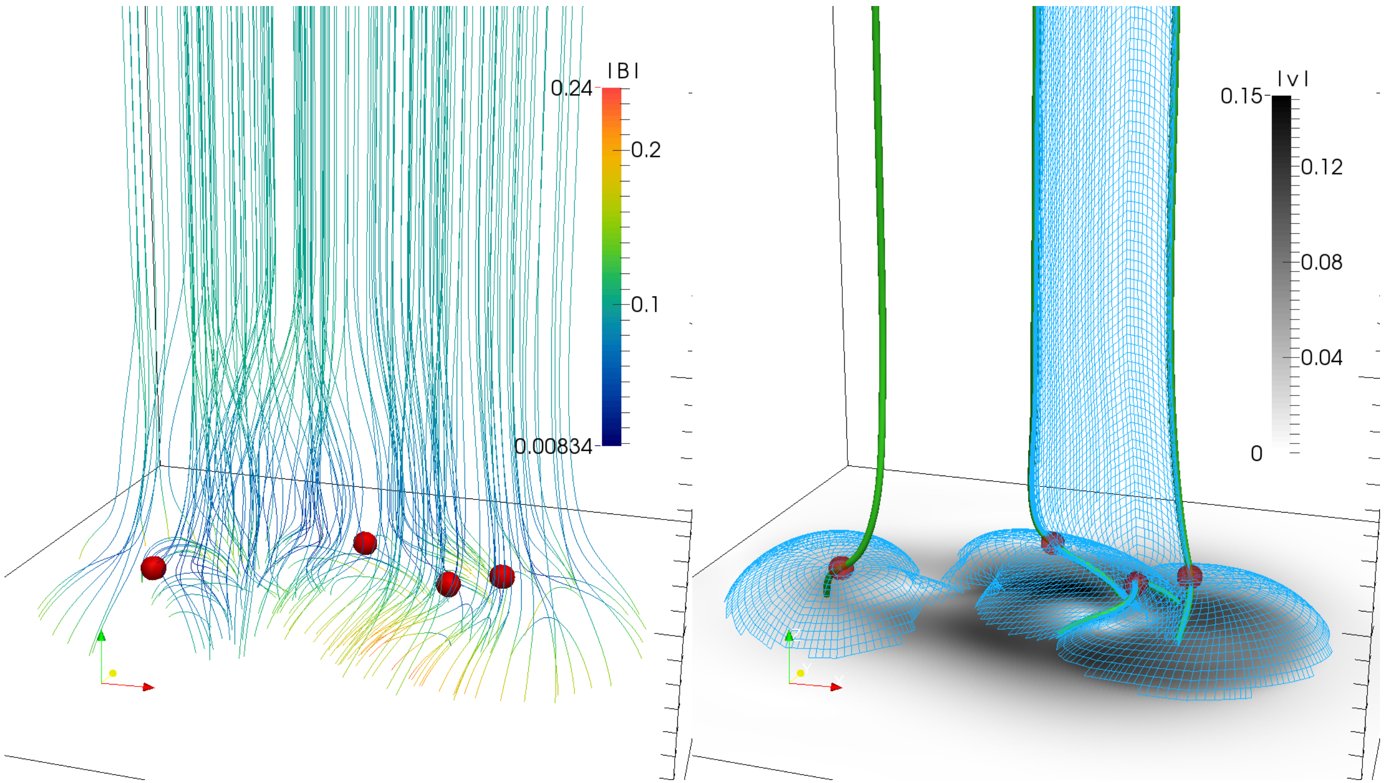
Magnetic field topology and electric current formation in plasma.

Simon Candelaresi



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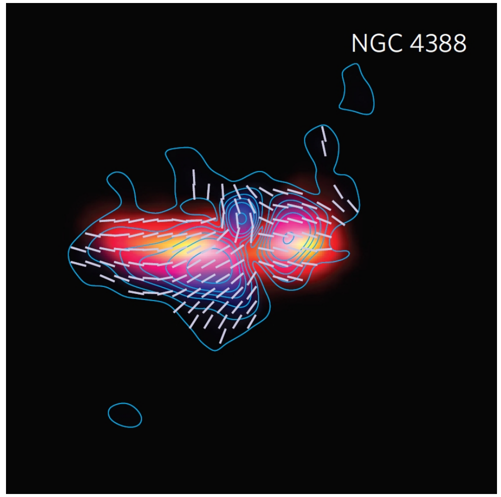
Magnetic Fields in Nature

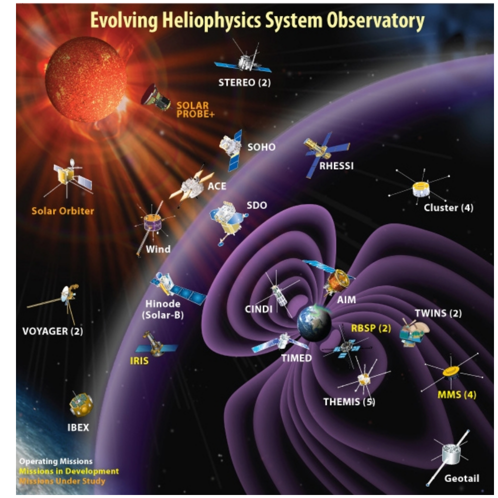
Galaxies:
10e-6 G



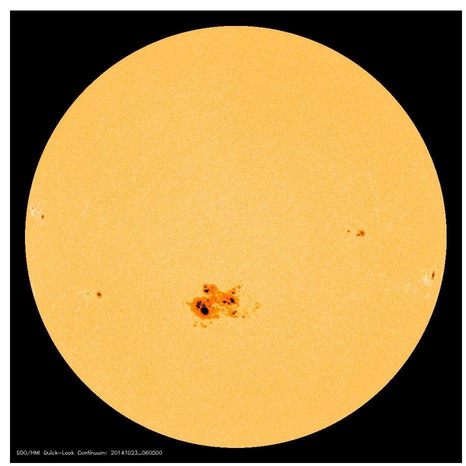
Pfrommer (2010)

Earth:
0.1-1G

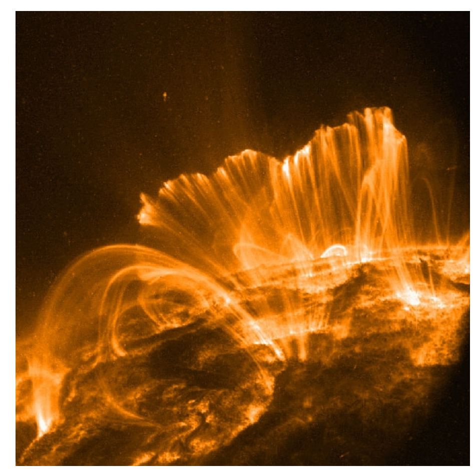
(NASA)



Sun:
2-2,000G
$\beta = 0.01$
Rm = 10^6 - 10^12



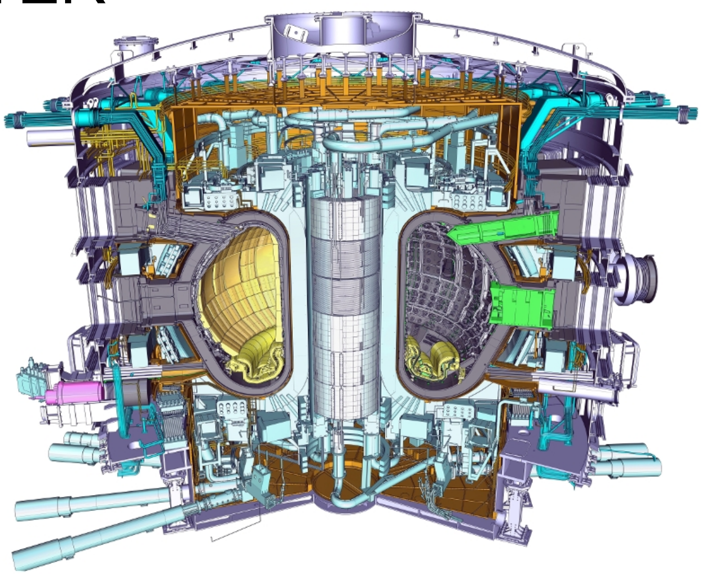
Continuum
2014-10-23 *(NASA)*

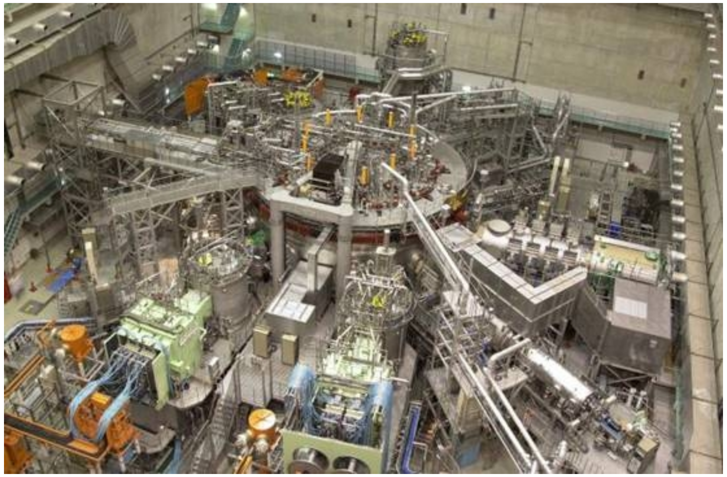


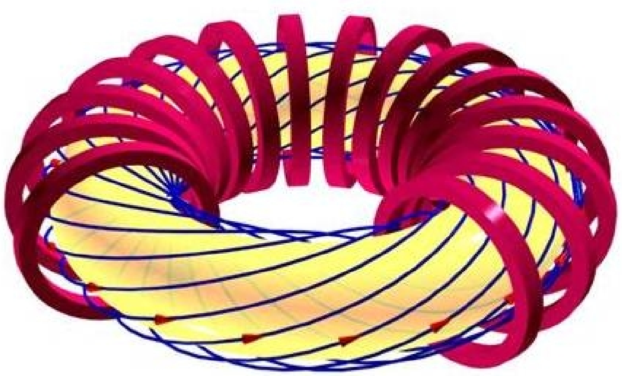
Coronal Loops *(NASA)*

Confining Plasma

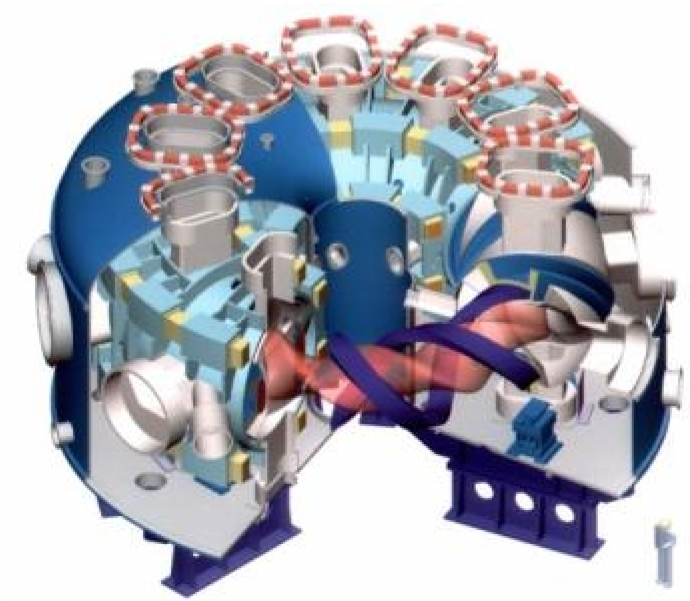
ITER







Team TONUS (2014)



Large Helical Device, Toki (Japan)

Field's Environment in the Corona

Magnetically dominated:

magnetic pressure >> thermal pressure

$$B^2/(2\mu_0) \gg nk_\mathrm{B}T$$

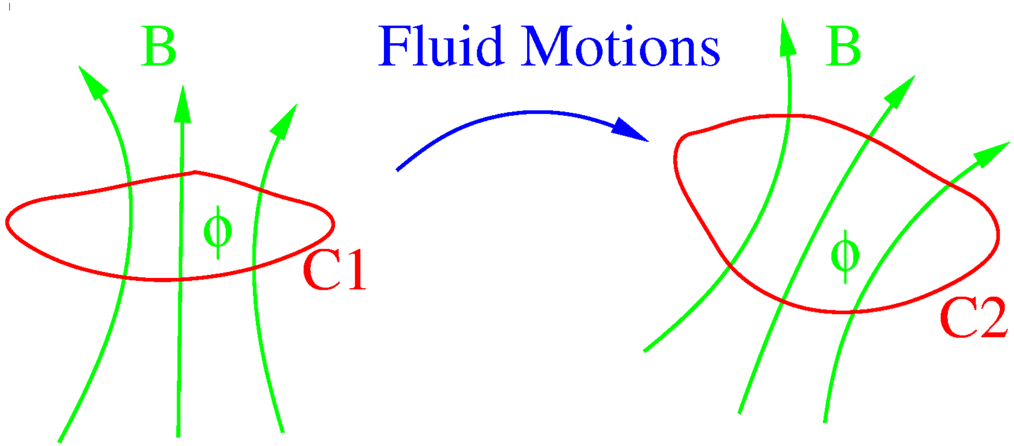
$$\beta = 2\mu_0 \frac{nk_\mathrm{B}T}{B^2} \ll 1$$

Solar corona: $\beta \approx 0.01$

Frozen-in magnetic flux:

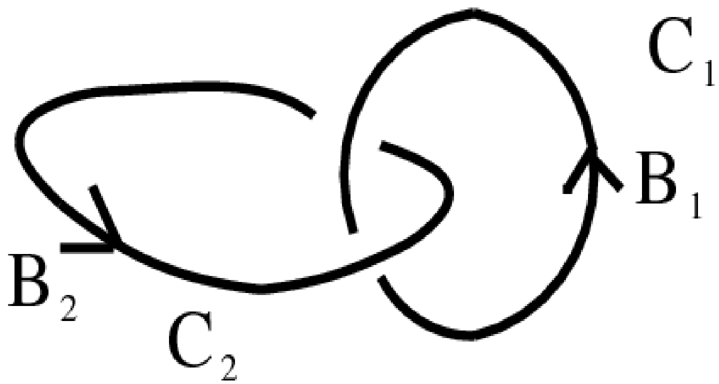
magnetic resistivity small: $t_\mathrm{dissipation} \gg t_\mathrm{dynamical}$

⇨ Magnetic field is *frozen-in* to the fluid.

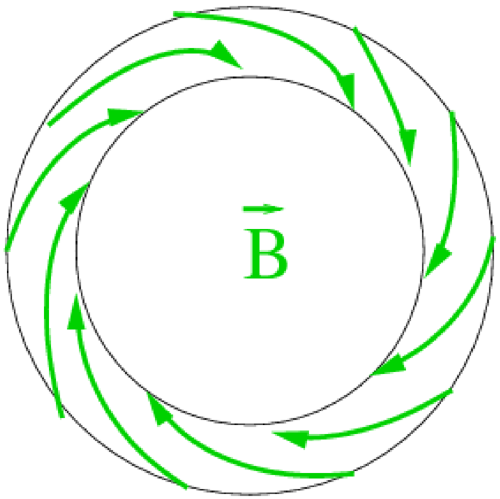


Batchelor (1950)

Topologies of Magnetic Fields

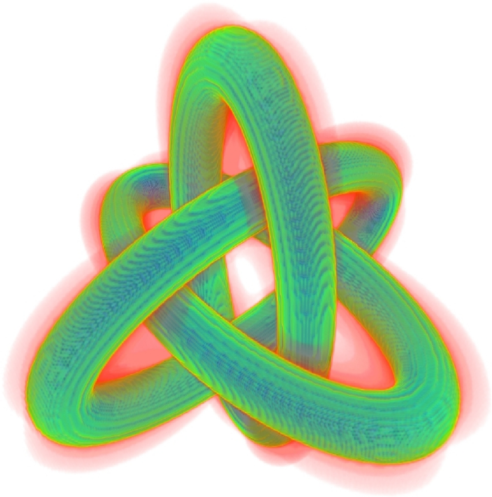


Hopf link

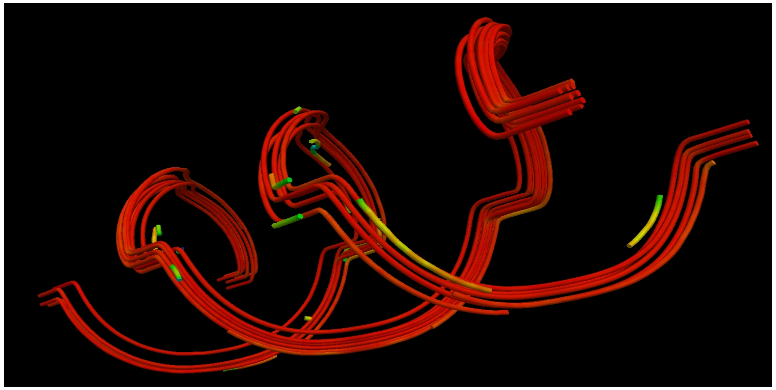


twisted field

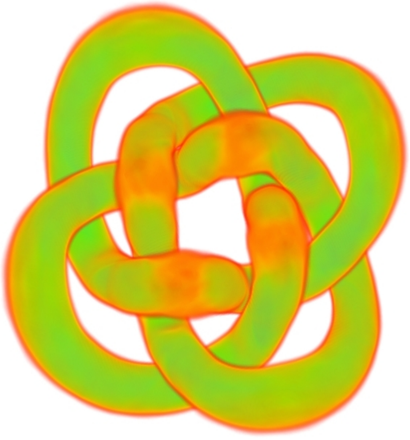
trefoil knot



Borromean rings



magnetic braid



IUCAA knot

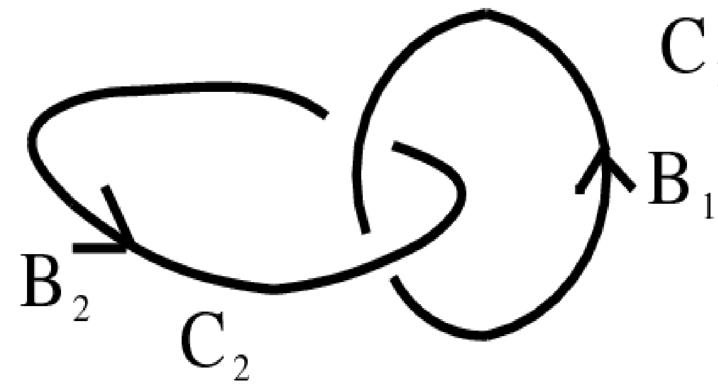
Magnetic Field Topology

Measure for the topology:

$$H_{\mathrm{M}} = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 2n\phi_1\phi_2$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B} \qquad \phi_i = \int_{S_i} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$$

$n =$ number of mutual linking



Moffatt (1969)

Conservation of magnetic helicity:

$$\lim_{\eta \to 0} \frac{\partial}{\partial t} \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0 \qquad \eta = \text{magnetic resistivity}$$

Realizability condition:

$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$

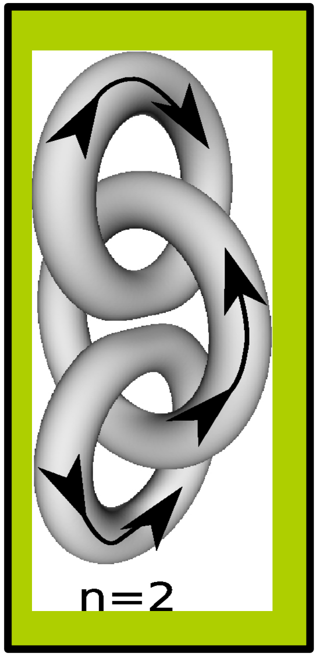
Arnold (1974)

Magnetic energy is bound from below by magnetic helicity.

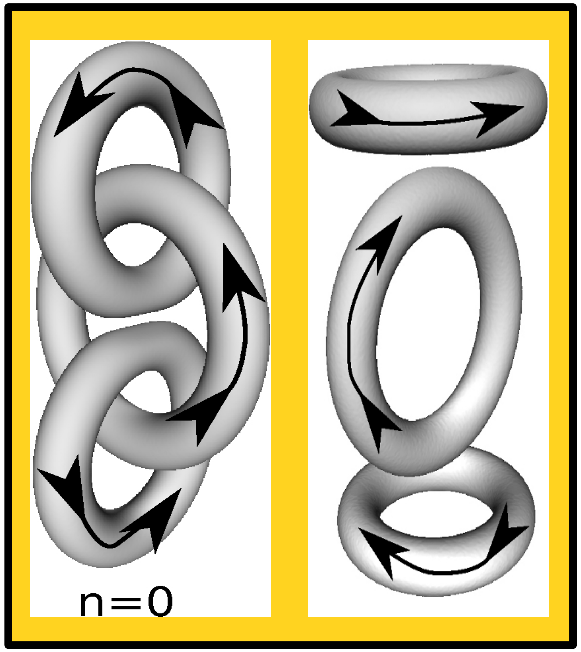
Interlocked Flux Rings

actual linking vs. magnetic helicity

$H_\mathrm{M} \neq 0$



$H_\mathrm{M} = 0$



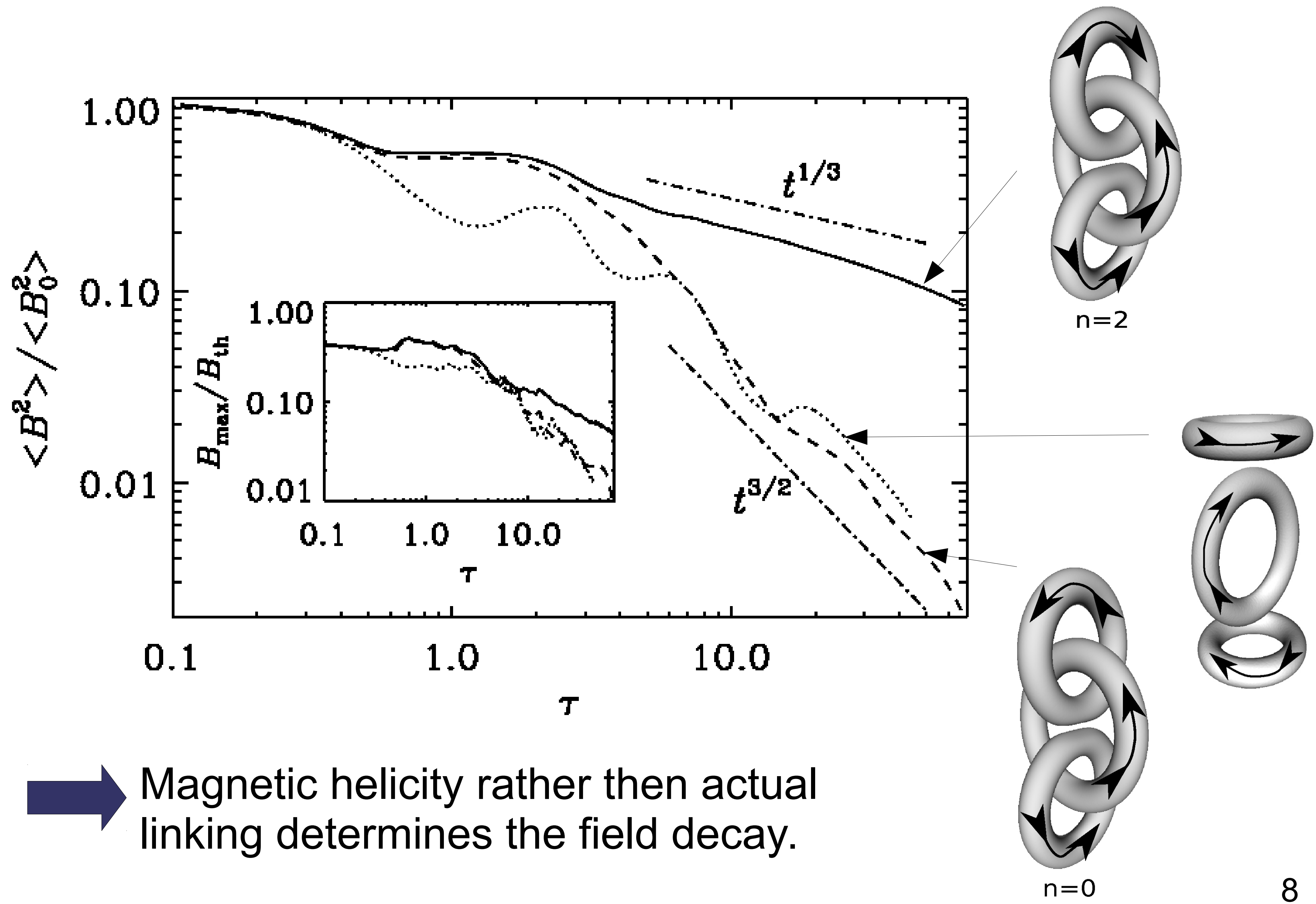
- initial condition: flux tubes
- isothermal compressible gas
- viscous medium
- periodic boundaries

(Del Sordo et al. 2010)

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A} \qquad \frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_\mathrm{S}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_\mathrm{visc}$$

Interlocked Flux Rings



Magnetic helicity rather then actual linking determines the field decay.

Stability Criteria

Ideal MHD: $\eta = 0$

↓

Induction equation: $\dfrac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B})$

	constraint	equilibrium
Woltjer (1958):	$\dfrac{\partial}{\partial t} \displaystyle\int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 0$	$\boldsymbol{\nabla} \times \boldsymbol{B} = \alpha \boldsymbol{B}$
Taylor (1974):	$\dfrac{\partial}{\partial t} \displaystyle\int_{\tilde{V}} \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 0$	$\boldsymbol{\nabla} \times \boldsymbol{B} = \alpha(a, b) \boldsymbol{B}$

$\alpha(a, b)$: constant along field line

V total volume $\quad$ $\tilde{V}$ volume along magnetic field line

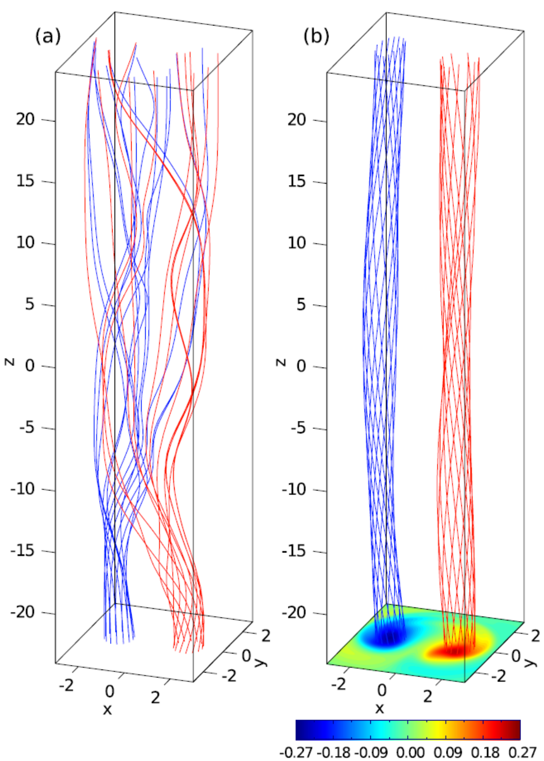
Taylor Relaxation

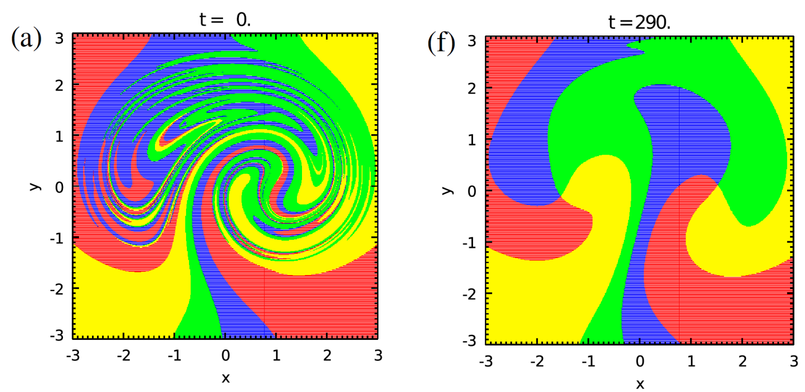
Field line magnetic helicity conservation

final state is non-linear force-free: $\nabla \times \mathbf{B} = \lambda(a,b)\mathbf{B}$

Taylor (1974)

Does the system always reach this state?





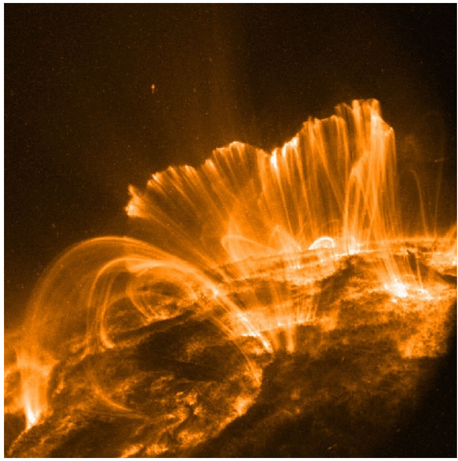
Yeates (2010)

Not necessarily. Additional topological degree must be conserved.

Force-Free Magnetic Fields

Solar corona: low plasma beta and magnetic resistivity

NASA

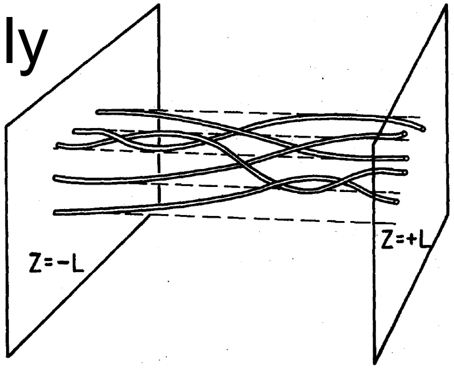


- Force-free magnetic fields
- Minimum energy state

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \;\Leftrightarrow\; \nabla \times \mathbf{B} = \alpha \mathbf{B}$$

Parker: Equilibrium with the same topology exists only if the twist varies uniformly along the field lines.
Strongly braided fields → topological dissipation.

(Parker 1972)



Braided fields from foot point motion complex enough. *(Parker 1983)*

Solutions possible with filamentary current structures (sheets).

(Mikic 1989, Low 2010)

Methods

Ideal (non-resistive) evolution
Frozen in magnetic field ⇒ use Lagrangian method

(Batchelor, 1950)

Preserves topology and divergence-freeness.

Magneto-frictional term: $\mathbf{u} = \mathbf{J} \times \mathbf{B}$ $\qquad \mathbf{J} = \nabla \times \mathbf{B}$

⇒ $\frac{\mathrm{d}E_{\mathrm{M}}}{\mathrm{d}t} < 0$ *(Craig and Sneyd 1986)*

Fluid with pressure: $\mathbf{u} = \mathbf{J} \times \mathbf{B} - \beta \nabla \rho$

Fluid with inertia: $\mathrm{d}\mathbf{u}/\mathrm{d}t = (\mathbf{J} \times \mathbf{B} - \nu \mathbf{u} - \beta \nabla \rho)/\rho$

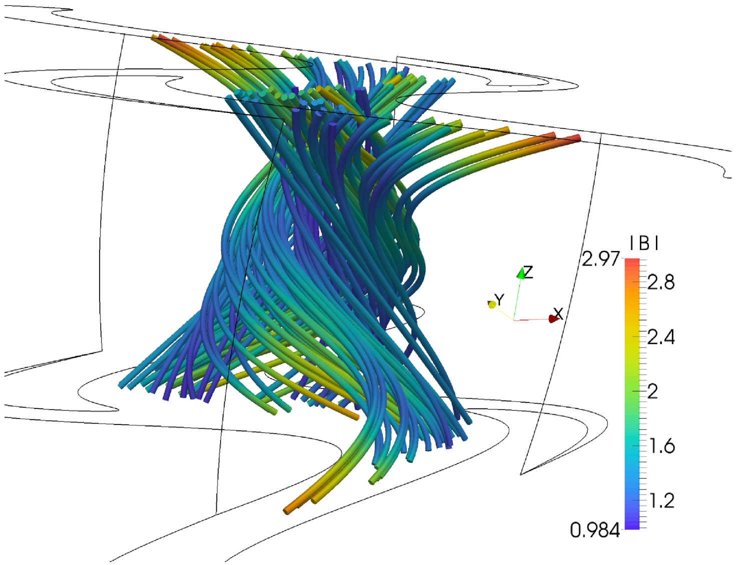
For $\mathbf{J} = \nabla \times \mathbf{B}$ use mimetic numerical operators.

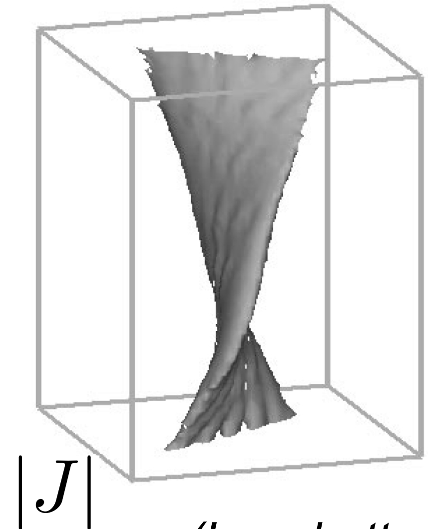
(Hyman, Shashkov 1997)

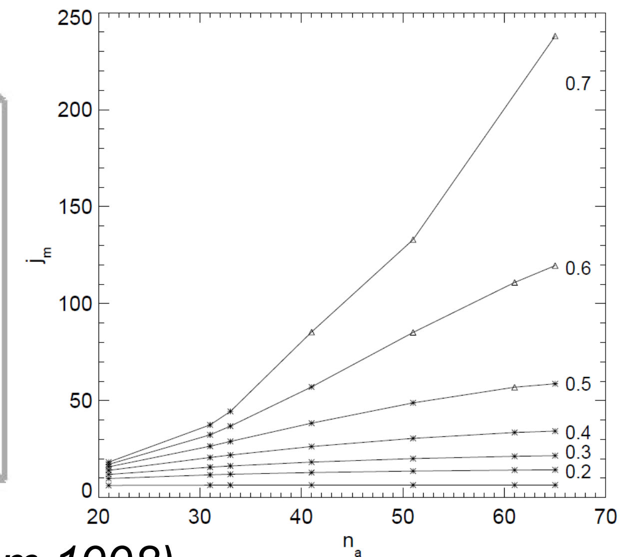
Own GPU code GLEMuR: (https://github.com/SimonCan/glemur)

(Candelaresi et al. 2014)

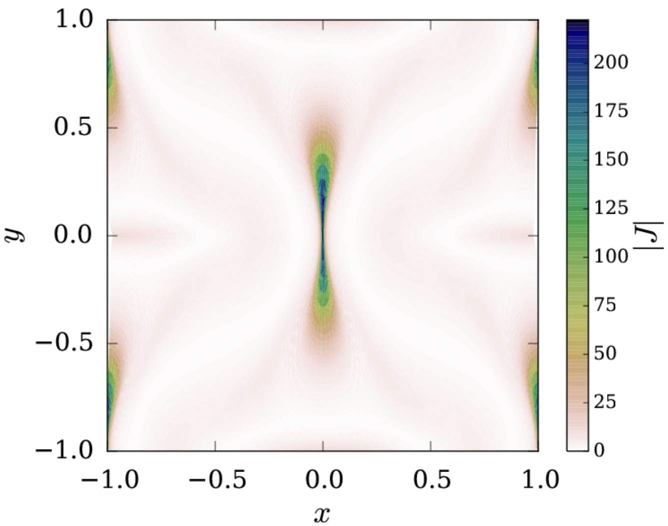
Distorted Magnetic Fields

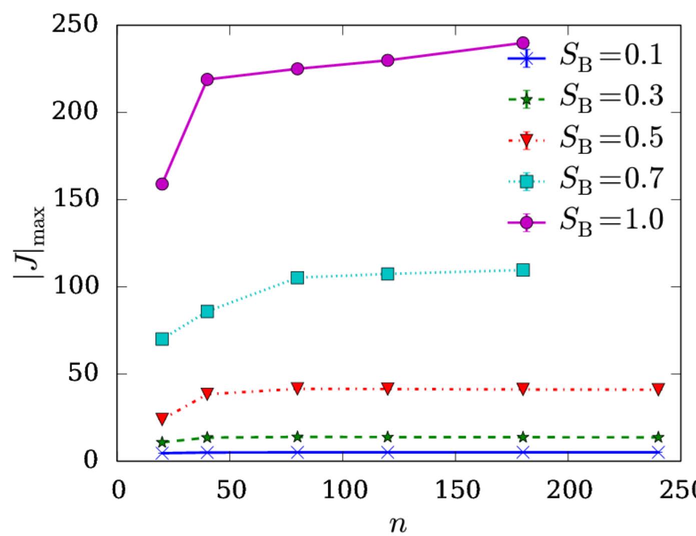






(Longbottom 1998)





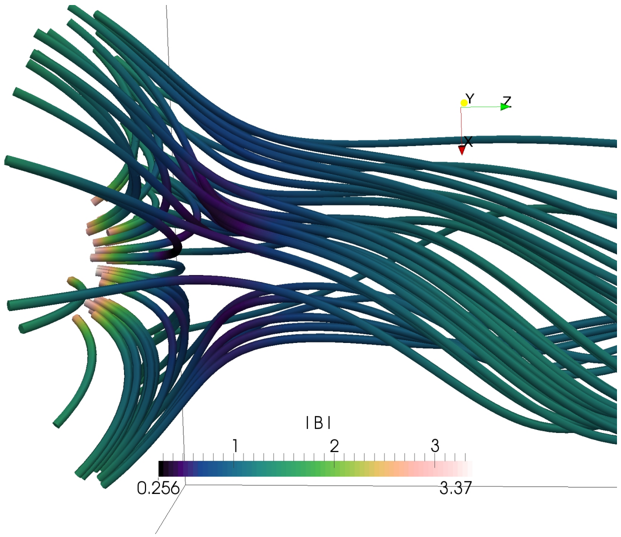
- resolved current concentrations
- shear leads to strong currents

(Candelaresi et al. 2015)

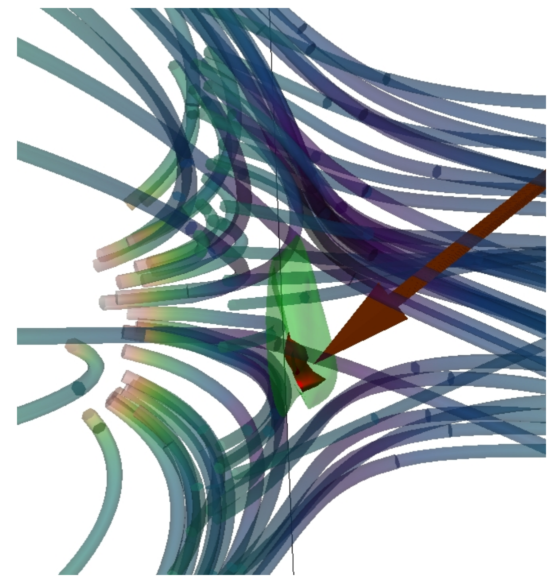
Magnetic Nulls

Singular current sheets observed at magnetic nulls (B = 0)

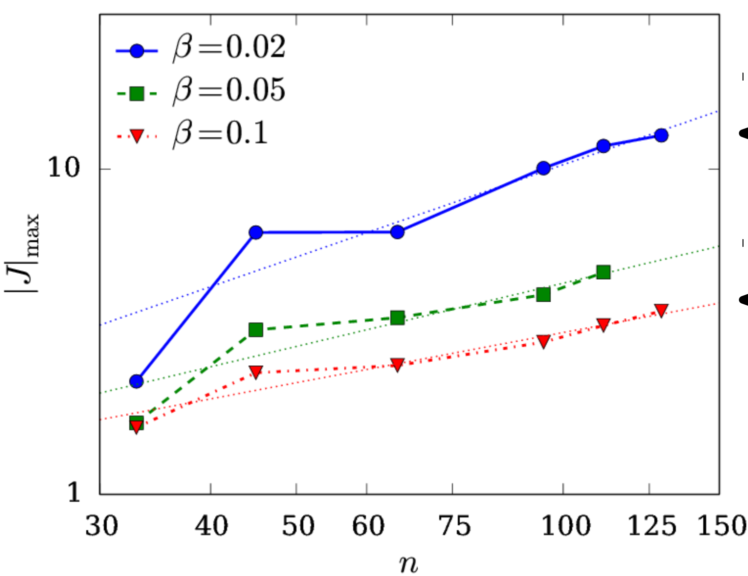
(Syrovatskiĭ 1971; Pontin & Craig 2005; Fuentes-Fernández & Parnell 2012, 2013; Craig & Pontin 2014)



$$\mathbf{u} = \mathbf{J} \times \mathbf{B}$$

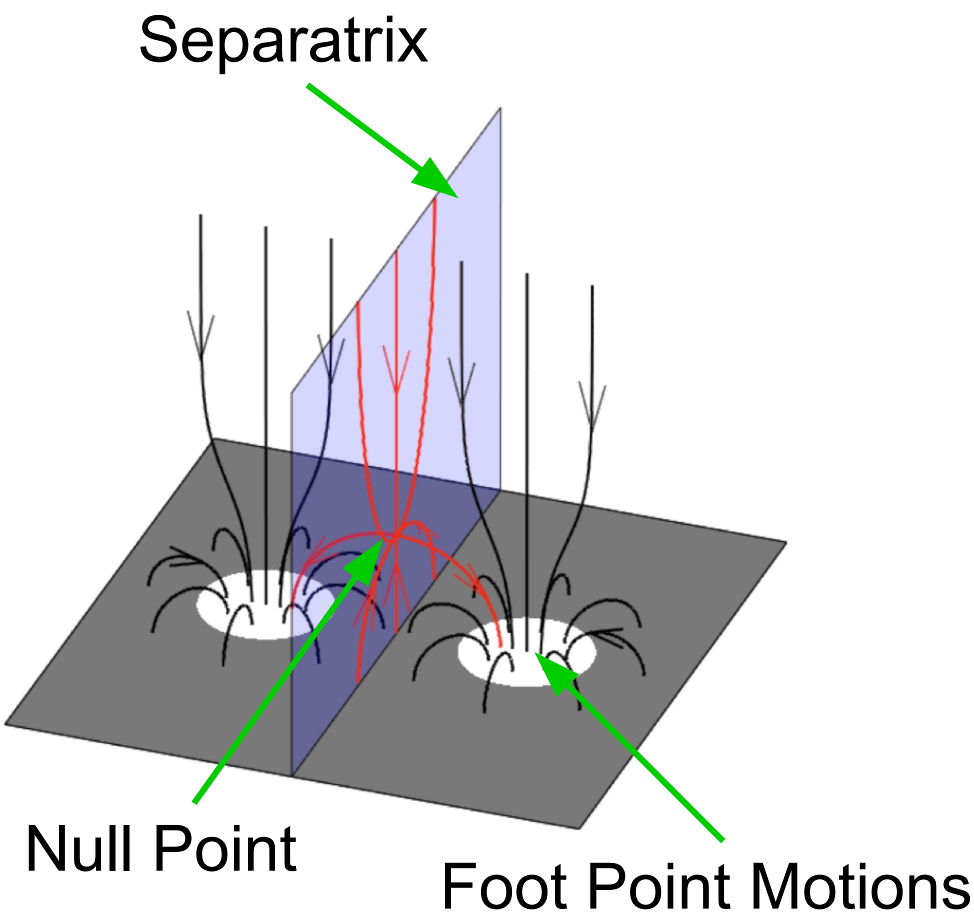


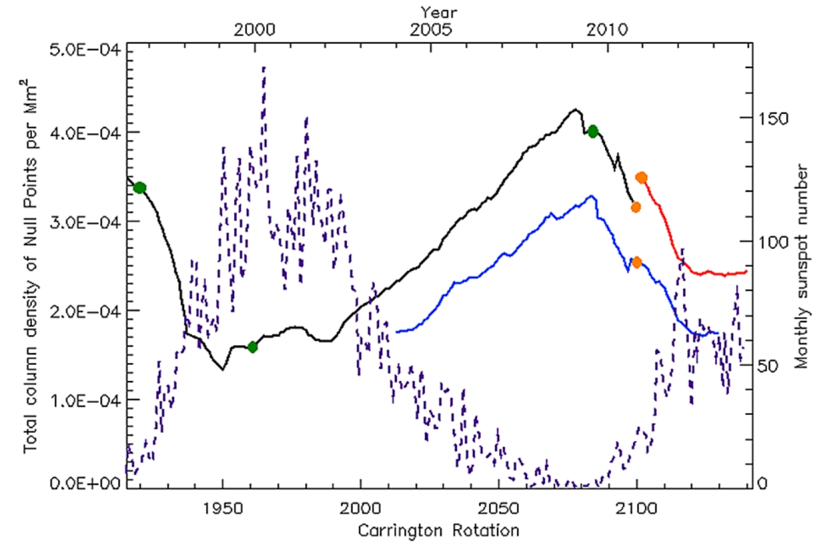
$$\mathbf{u} = \mathbf{J} \times \mathbf{B} - \beta \nabla \rho$$

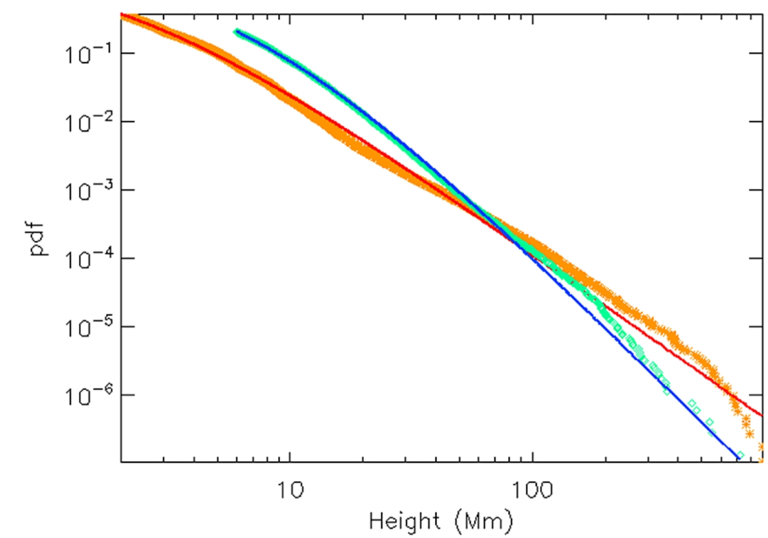


- singular current sheets at magnetic nulls
- Pressure cannot balance singularity.

Magnetic Carpet







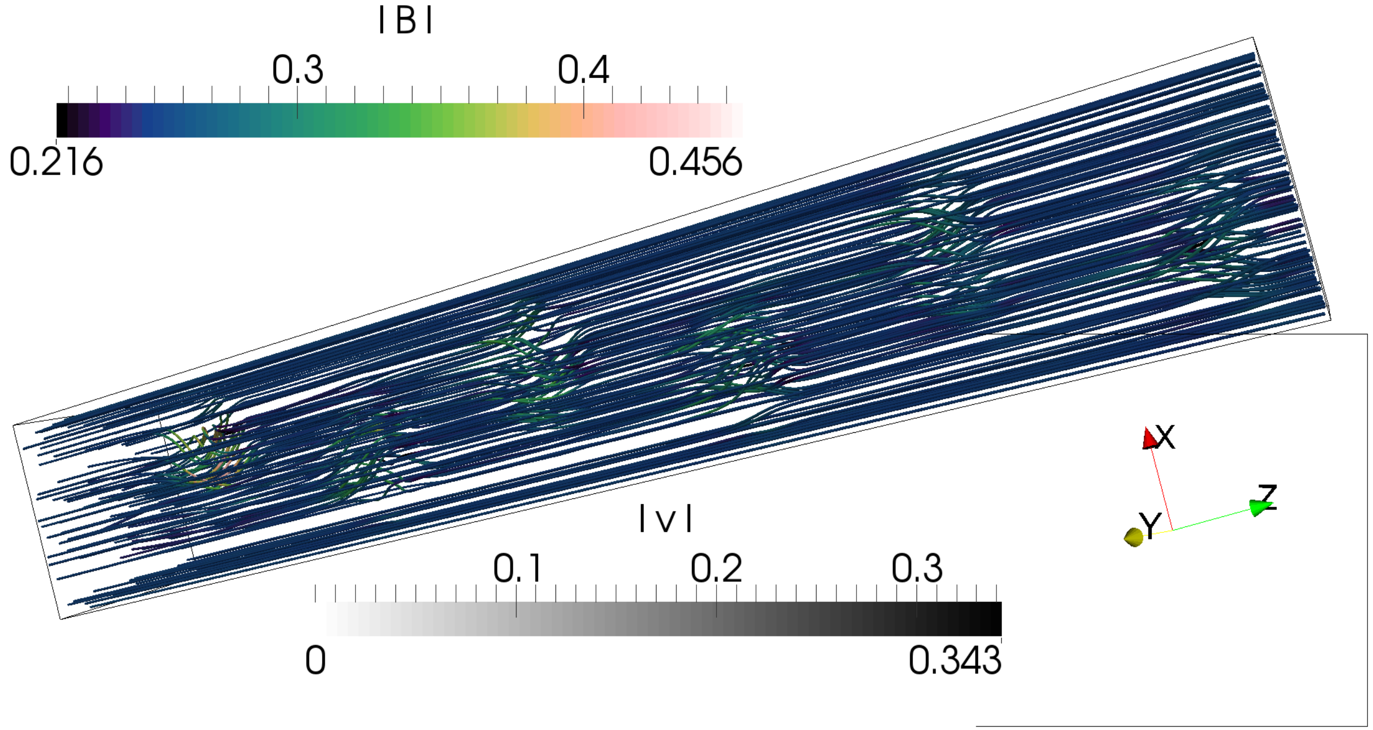
(Richard 2015)

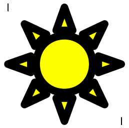
Questions: How do disturbances travel into the domain?
Reconnection at null point?
Propagation in presence of nulls?

E3 Experiments

full resistive MHD simulations with the PencilCode

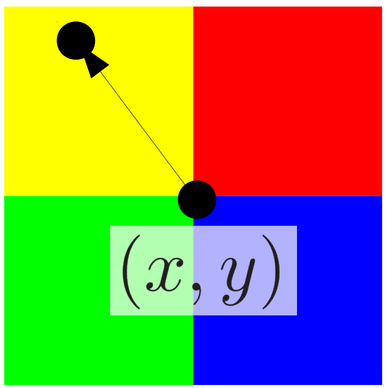
initially homogeneous field, E3 type of boundary driving

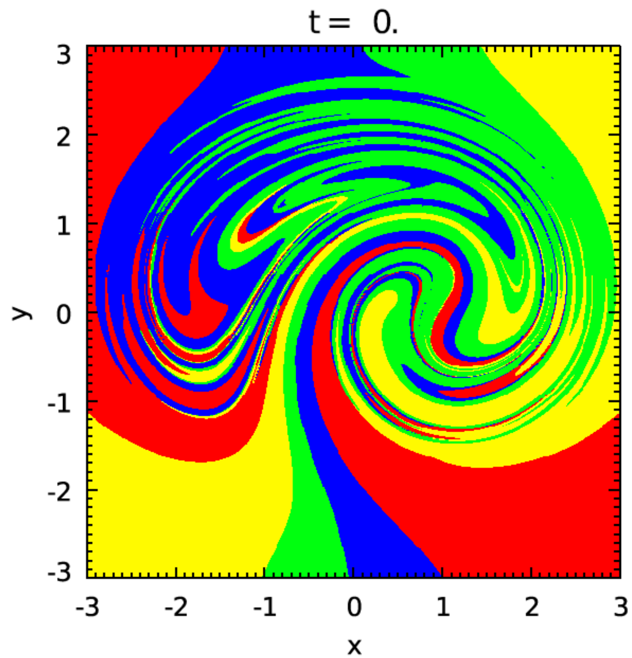


 Braid propagates into domain.

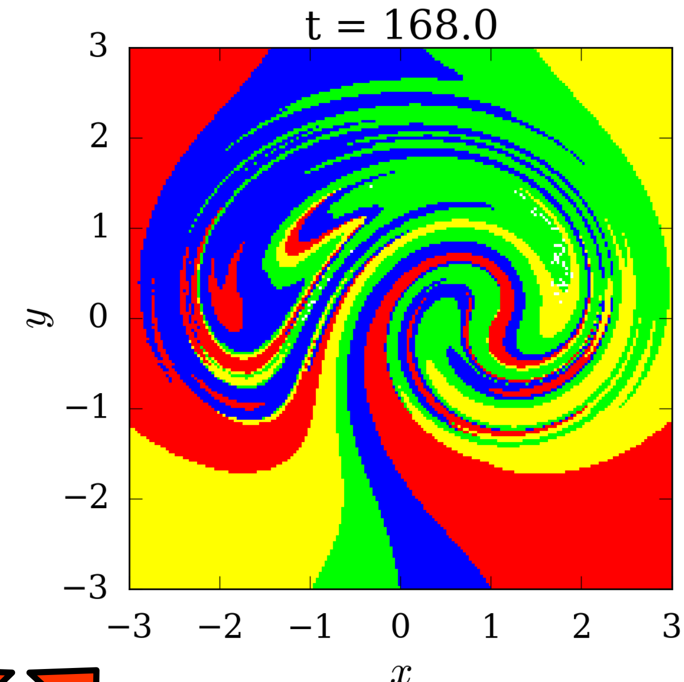
E3 Experiments

field line mapping





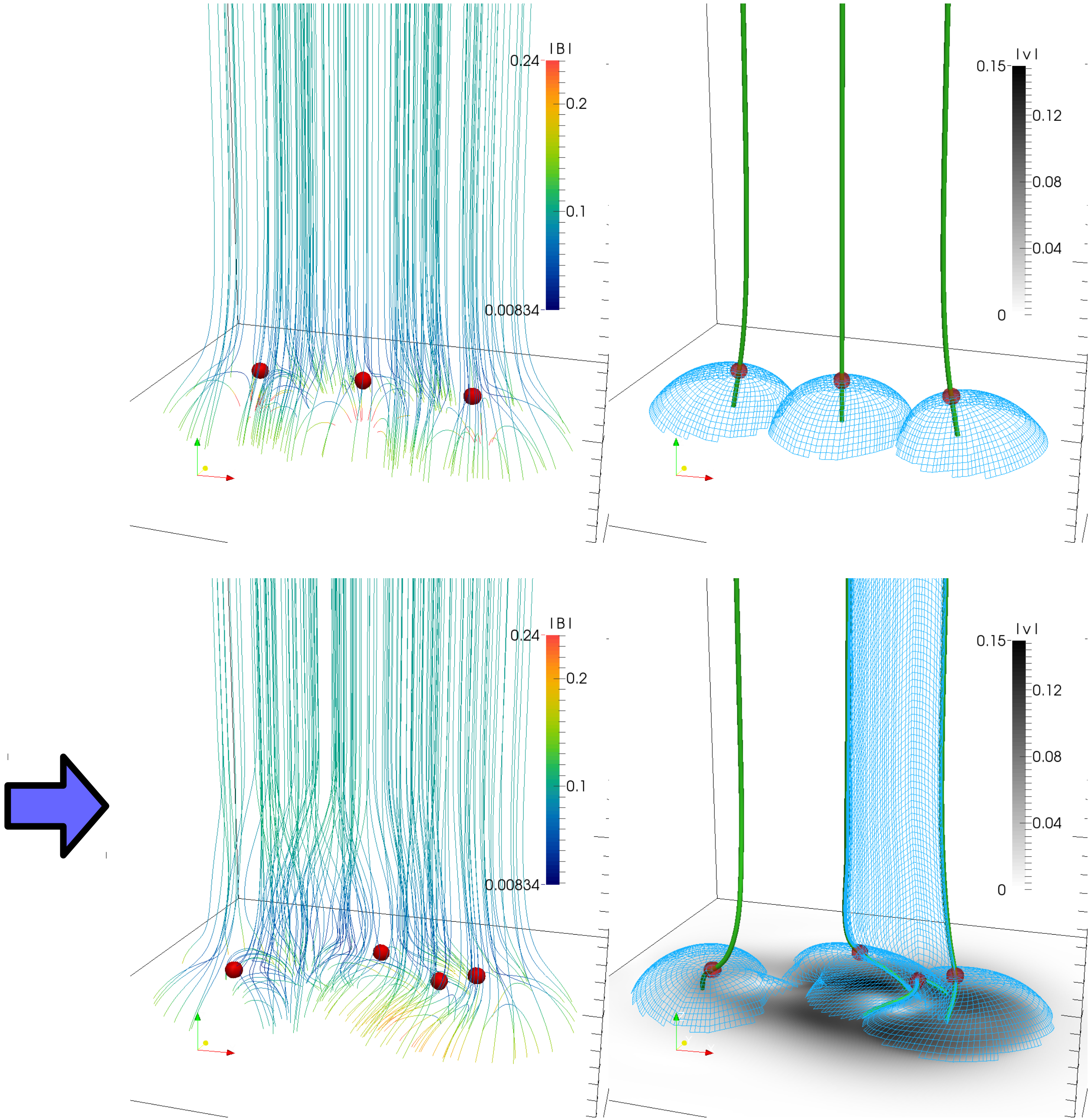
(Yeates et al. 2010)



vs.

field line connectivity with foot point motions

Magnetic Skeleton



Conclusions

- Topology preserving relaxation of magnetic fields.
- Current concentrations not singular.
- Current increases strongly with field complexity.
- Singular currents at magnetic nulls.
- Braiding through photospheric foot point motion.
- Null point disruption through boundary motions.

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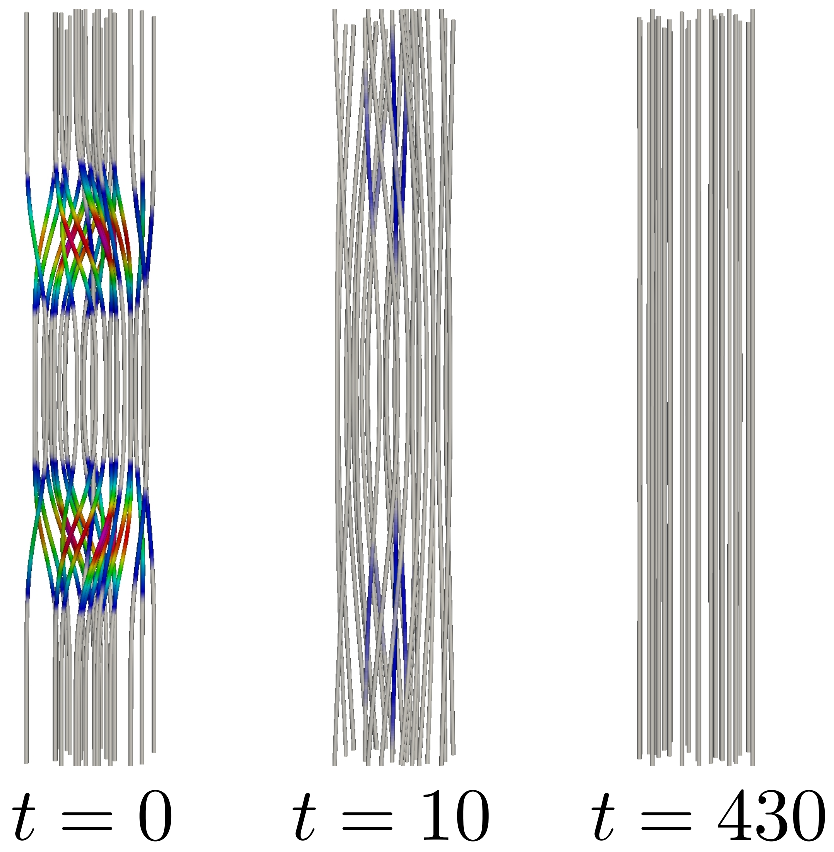
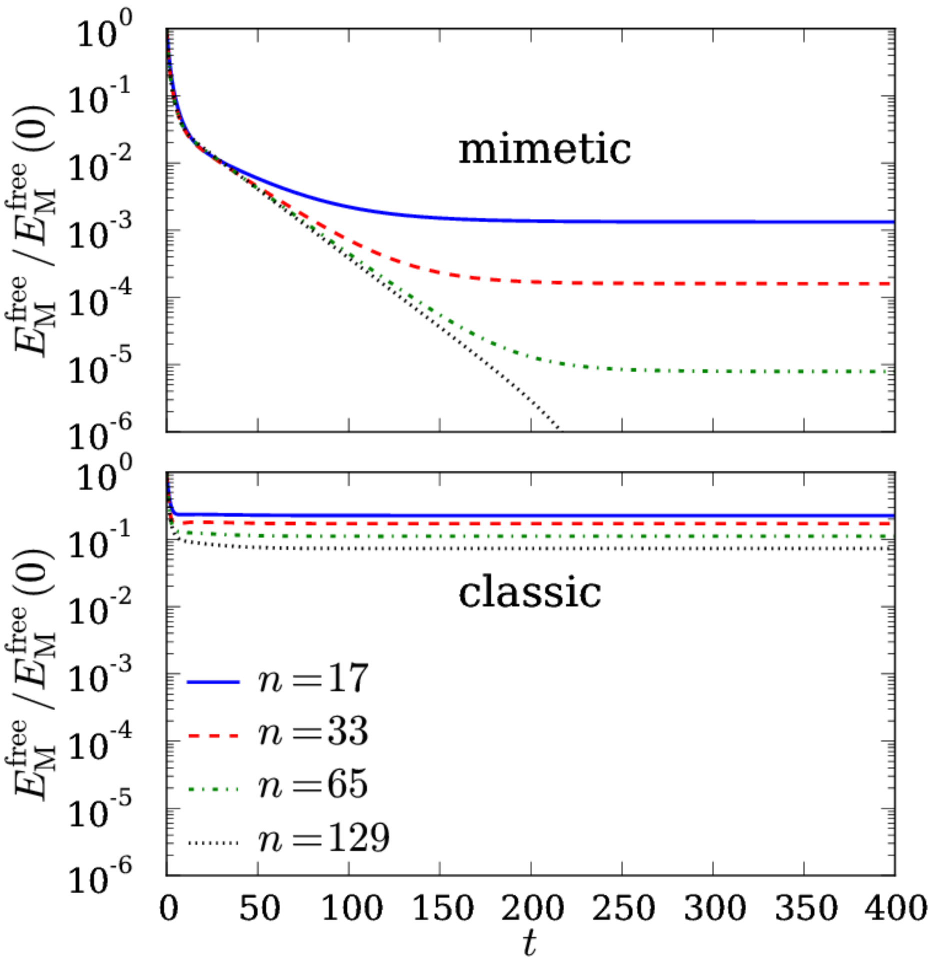
Simply Twisted Fields

Magnetic streamlines:





(Candelaresi et al. 2014)