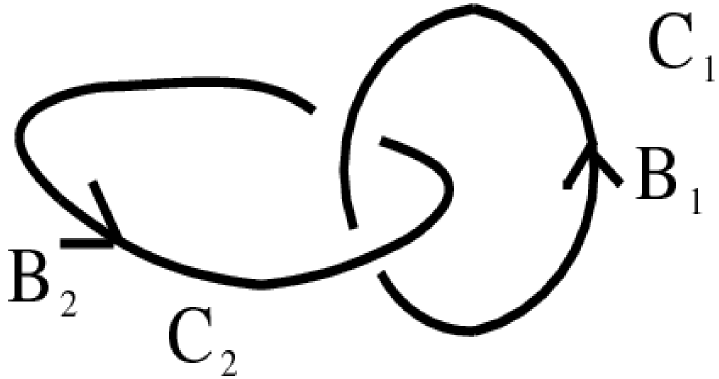




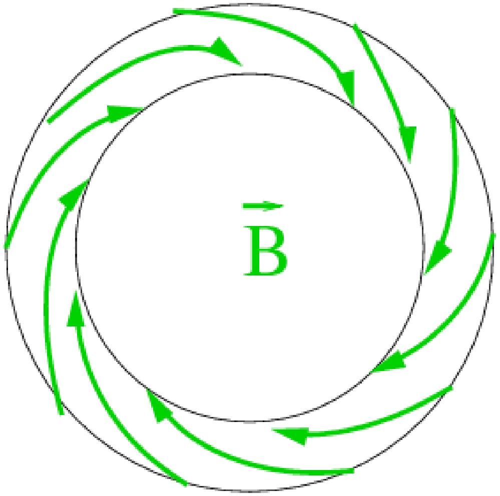
(Thiffeault et al. 2006)

- Field line tangling in solar magnetic fields.
- Study the tangling of solar magnetic field lines.

Topologies of Magnetic Fields

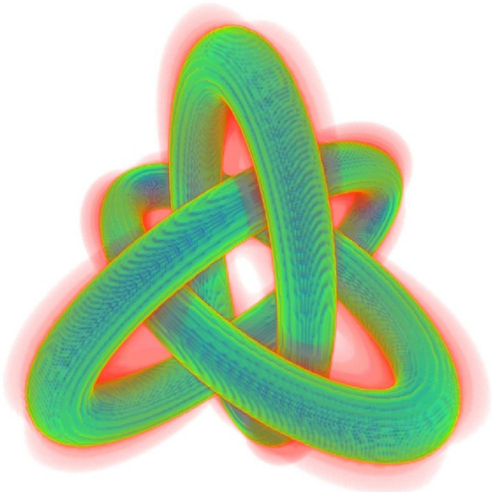


Hopf link

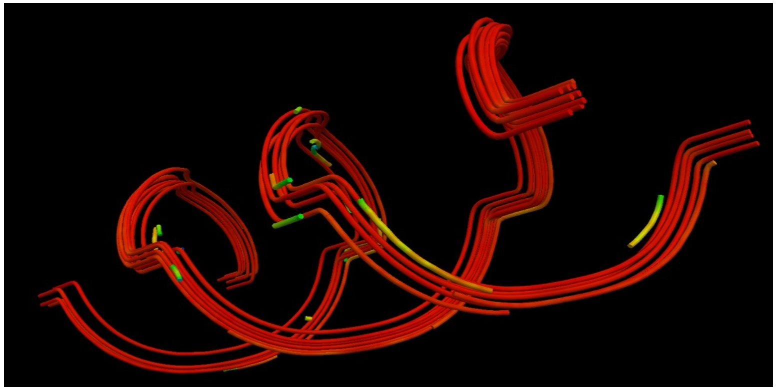


twisted field

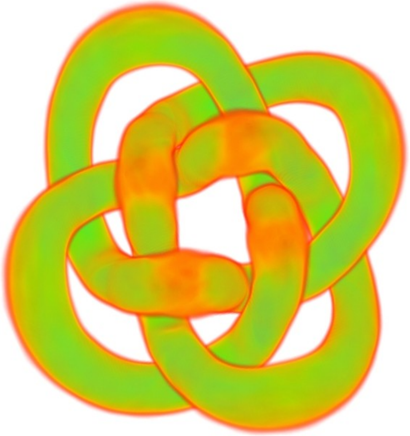
trefoil knot



Borromean rings



magnetic braid



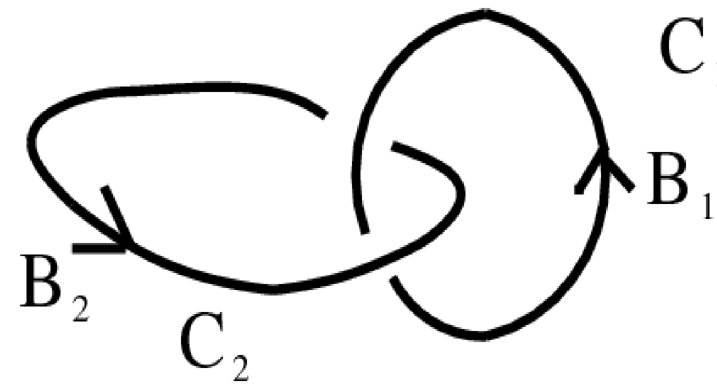
IUCAA knot

Magnetic Helicity

Measure for the topology:

$$H_\mathrm{M} = \int_V \boldsymbol{A} \cdot \boldsymbol{B}\ \mathrm{d}V = 2n\phi_1\phi_2$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B} \qquad \phi_i = \int_{S_i} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$$



$n =$ number of mutual linking

Conservation of magnetic helicity:

$$\lim_{\eta \to 0} \frac{\partial}{\partial t} \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0 \qquad \eta = \text{magnetic resistivity}$$

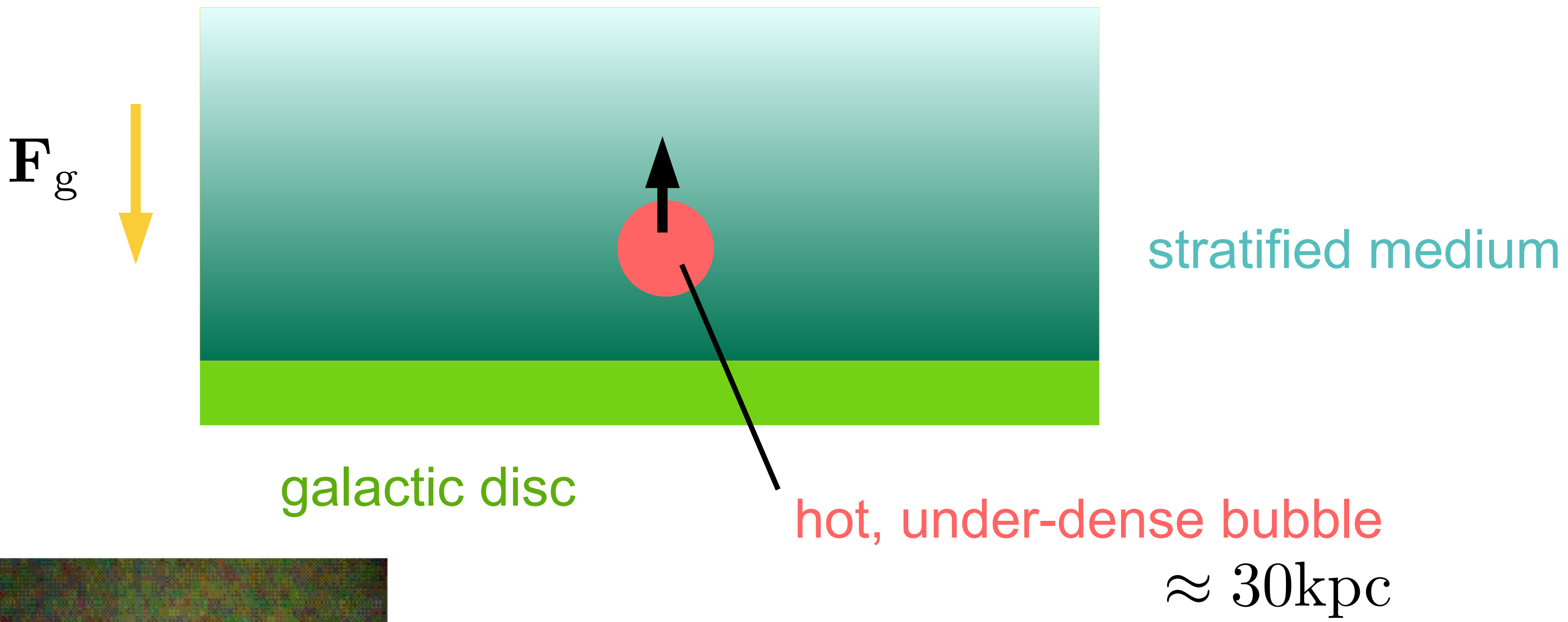
$$\frac{\partial}{\partial t} \int_V \boldsymbol{A} \cdot \boldsymbol{B}\ \mathrm{d}V = -2\eta \int_V \boldsymbol{J} \cdot \boldsymbol{B}$$

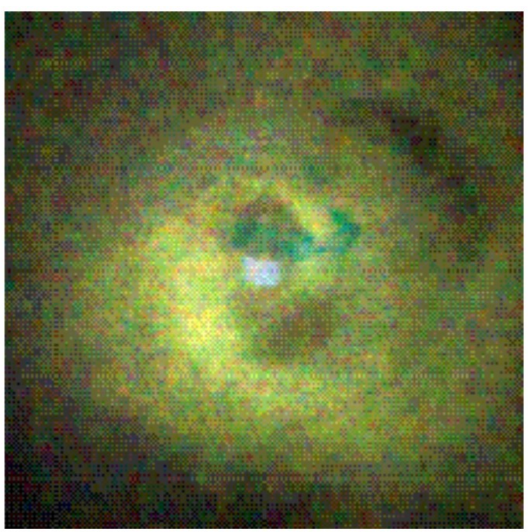
Realizability condition:

$$E_\mathrm{m}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.

Intergalactic Bubbles



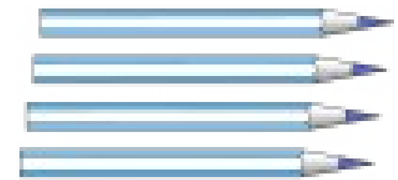


(Fabian et al. 2000)

- Bubbles rise buoyantly through density difference.
- Bubbles' age is several tens of millions of years.

Numerical Experiments

Full resistive magnetohydrodynamics simulations with the PencilCode.

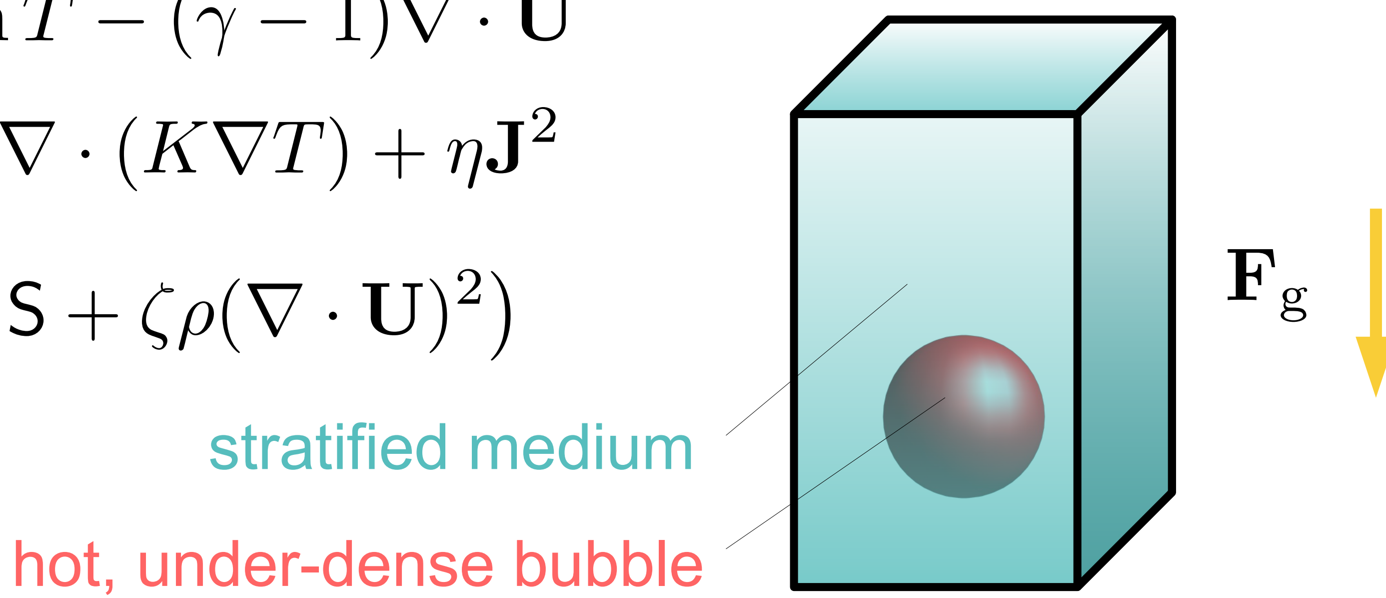


$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \eta \nabla^2 \mathbf{A}$$

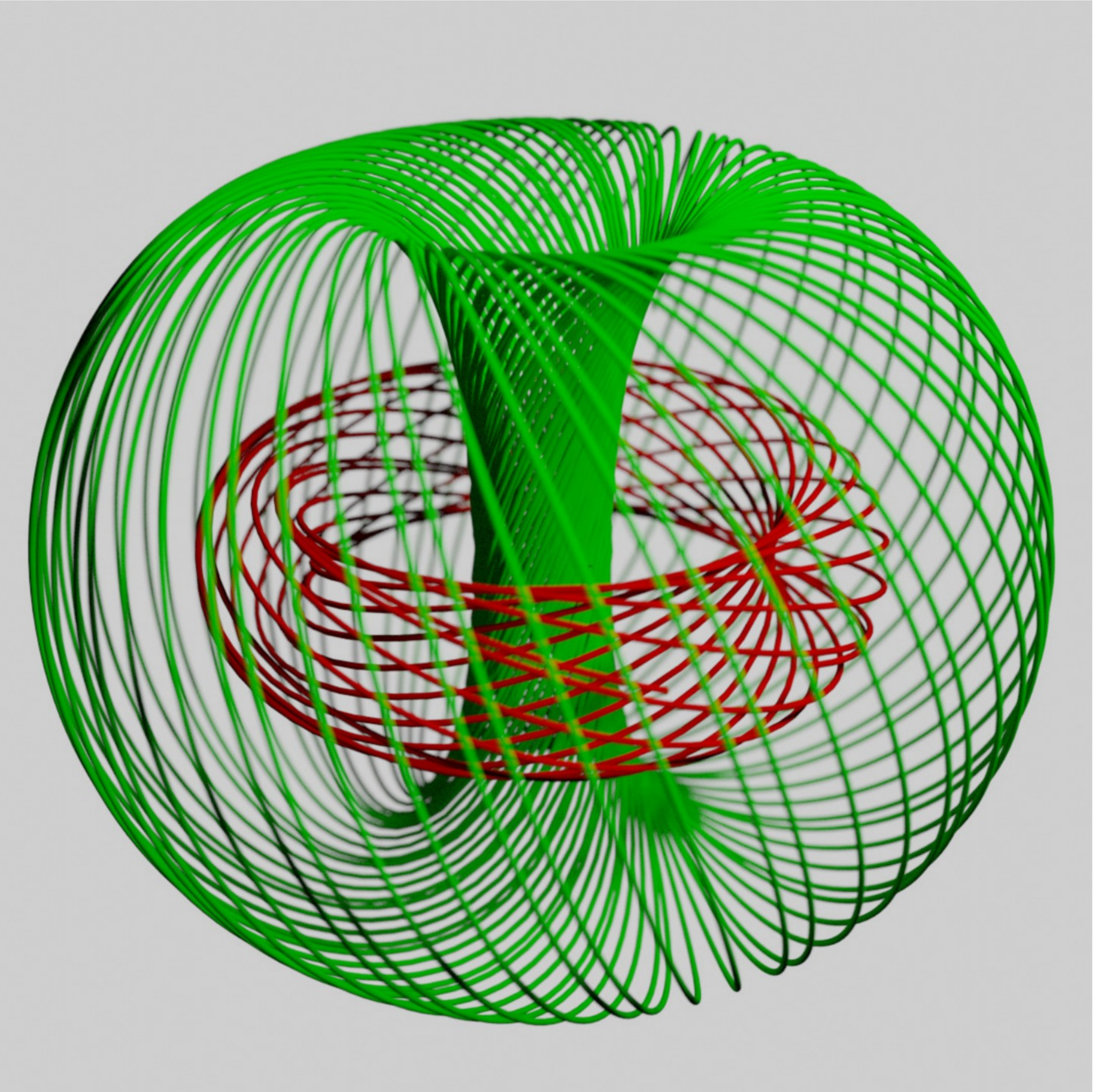
$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -c_\mathrm{S}^2 \nabla \left(\frac{\ln T}{\gamma} \ln \rho \right) + \mathbf{J} \times \mathbf{B}/\rho - \mathbf{g} + \mathbf{F}_\mathrm{visc}$$

$$\frac{\partial \ln T}{\partial t} = -\mathbf{U} \cdot \nabla \ln T - (\gamma - 1) \nabla \cdot \mathbf{U} + \frac{1}{\rho c_V T} \left(\nabla \cdot (K \nabla T) + \eta \mathbf{J}^2 + 2\rho\nu \mathsf{S} \otimes \mathsf{S} + \zeta \rho (\nabla \cdot \mathbf{U})^2 \right)$$

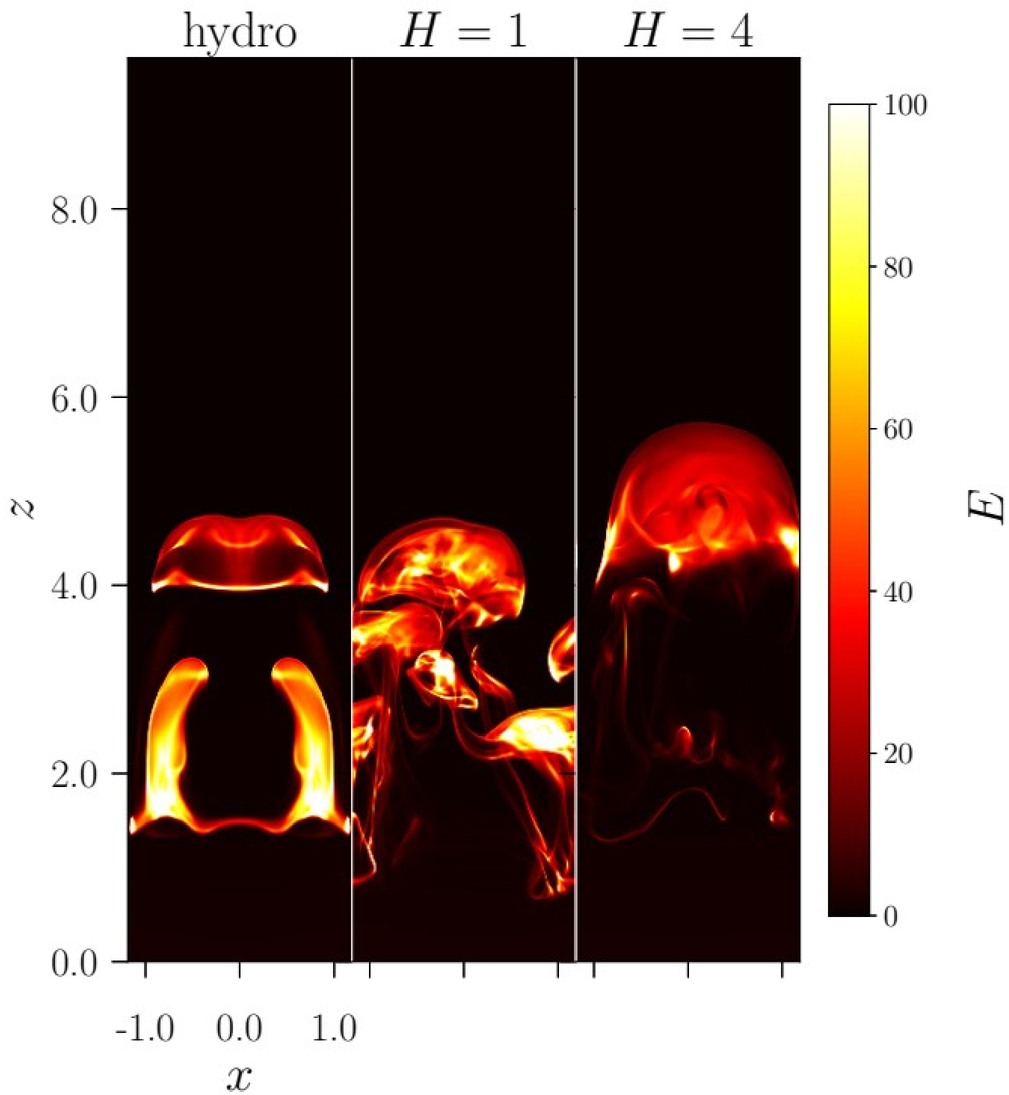
$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\nabla \cdot \mathbf{U}$$

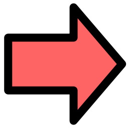


Initial Condition: Spheromak



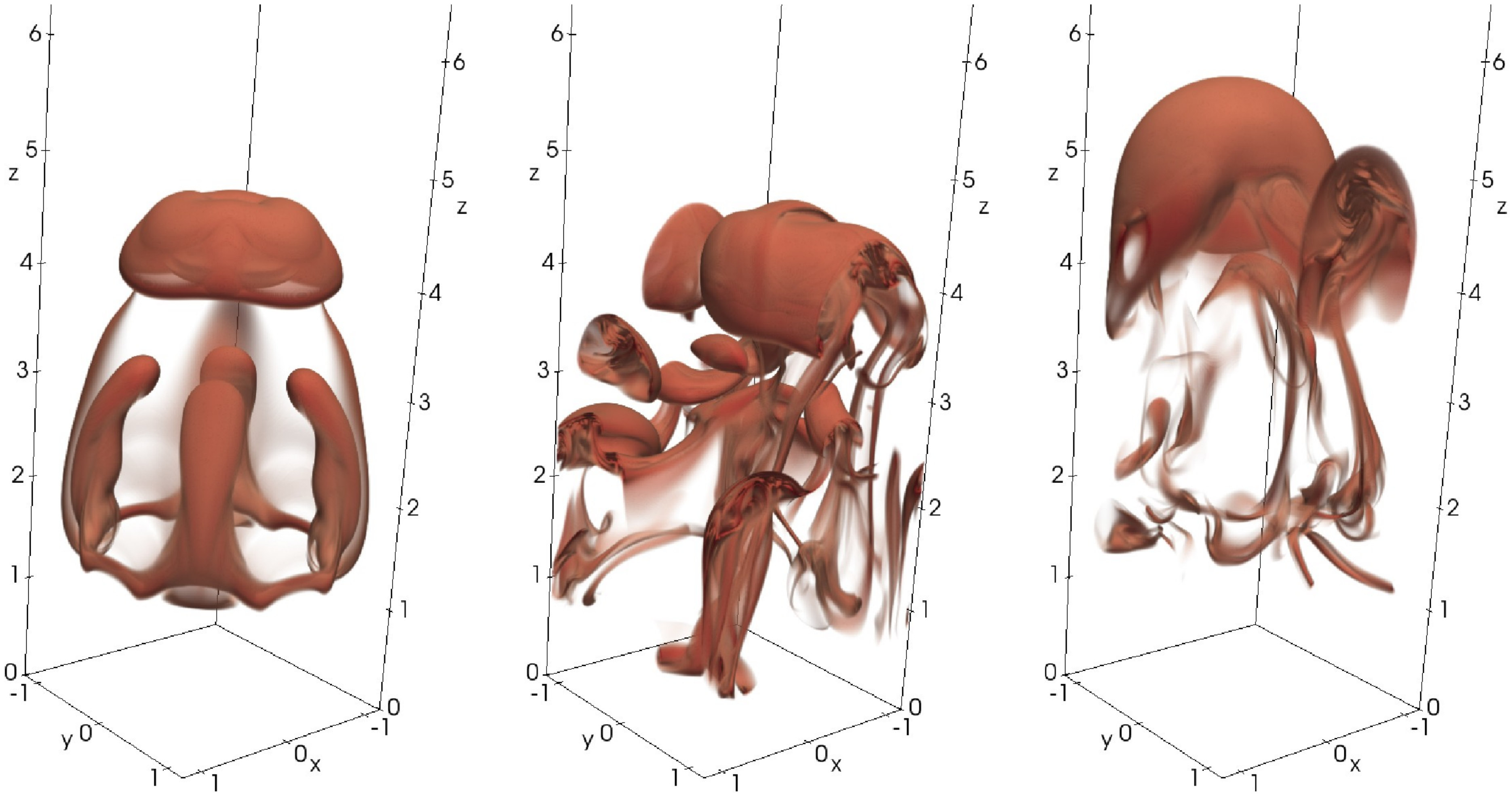
Thermal Emission





Magnetic helicity stabilises the bubbles.

Temperature Iso-Surfaces

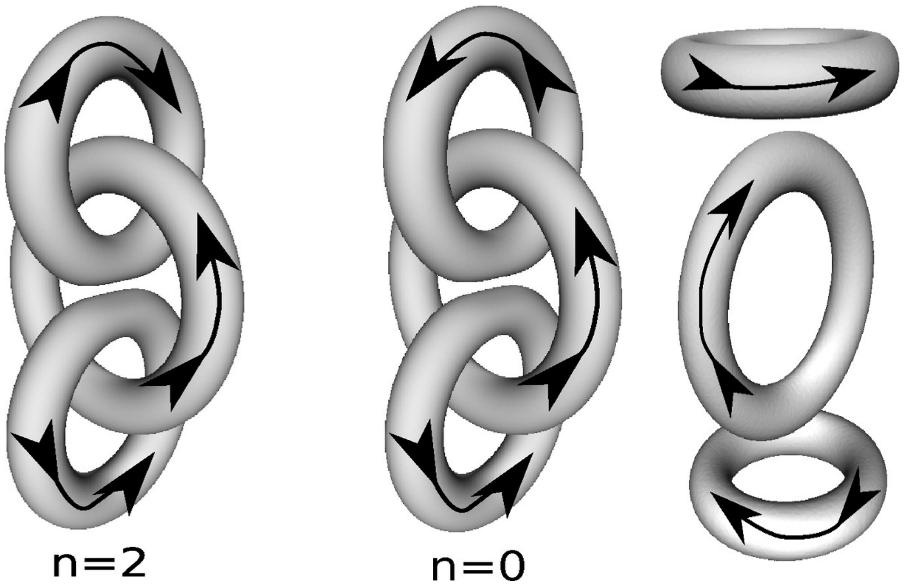


hydro | low helicity | high helicity

Interlocked Flux Rings

actual linking vs. magnetic helicity

$H_\mathrm{M} \neq 0$ $\qquad$ $H_\mathrm{M} = 0$



- initial condition: flux tubes
- isothermal compressible gas
- viscous medium
- periodic boundaries

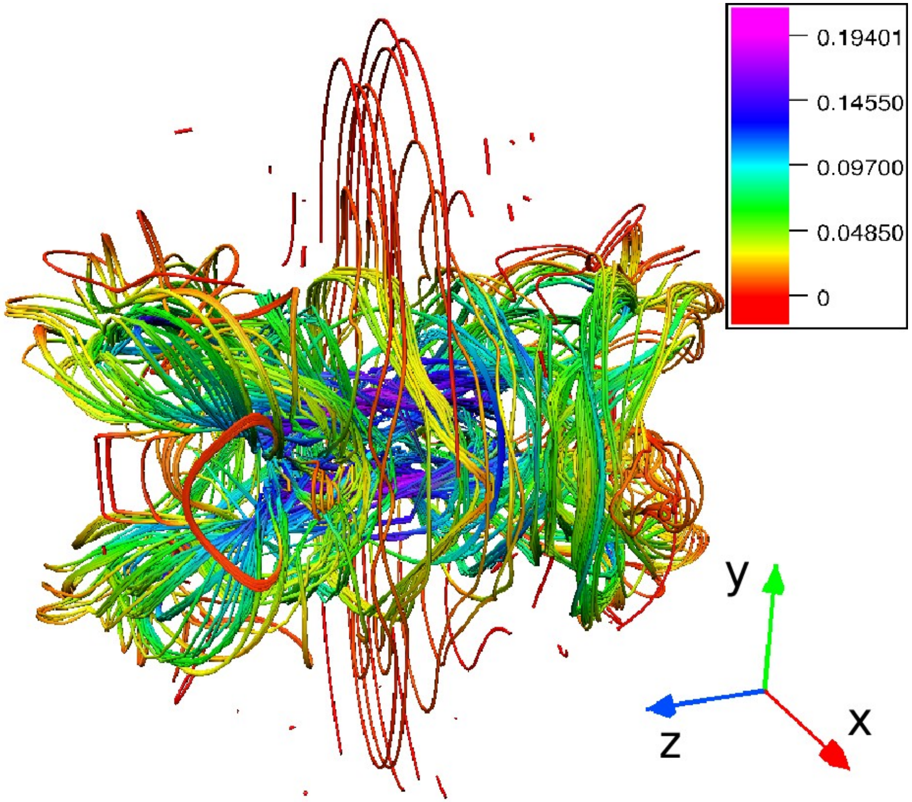
(Del Sordo et al. 2010)

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A} \qquad \frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

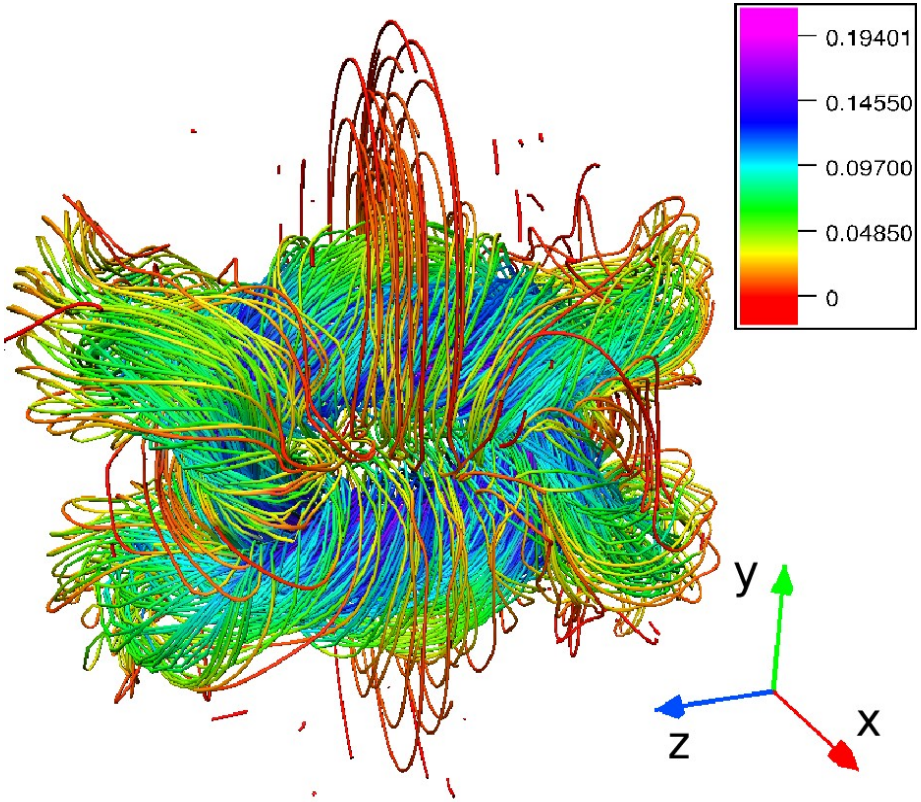
$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_\mathrm{S}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_\mathrm{visc}$$

Interlocked Flux Rings

$\tau = 4$

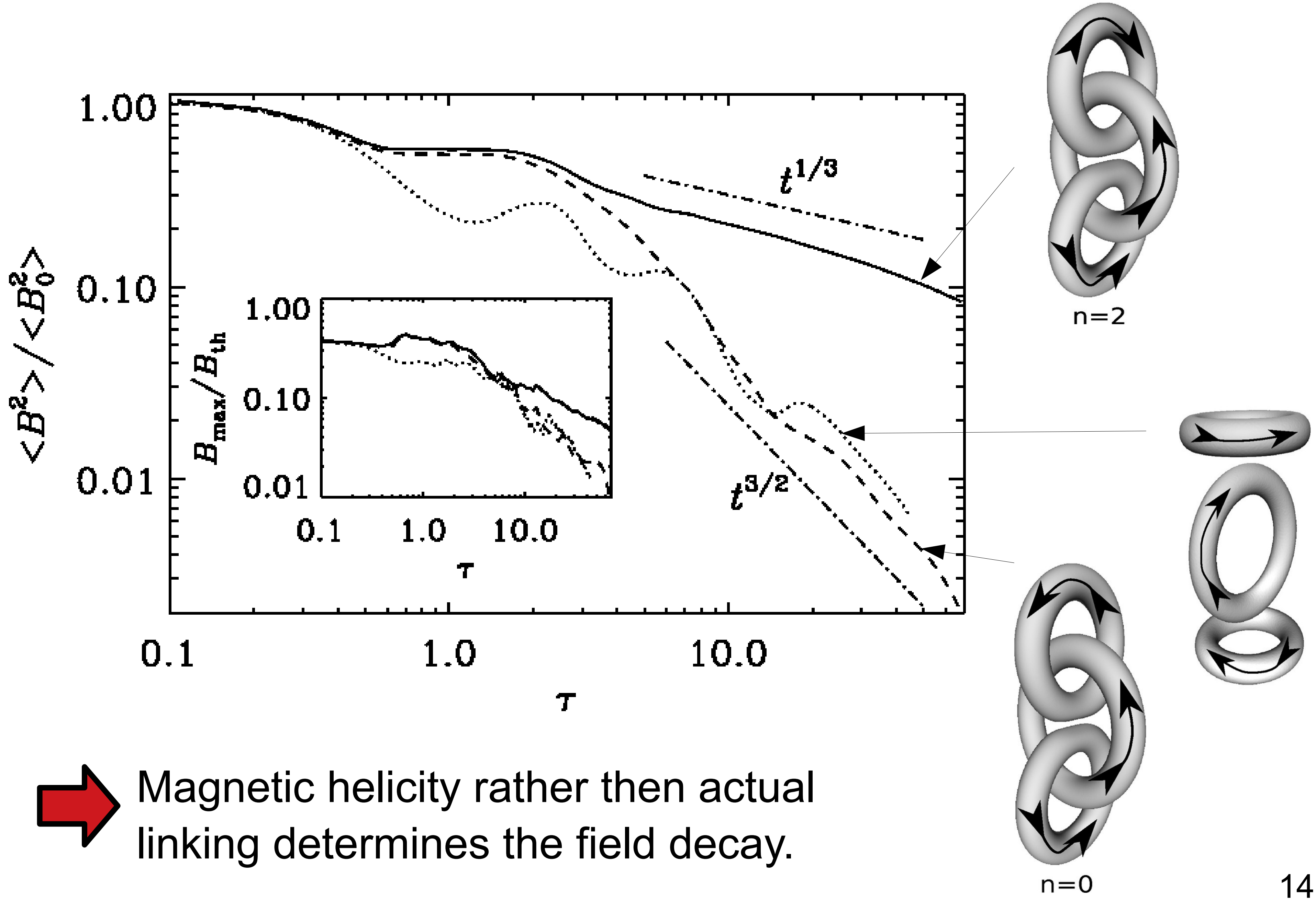


$H_{\mathrm{M}} = 0$



$H_{\mathrm{M}} \neq 0$

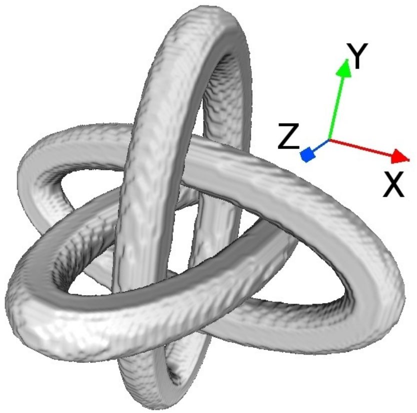
Interlocked Flux Rings



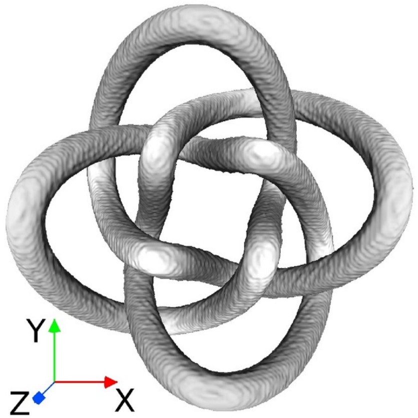
➡ Magnetic helicity rather then actual linking determines the field decay.

IUCAA Knot and Borromean Rings

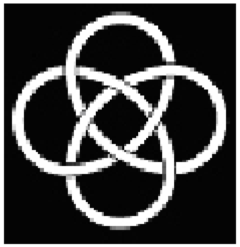
- Is magnetic helicity sufficient?
- Higher order invariants?



Borromean rings



IUCAA knot

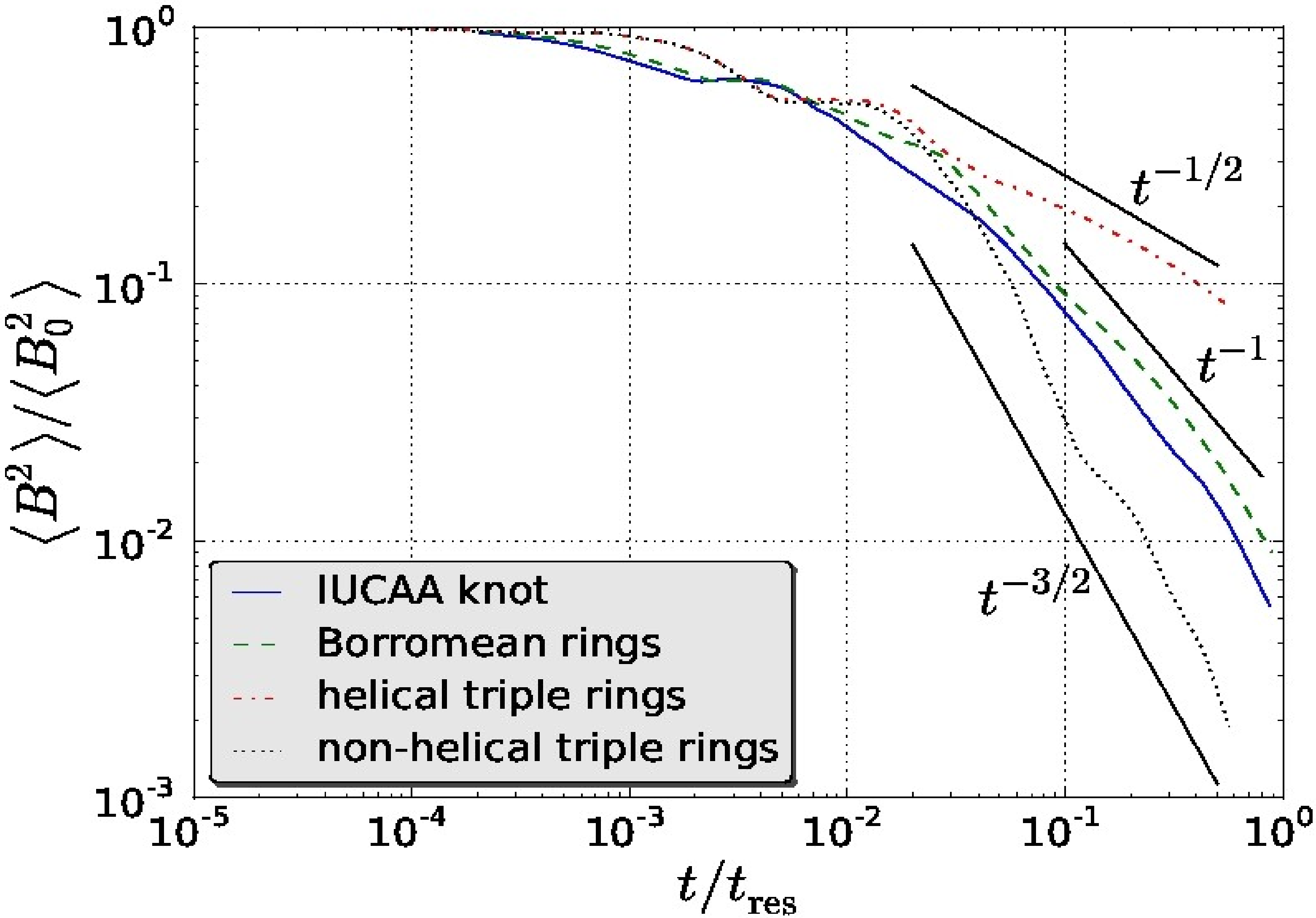


$$H_{\mathrm{M}} = 0$$

(Candelaresi and Brandenburg 2011)

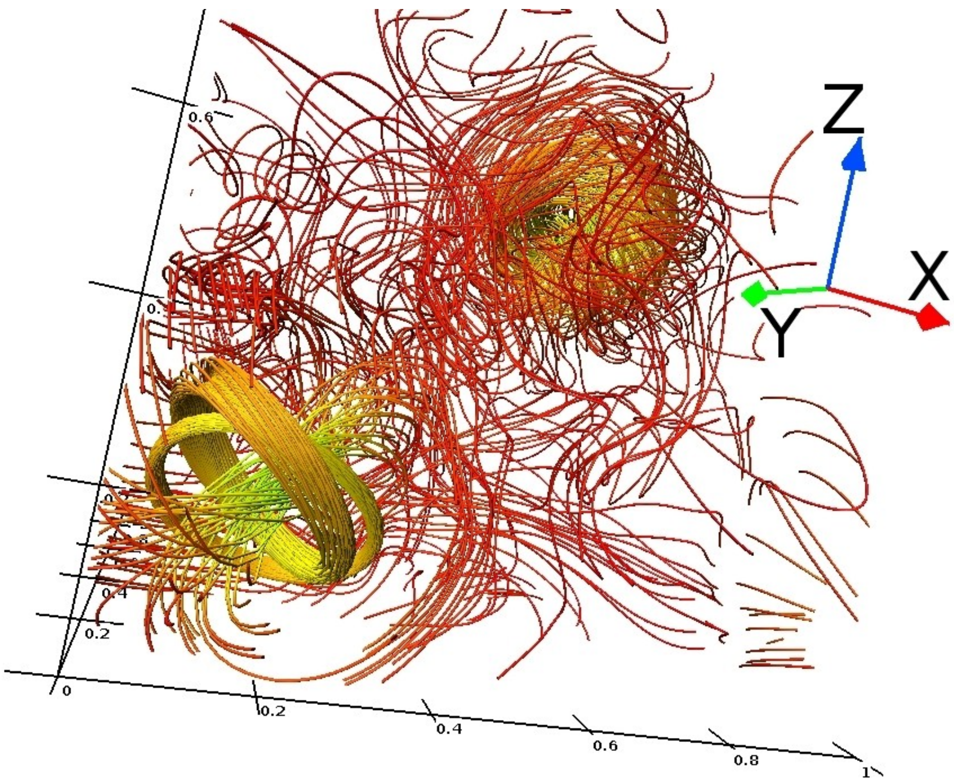
IUCAA = The Inter-University Centre for Astronomy and Astrophysics, Pune, India

Magnetic Energy Decay

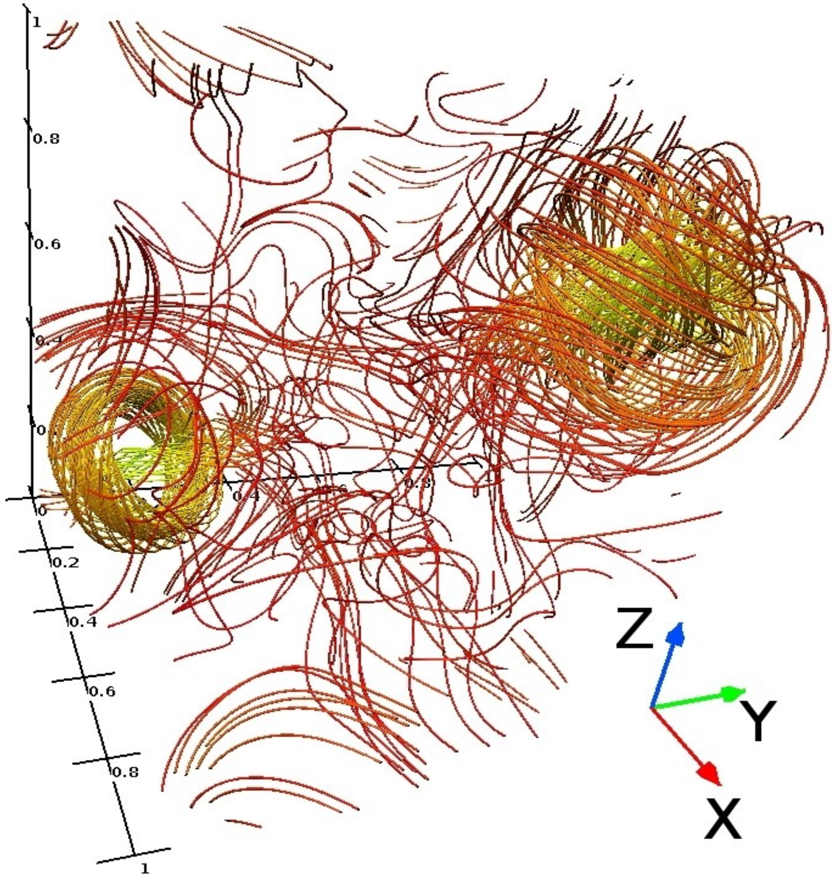


Higher order invariants?

Borromean Rings



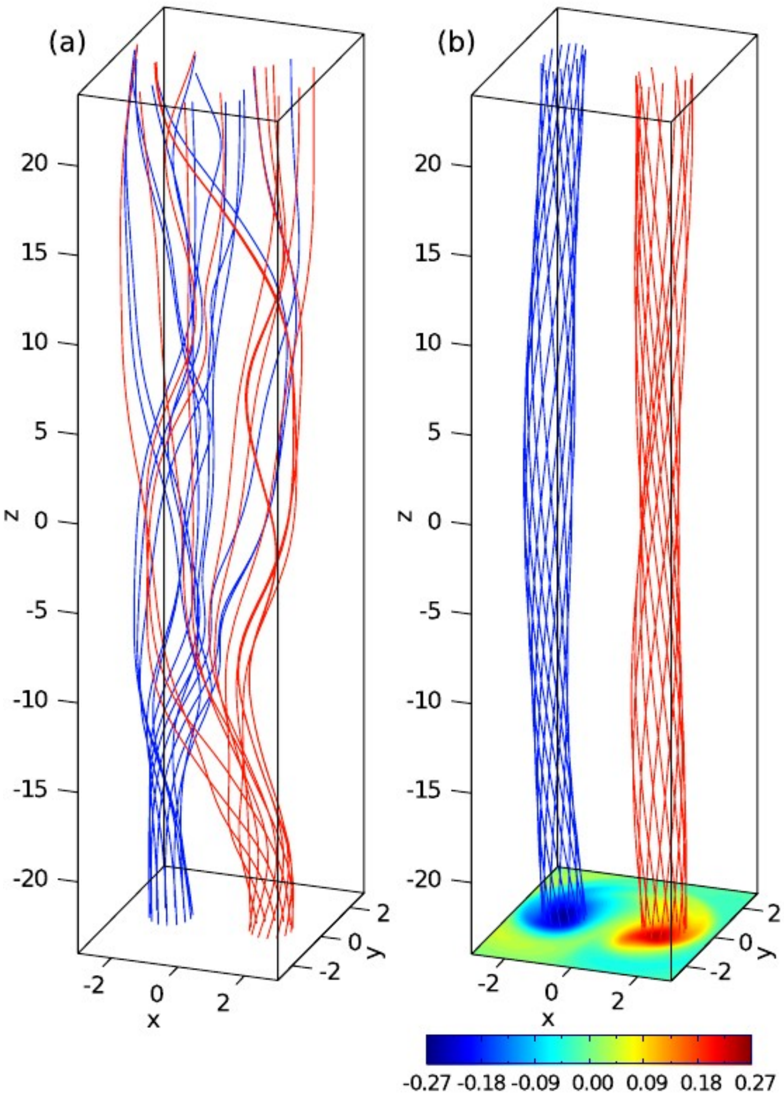
$t = 70$



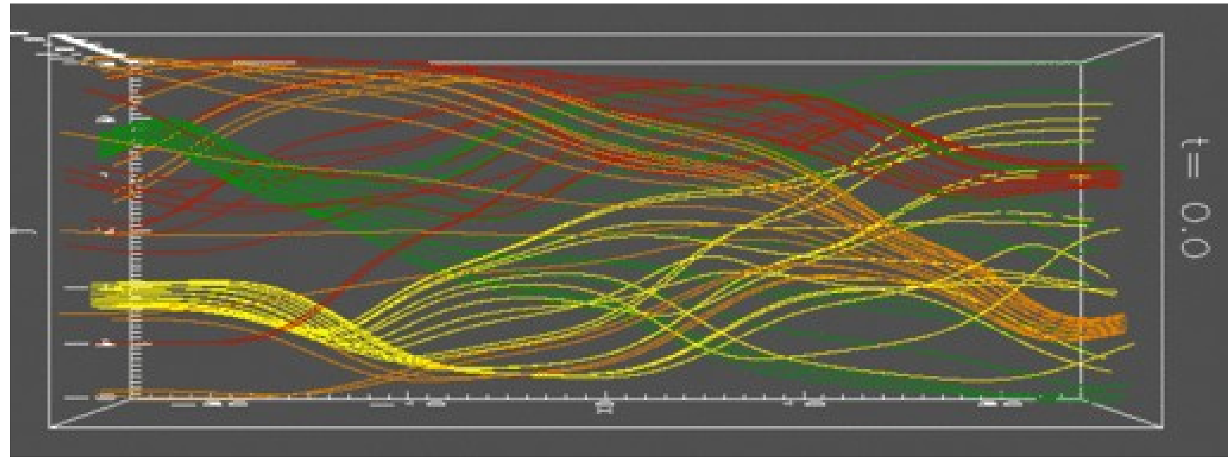
$t = 78$

3 rings ➡ Twisted ring + interlocked rings ➡ 2 twisted rings

Magnetic Braid



(Yeates 2011)

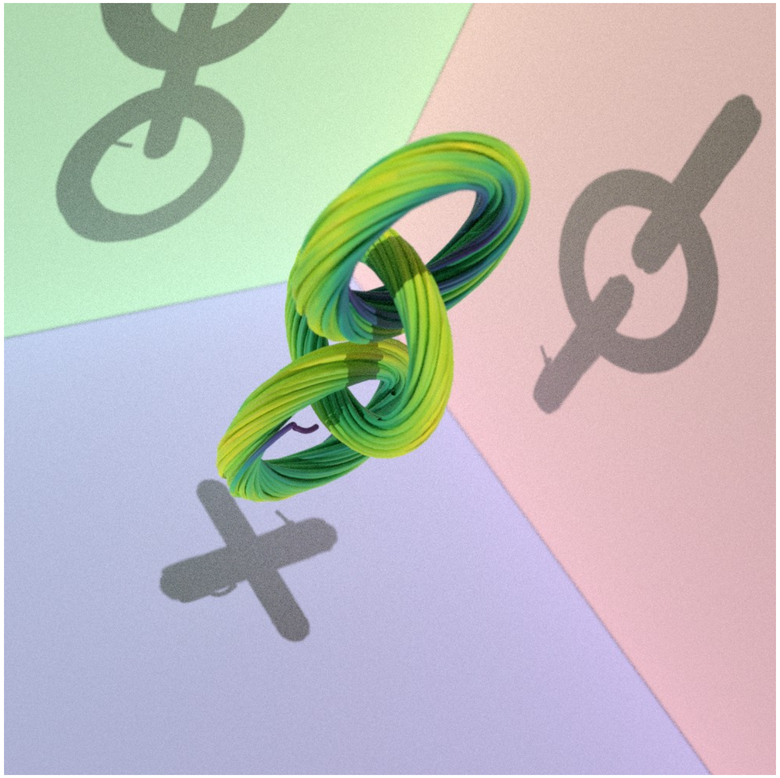


(Wilmot-Smith 2010)

- Periodic braid topologically equivalent to Borromean rings.
- Separation into two twisted field regions.
- Conserved invariants like fixed point index and field line helicity.

Magnetic Fields with a Twist

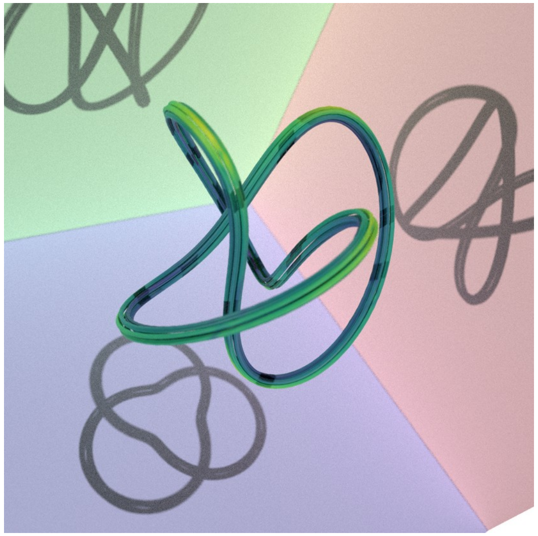
- Helical fields can be made non-helical by twisting the field lines.
- Non-elical fields can be made helical by twisting the field lines.
- Simulated twisted knots and links in MHD (Pencil Code).



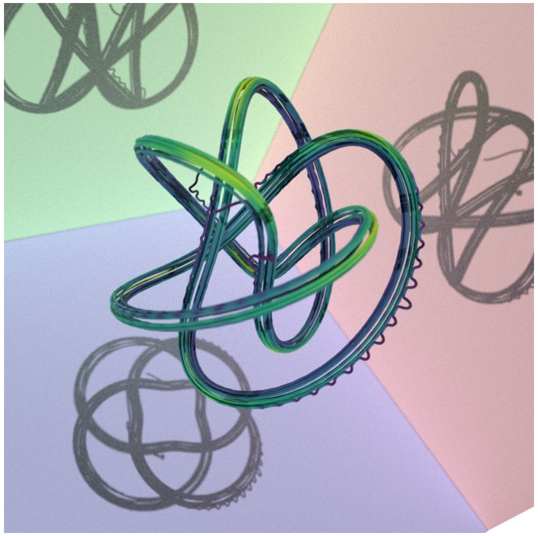
$$E_{\mathrm{M}}(t) = ? \qquad \frac{\mathrm{d}}{\mathrm{d}t} H_{\mathrm{m}} = ? \qquad \int_V J \cdot B \, \mathrm{d}V = ?$$

Knots and Links

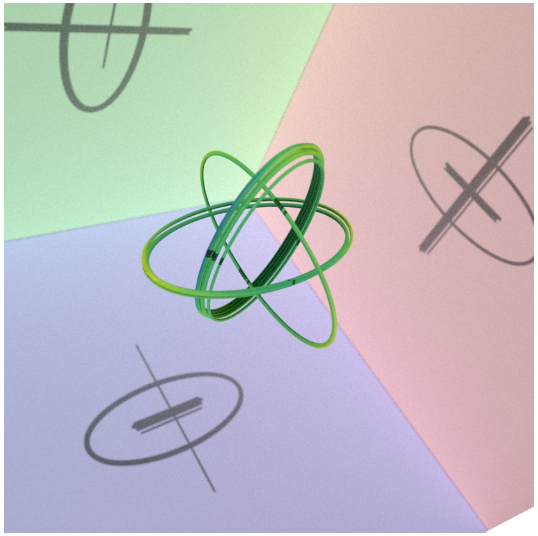
trefoil



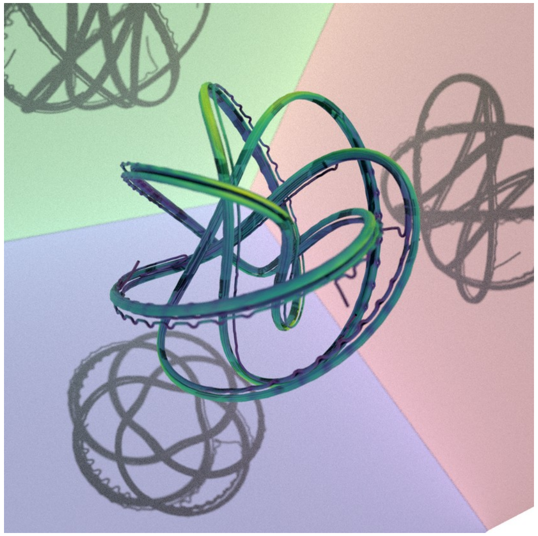
4-foil



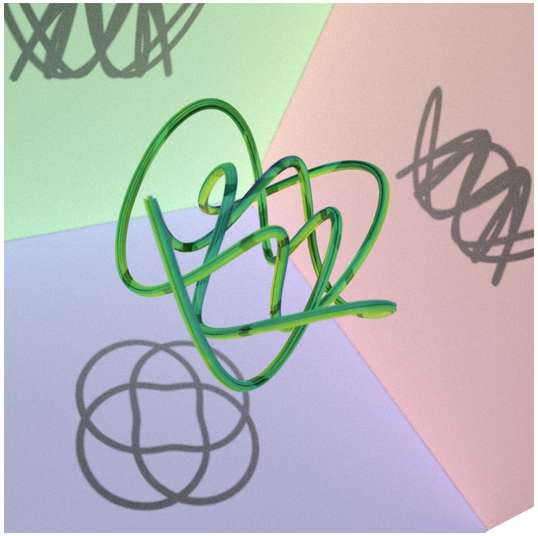
Borromean rings



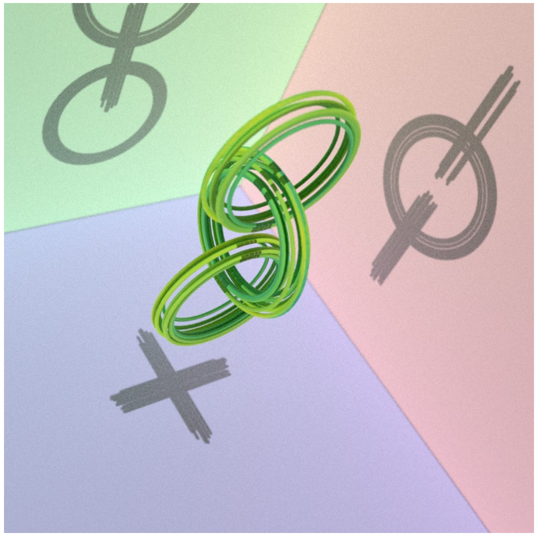
5-foil



IUCAA



triple rings

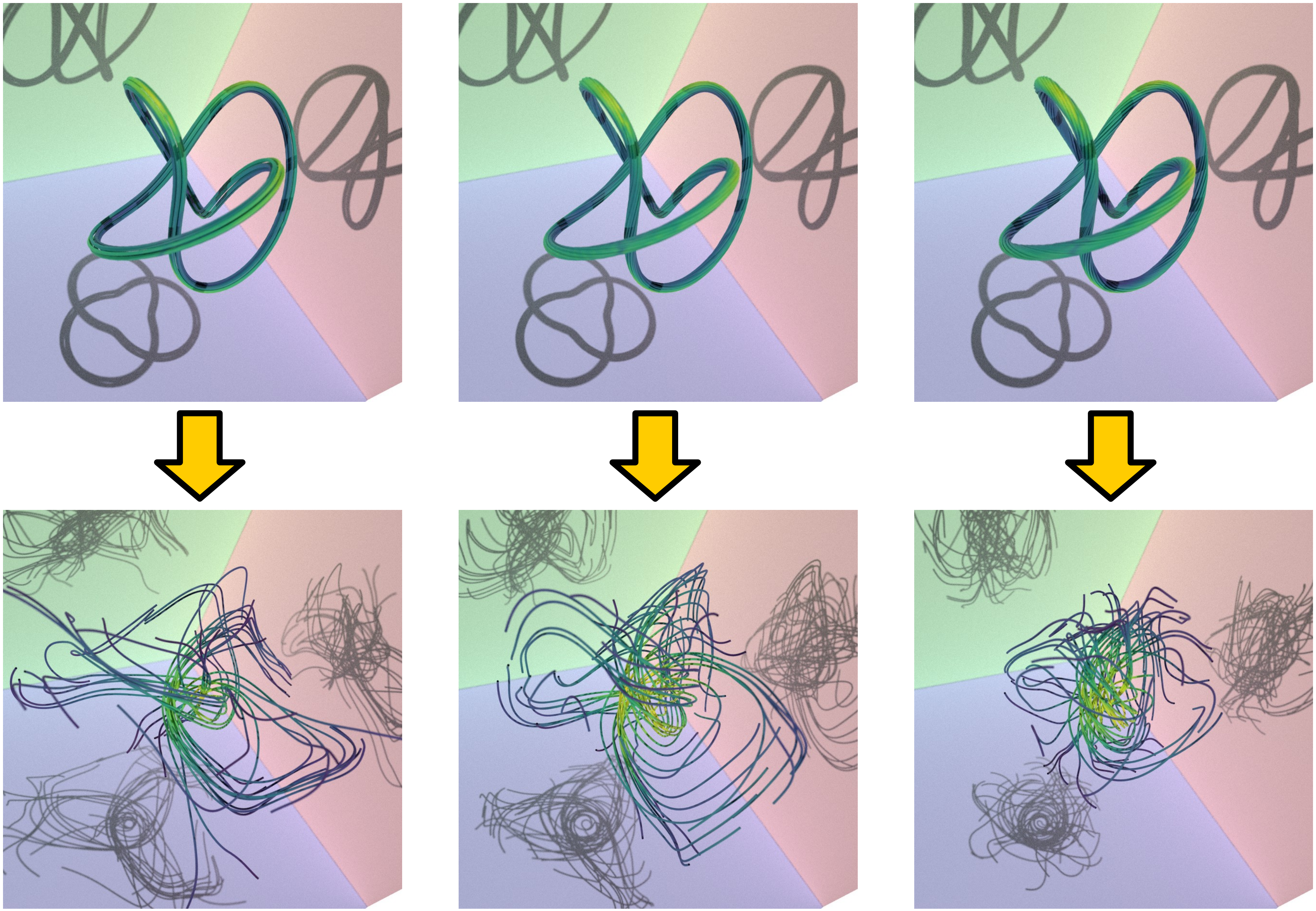


Twisted Trefoil Knot

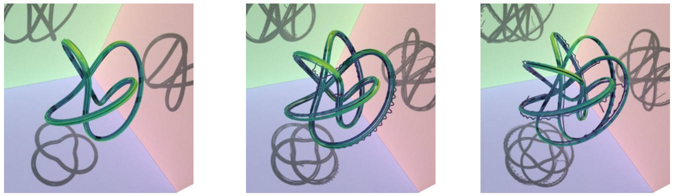
$H_{\mathrm{m}} = 1, \mathrm{tw} = 0$ | $H_{\mathrm{m}} = 0, \mathrm{tw} = 1$ | $H_{\mathrm{m}} = -1, \mathrm{tw} = 2$

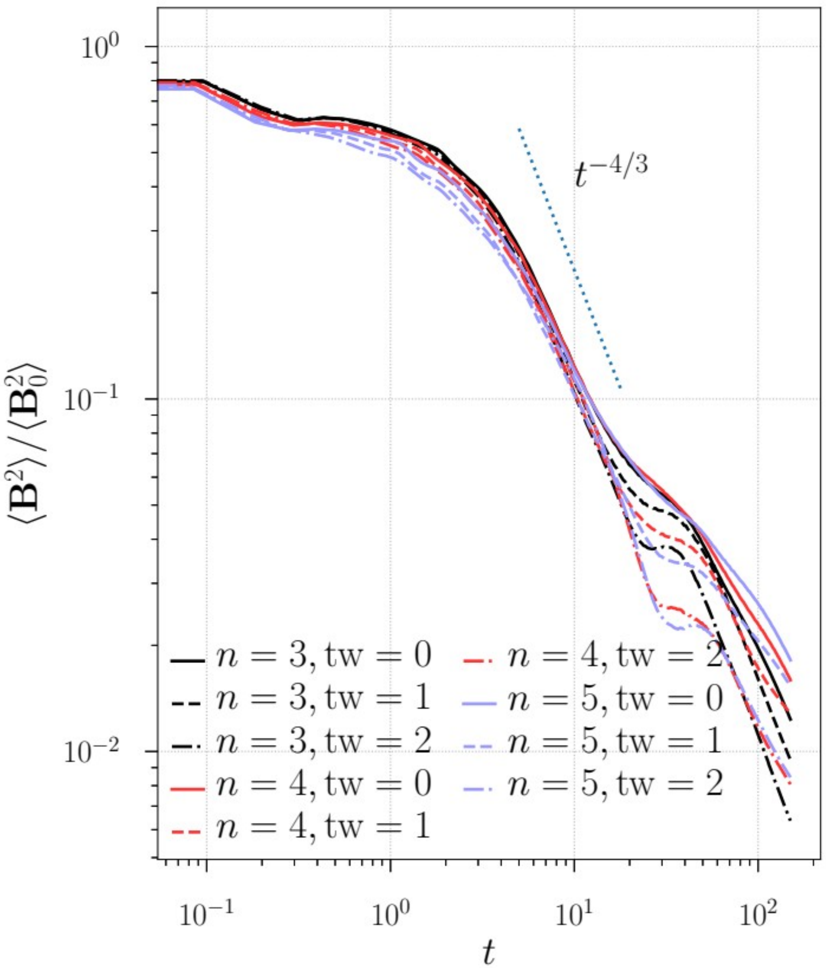
$t = 0$

$t = 150$

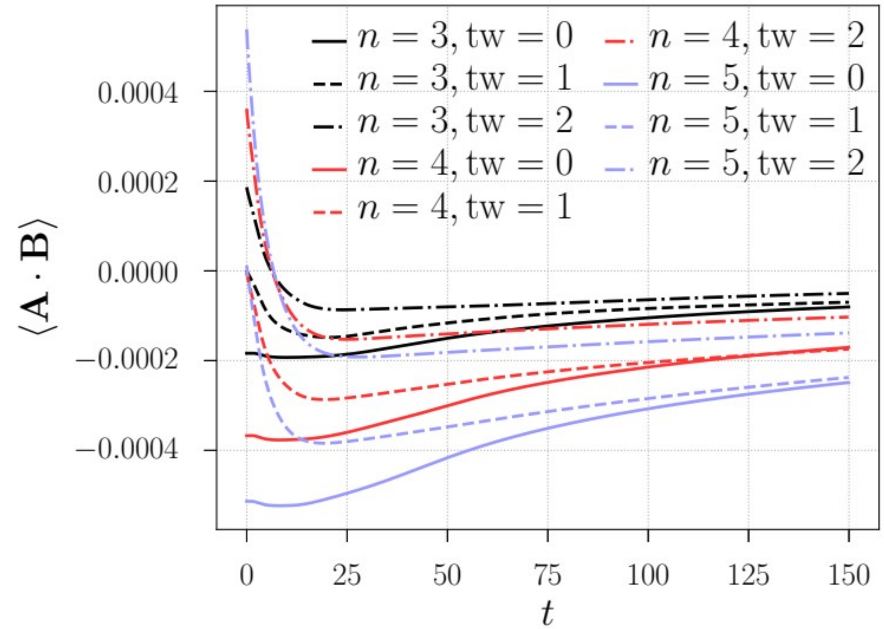


Knots

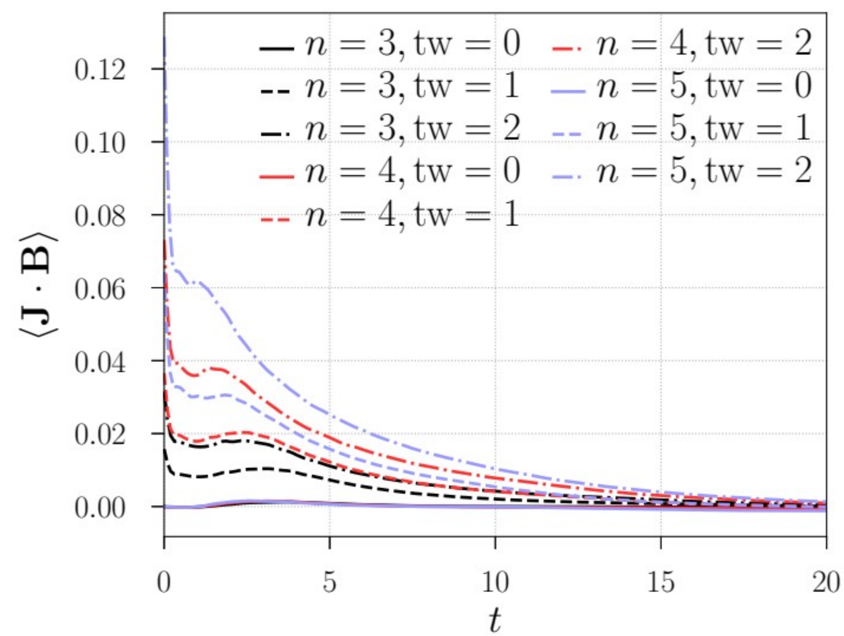




$\langle \mathbf{B}^2 \rangle / \langle \mathbf{B}_0^2 \rangle$
10^0
10^{-1}
10^{-2}
$t^{-4/3}$
$n = 3, \mathrm{tw} = 0$
$n = 3, \mathrm{tw} = 1$
$n = 3, \mathrm{tw} = 2$
$n = 4, \mathrm{tw} = 0$
$n = 4, \mathrm{tw} = 1$
$n = 4, \mathrm{tw} = 2$
$n = 5, \mathrm{tw} = 0$
$n = 5, \mathrm{tw} = 1$
$n = 5, \mathrm{tw} = 2$
10^{-1}
10^0
10^1
10^2
t



$\langle \mathbf{A} \cdot \mathbf{B} \rangle$
0.0004
0.0002
0.0000
−0.0002
−0.0004
$n = 3, \mathrm{tw} = 0$
$n = 3, \mathrm{tw} = 1$
$n = 3, \mathrm{tw} = 2$
$n = 4, \mathrm{tw} = 0$
$n = 4, \mathrm{tw} = 1$
$n = 4, \mathrm{tw} = 2$
$n = 5, \mathrm{tw} = 0$
$n = 5, \mathrm{tw} = 1$
$n = 5, \mathrm{tw} = 2$
0
25
50
75
100
125
150
t



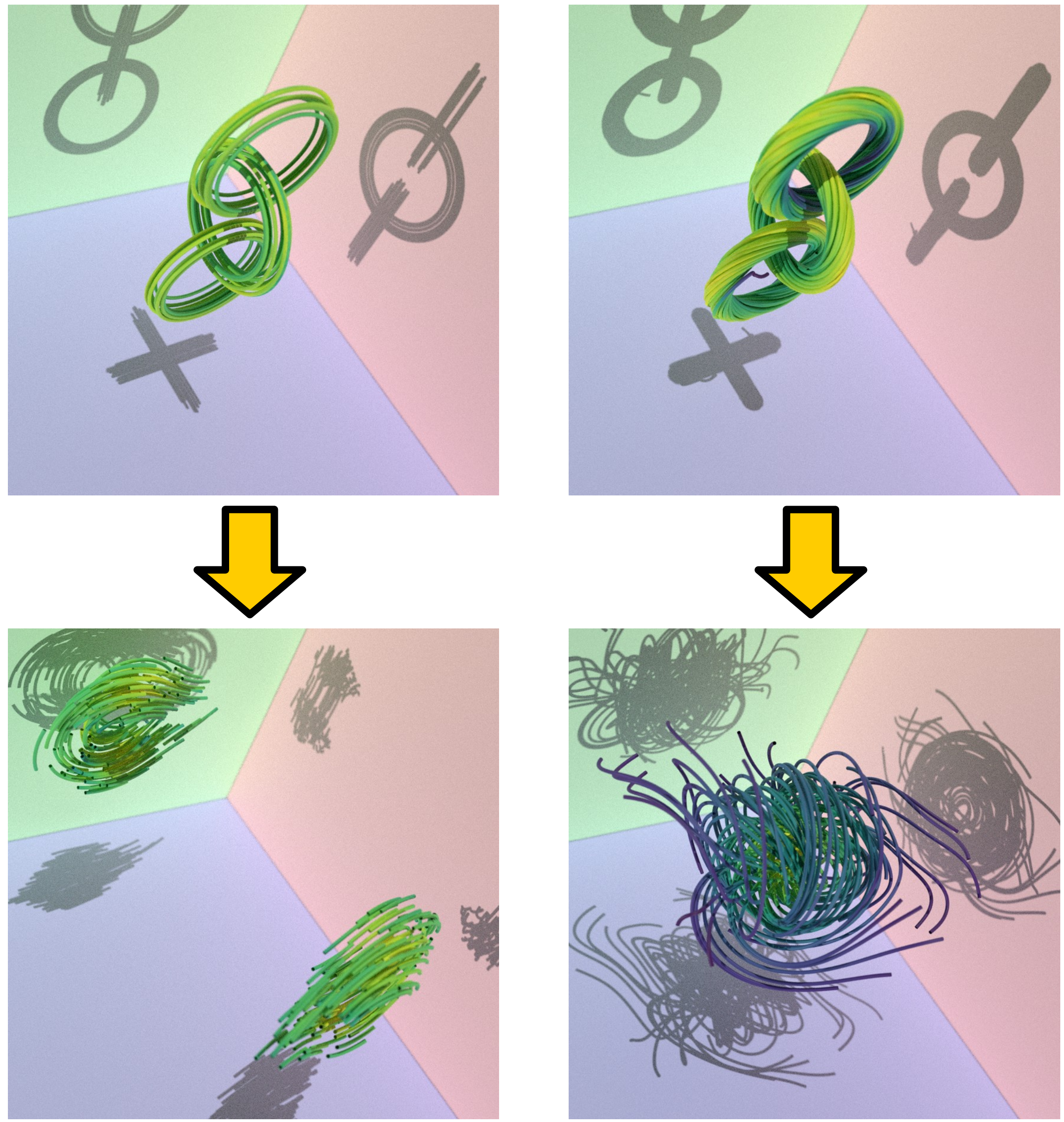
$\langle \mathbf{J} \cdot \mathbf{B} \rangle$
0.12
0.10
0.08
0.06
0.04
0.02
0.00
$n = 3, \mathrm{tw} = 0$
$n = 3, \mathrm{tw} = 1$
$n = 3, \mathrm{tw} = 2$
$n = 4, \mathrm{tw} = 0$
$n = 4, \mathrm{tw} = 1$
$n = 4, \mathrm{tw} = 2$
$n = 5, \mathrm{tw} = 0$
$n = 5, \mathrm{tw} = 1$
$n = 5, \mathrm{tw} = 2$
0
5
10
15
20
t

Triple Rings (non-helical)

$H_{\mathrm{m}} = 0, \mathrm{tw} = 0$ $\quad$ $H_{\mathrm{m}} = 1, \mathrm{tw} = 1$

$t = 0$

$t = 150$

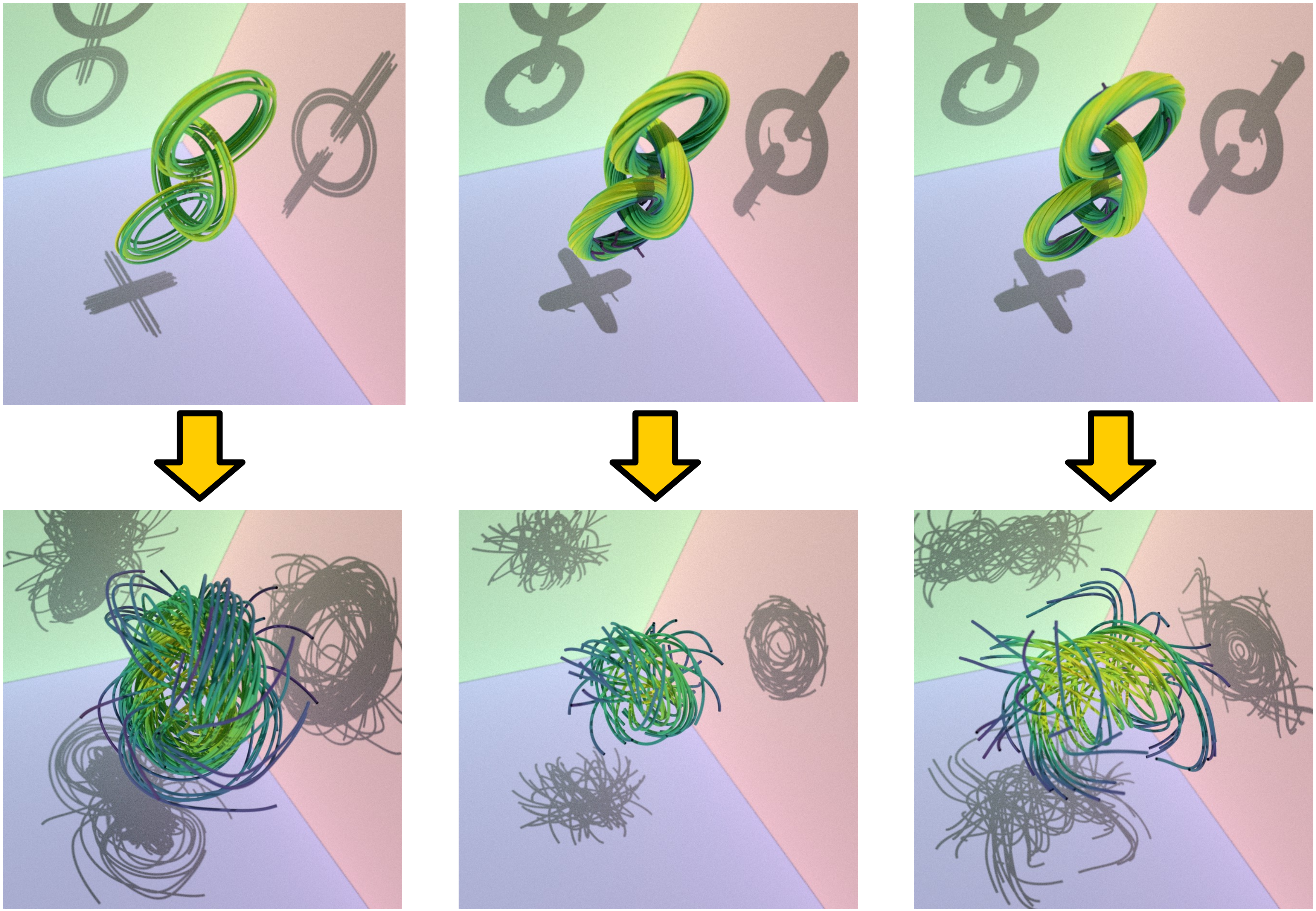


Triple Rings (helical)

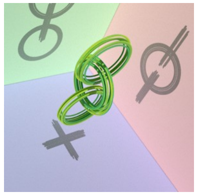
$H_{\mathrm{m}} = 1, \mathrm{tw} = 0$ | $H_{\mathrm{m}} = 2, \mathrm{tw} = 1$ | $H_{\mathrm{m}} = 0, \mathrm{tw} = -1$

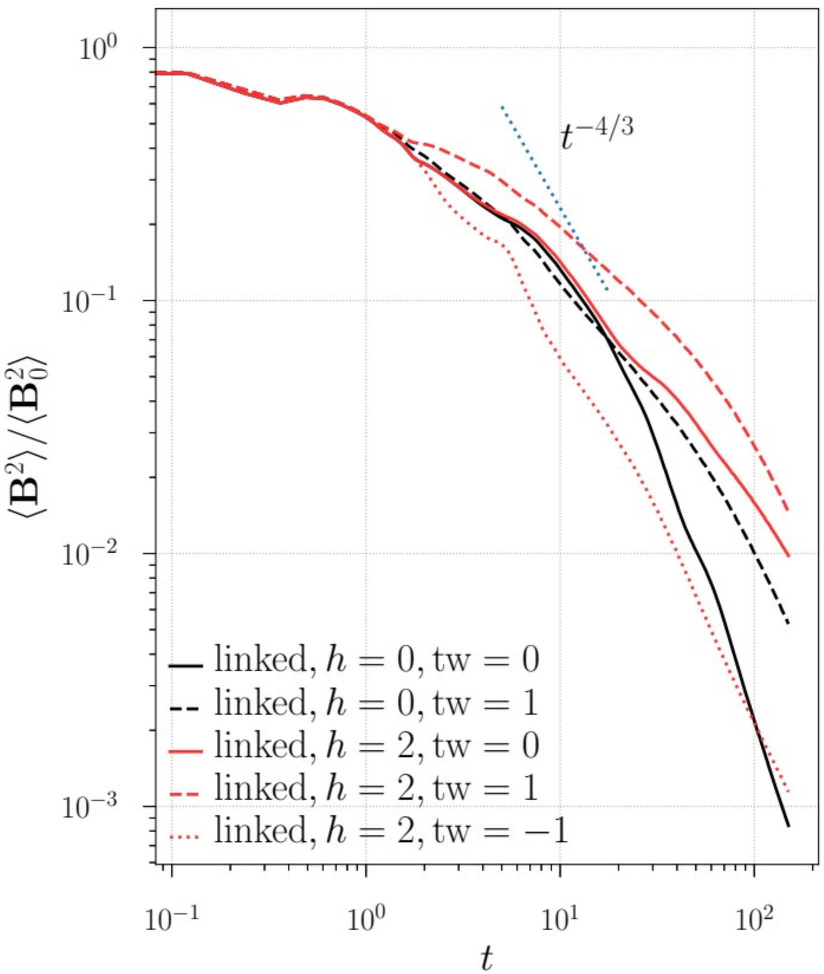
$t = 0$

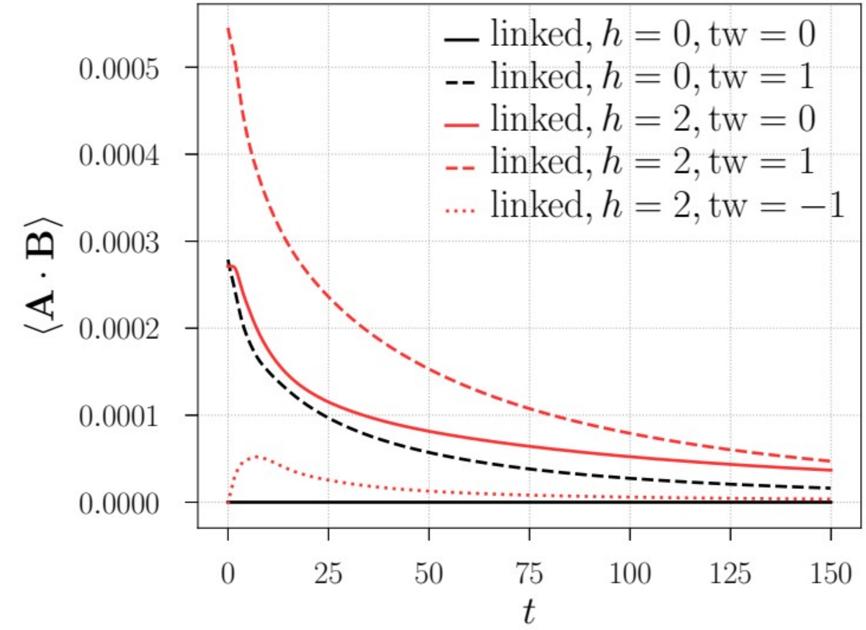
$t = 150$

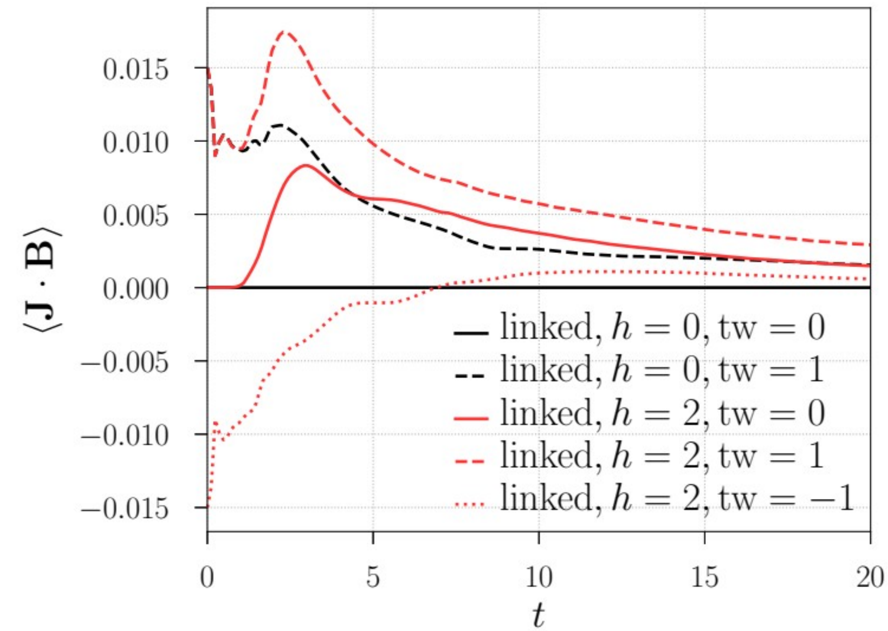


Triple Rings







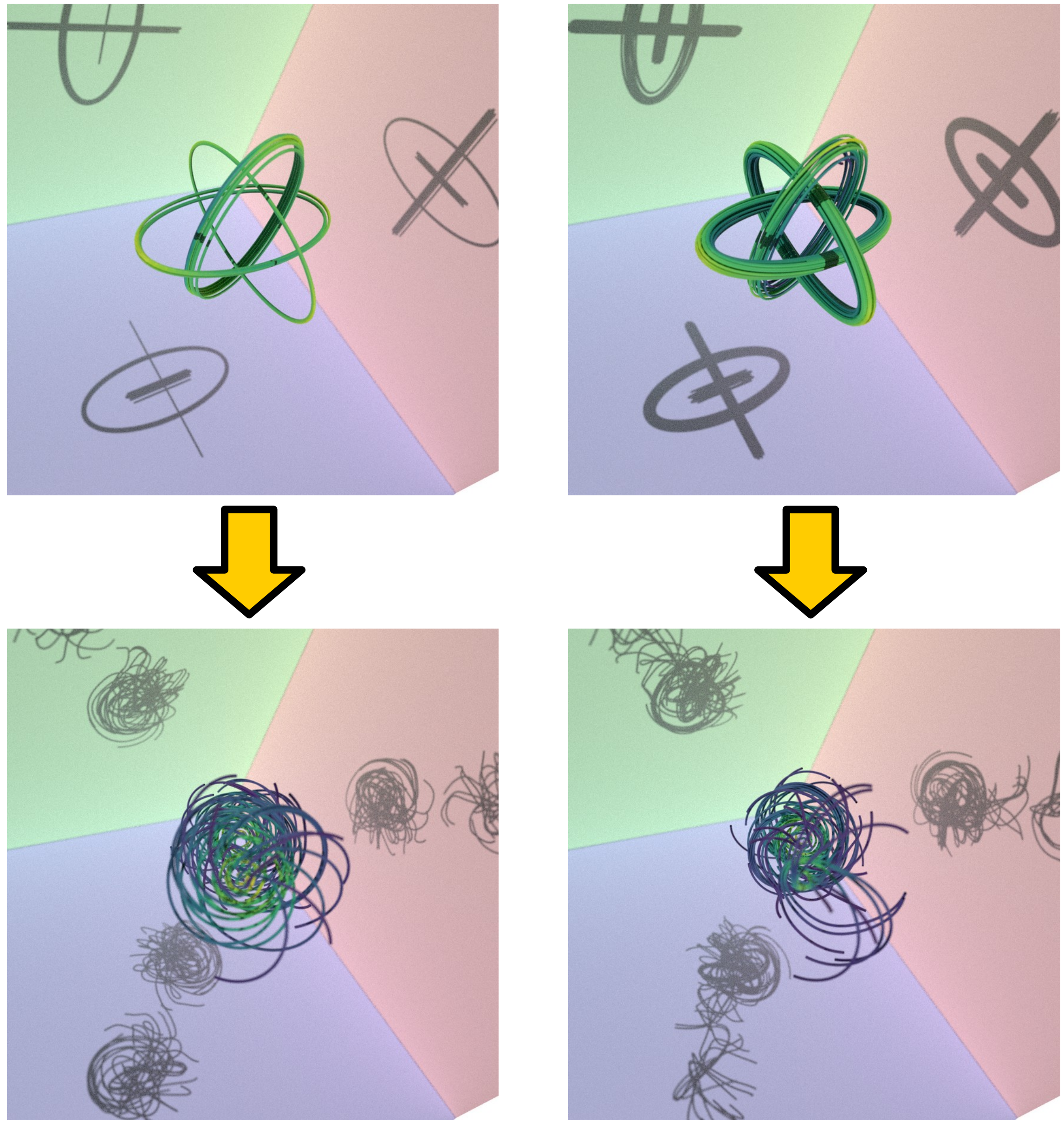


Borromean Rings

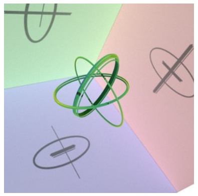
$H_{\mathrm{m}} = 0, \mathrm{tw} = 0$ $\quad$ $H_{\mathrm{m}} = 1, \mathrm{tw} = 1$

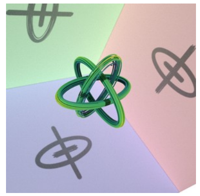
$t = 0$

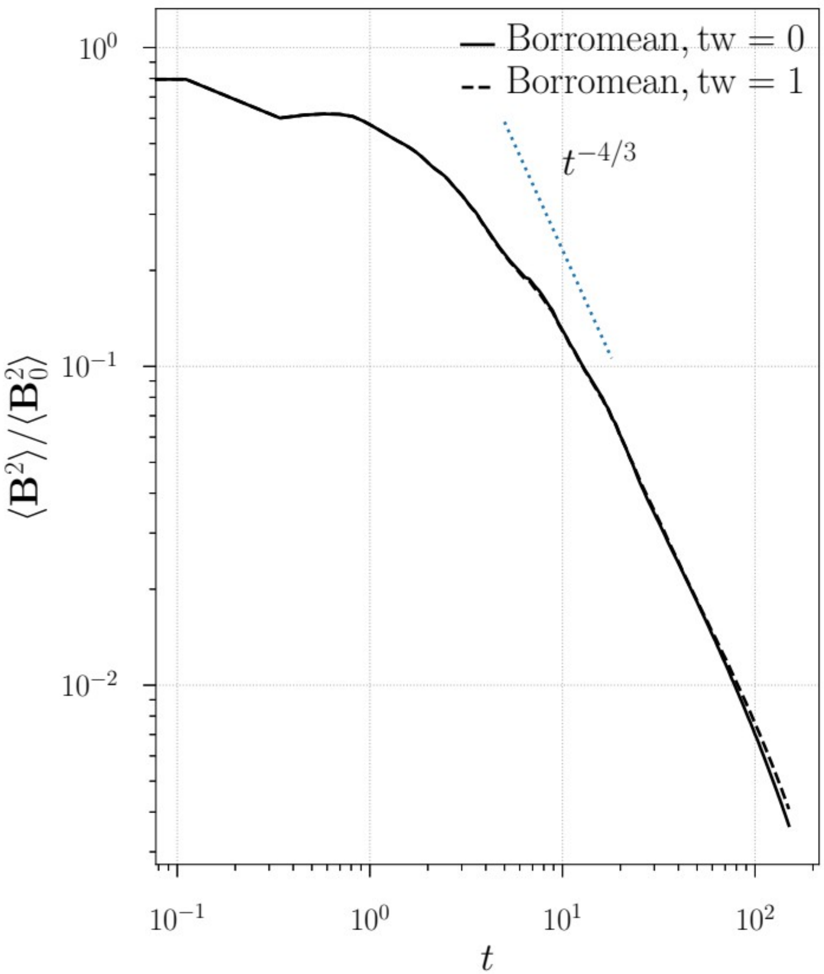
$t = 150$

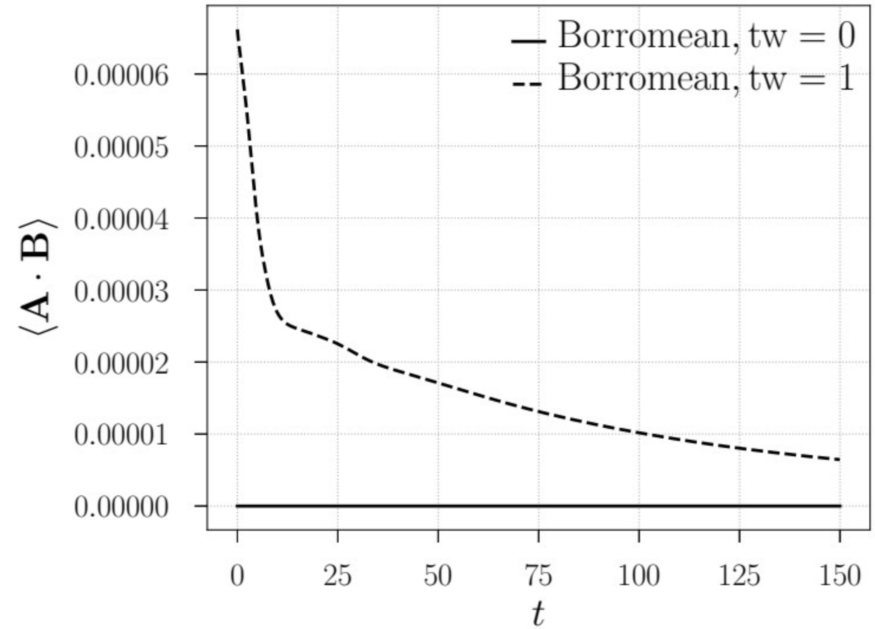


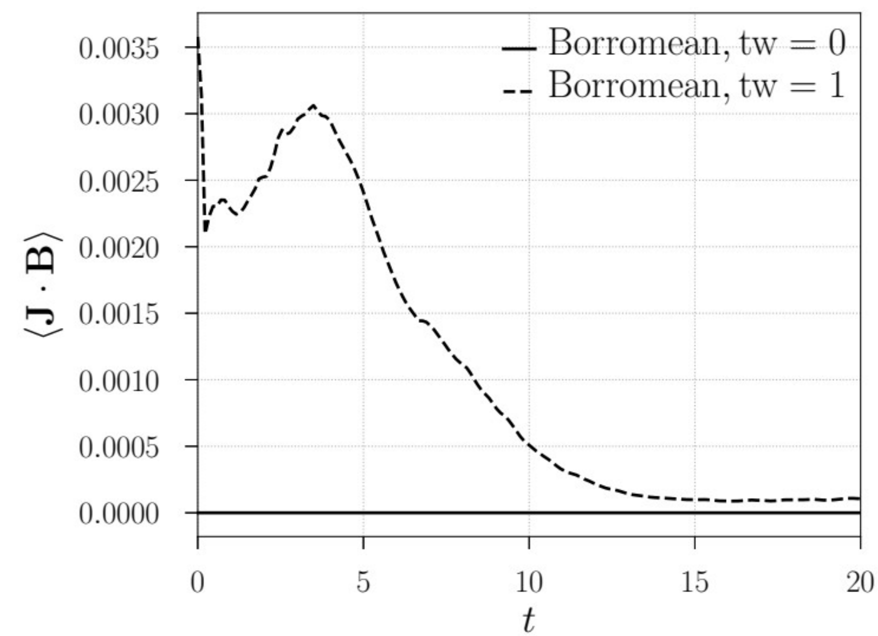
Borromean Rings









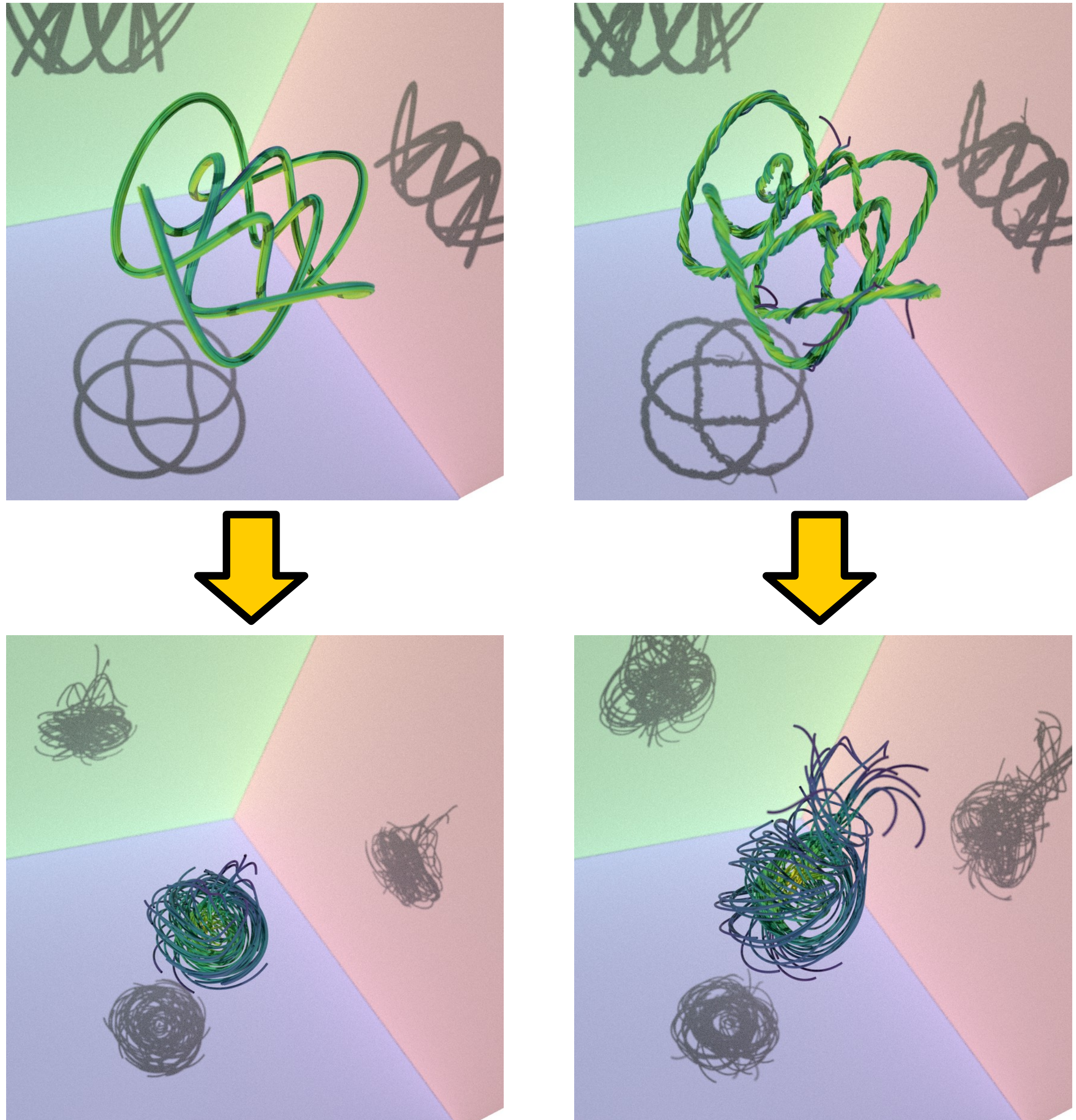


IUCAA Knots

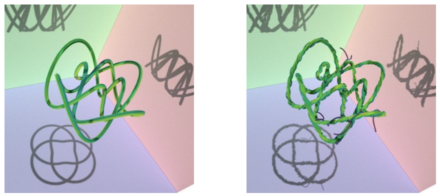
$H_{\rm m} = 0, {\rm tw} = 0$ $\quad$ $H_{\rm m} = 1, {\rm tw} = 1$

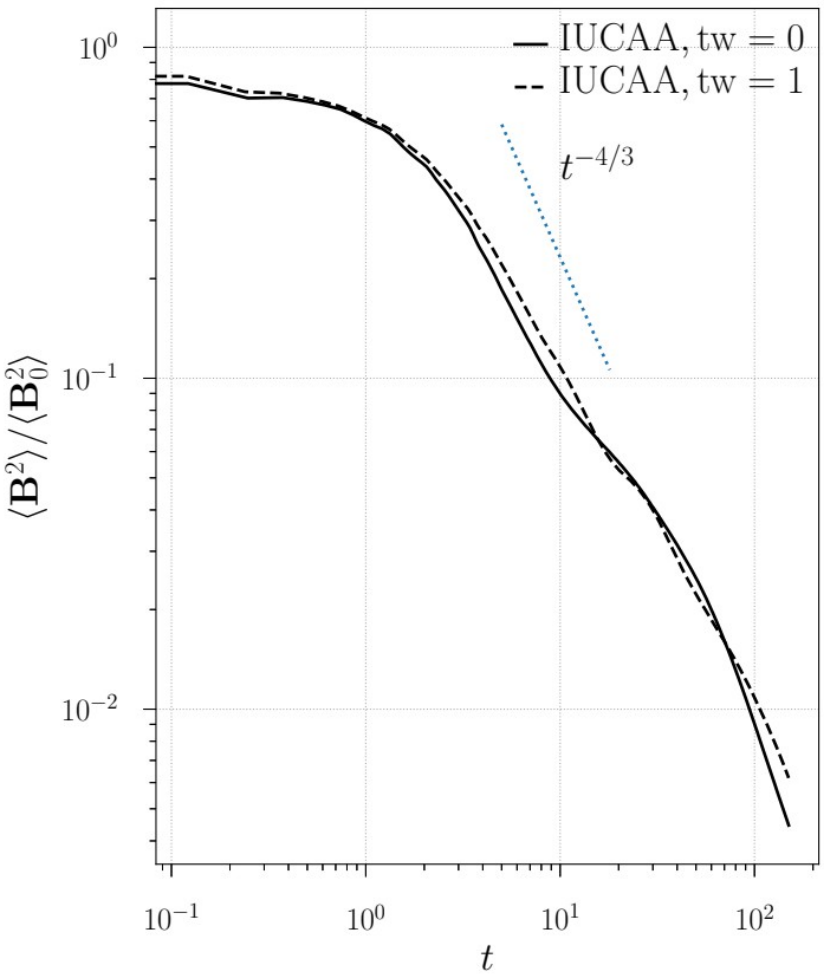
$t = 0$

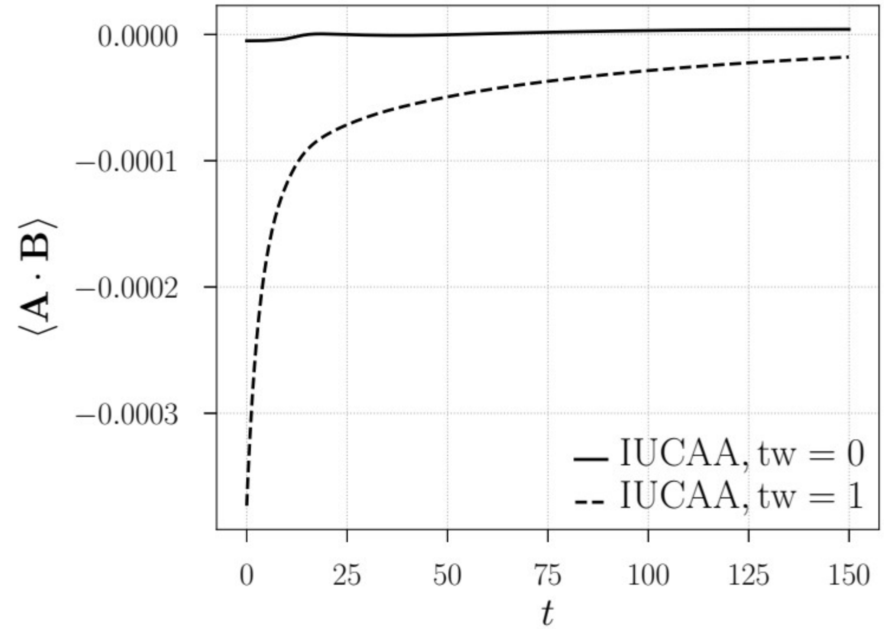
$t = 150$

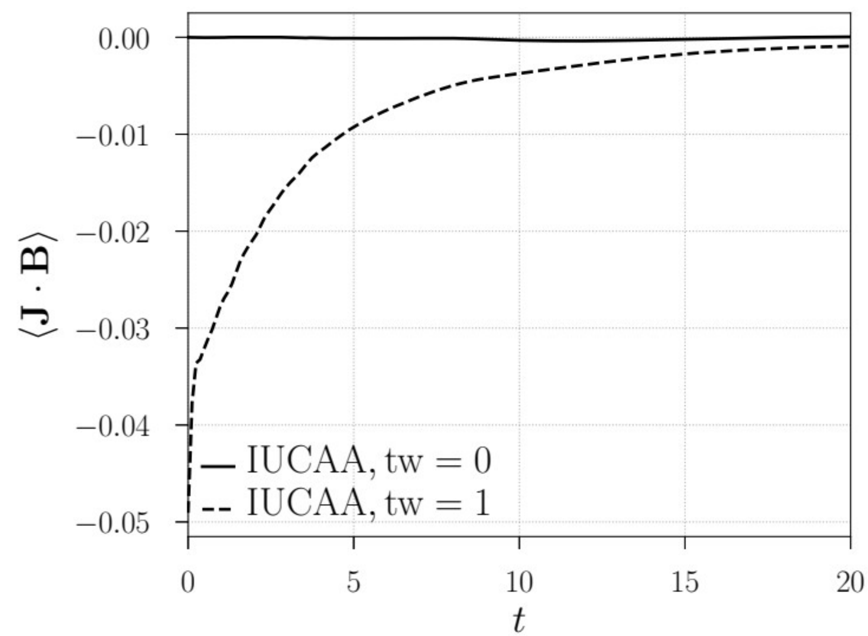


IUCAA Knot




IUCAA, tw = 0
IUCAA, tw = 1
$t^{-4/3}$
10^0
10^{-1}
10^{-2}
$\langle \mathbf{B}^2 \rangle / \langle \mathbf{B}_0^2 \rangle$
10^{-1} 10^0 10^1 10^2
t


0.0000
−0.0001
−0.0002
−0.0003
$\langle \mathbf{A} \cdot \mathbf{B} \rangle$
IUCAA, tw = 0
IUCAA, tw = 1
0 25 50 75 100 125 150
t


0.00
−0.01
−0.02
−0.03
−0.04
−0.05
$\langle \mathbf{J} \cdot \mathbf{B} \rangle$
IUCAA, tw = 0
IUCAA, tw = 1
0 5 10 15 20
t

Saffman Invariant

$$I_H = \int \mathrm{d}^3 r \langle h(x) h(x+r) \rangle$$

(Hosking 2021)

- Non-zero for non-helical turbulence.
- Gauge invariant.
- Conserved quantity.
- Current results only for isotropic homogeneous turbulence

Conclusions

- Magnetic helicity as constraint on plasma dynamics.
- Magnetic helicity leads to stability at small magnetic energy.
- Non-helical field exhibit intermediate energy decay (sometimes).
- Helicity alone not a good indicator. Consider helicity production.
- Saffman invariant for non-helical fields.

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