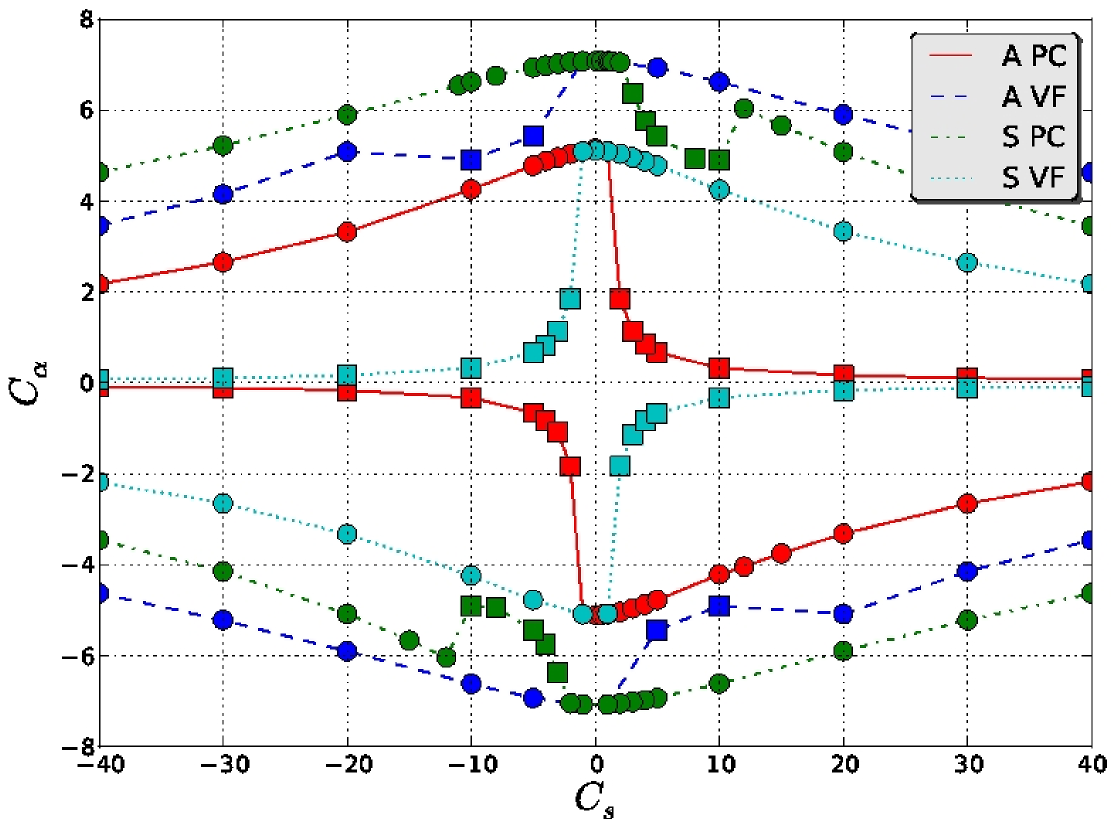
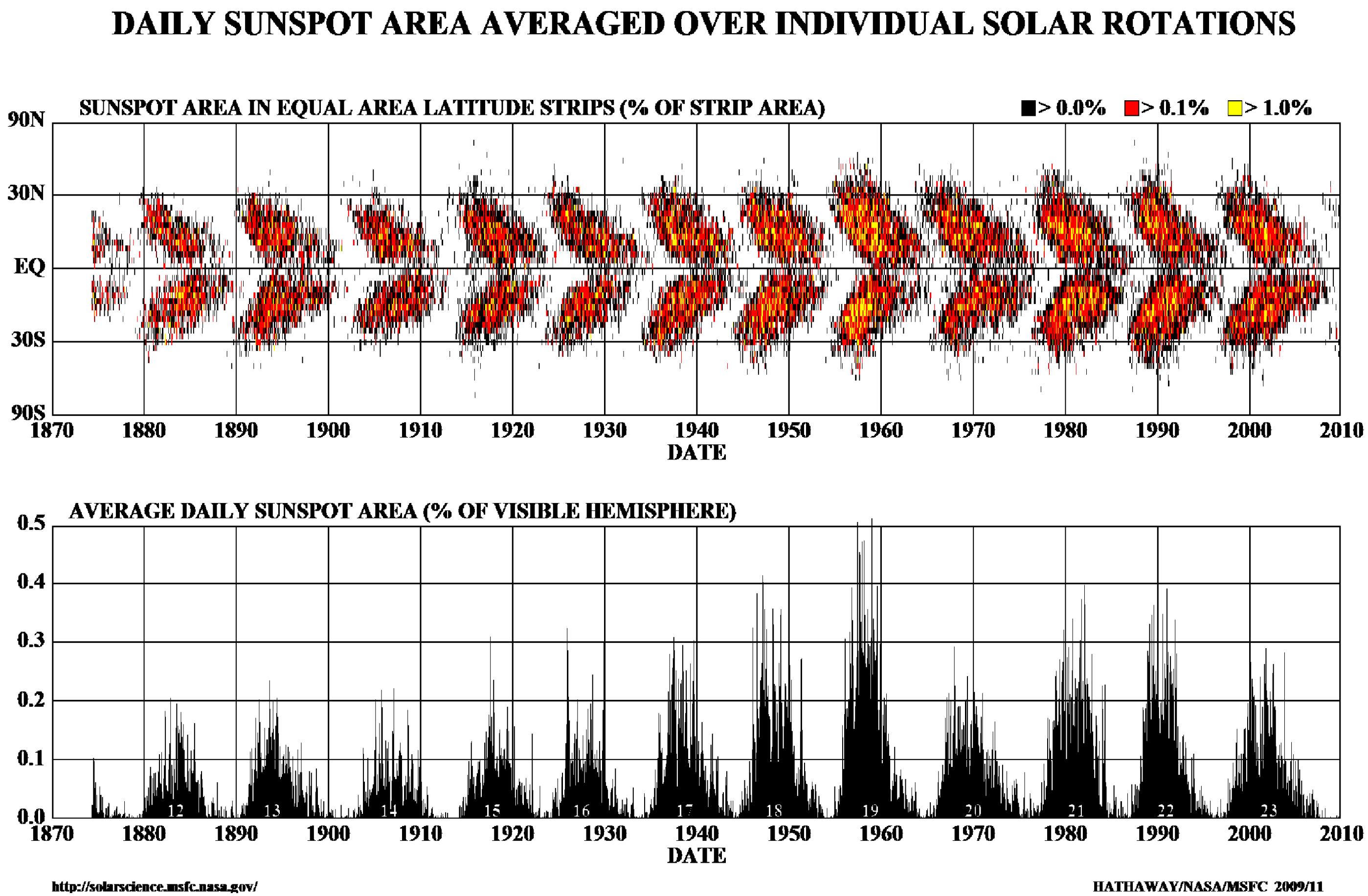
Magnetic helicity fluxes in dynamically quenched dynamos

Simon Candelaresi



Solar Magnetic Field

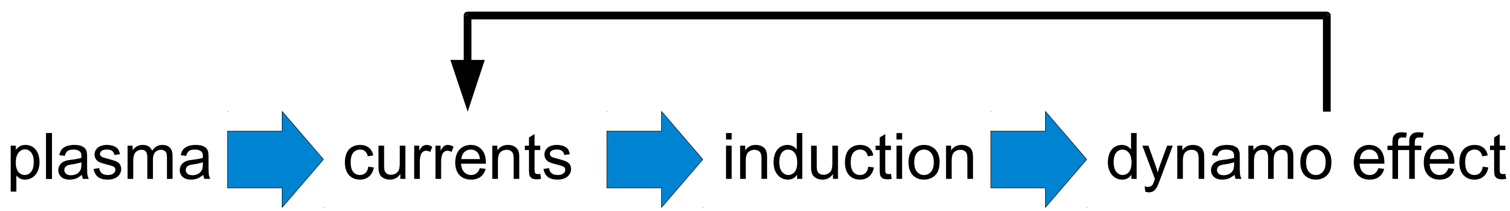
11 year cycle



蝶形図

→ dynamo working

Dynamo Mechanism



Equations of **magnetohydrodynamics** (MHD):

Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Momentum equation: $$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

Continuity equation: $$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Advective derivative: $$\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U} \cdot \boldsymbol{\nabla}$$

Mean-Field Formalism

Mean-field decomposition: $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$

Reynolds rules: $\overline{\boldsymbol{B}_1 + \mathbf{B}_2} = \overline{\boldsymbol{B}}_1 + \overline{\boldsymbol{B}}_2, \quad \overline{\overline{\boldsymbol{B}}} = \overline{\boldsymbol{B}}, \quad \overline{\boldsymbol{b}} = 0$

$$\overline{\partial_\mu \boldsymbol{B}} = \partial_\mu \overline{\boldsymbol{B}}, \quad \mu = 0, 1, 2, 3$$

Mean-field induction equations:

$$\partial_t \overline{\boldsymbol{B}} = \eta \nabla^2 \overline{\boldsymbol{B}} + \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}})$$

$$\partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{G}) + \boldsymbol{\nabla} \times (\boldsymbol{u} \times \overline{\boldsymbol{B}}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{b}$$

Electromotive force (emf): $\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$

$$\boldsymbol{G} = \boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}$$

Mofatt, H. K., 1978, Krause and Raedler, 1980

Electromotive Force

The EMF is assumed to be linear and homogeneous in $\overline{\boldsymbol{B}}$.

$$\mathcal{E}_i(x,t) = \mathcal{E}_i^{(0)}(x,t) + \int\int_\alpha K_{ij}(x,x',t,t')\overline{B}_j(x-x',t-t')\ \mathrm{d}^3x'\ \mathrm{d}t'$$

Taylor expansion:

$$\overline{B}_j(x',t) = \overline{B}_j(x,t) + (x'_k - x_k)\frac{\partial \overline{B}_j(x,t)}{\partial x_k} + \ldots$$

Assume local and instantaneous dependence of $\overline{\boldsymbol{\mathcal{E}}}$ on $\overline{\boldsymbol{B}}$.

$$\overline{\mathcal{E}}_i = \alpha_{ij}\overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k} + \ldots$$

For a turbulent system without preferred direction, i.e. $\boldsymbol{U} = 0$:

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_{\mathrm{t}}\nabla\times\overline{\boldsymbol{B}}$$

$$\partial_t\overline{\boldsymbol{B}} = \alpha\boldsymbol{\nabla}\times\overline{\boldsymbol{B}} + \eta_{\mathrm{T}}\boldsymbol{\nabla}^2\overline{\boldsymbol{B}}$$

Alpha-Effect

α effect: $\alpha = \alpha_{\rm K} + \alpha_{\rm M}$

$$\alpha_{\rm K} = -\tau\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}/3$$

$$\alpha_{\rm M} = \tau\overline{\boldsymbol{j}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \tau k^2\overline{\boldsymbol{a}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \overline{h}_{\rm m}$$

helically driven dynamo $\overline{h}_{\rm K,f} = \overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}$

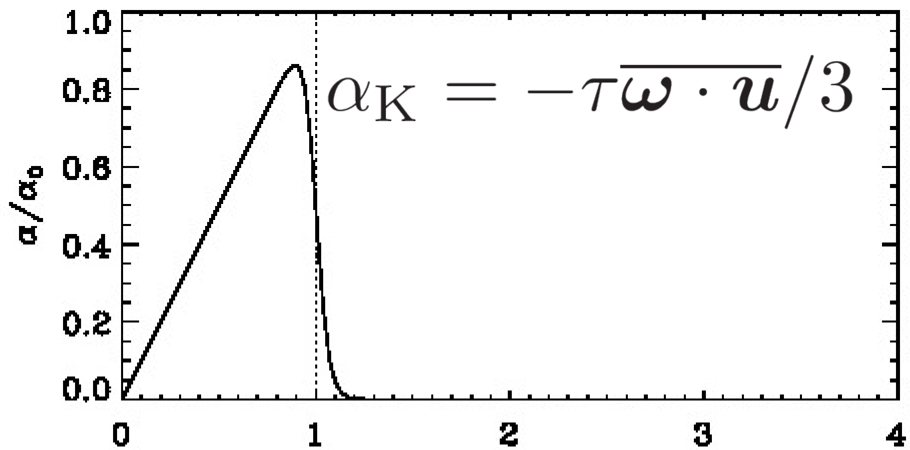
- production of magnetic helicity $\overline{h}_{\rm M,f} = \overline{\boldsymbol{a}\cdot\boldsymbol{b}}$
- total magnetic helicity conservation $\overline{h}_{\rm M,m} = \overline{\boldsymbol{A}}\cdot\overline{\boldsymbol{B}}$

$\overline{\boldsymbol{a}\cdot\boldsymbol{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^9$ galaxies: ${\rm Re}_{\rm M} = 10^{18}$

Magnetic Helicity Fluxes

$$\frac{\partial \alpha_\mathrm{M}}{\partial t} = -2\eta_\mathrm{t} k_\mathrm{f}^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}}}{B_\mathrm{eq}^2} + \frac{\alpha_\mathrm{M}}{R_\mathrm{m}} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

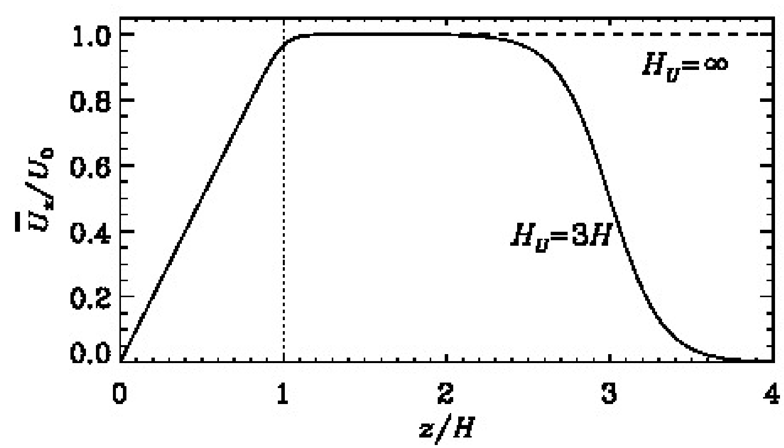


advective: $\alpha_\mathrm{M} \overline{\boldsymbol{U}}$

α diffusion $\kappa_\alpha \frac{\partial \alpha_\mathrm{M}}{\partial z}$

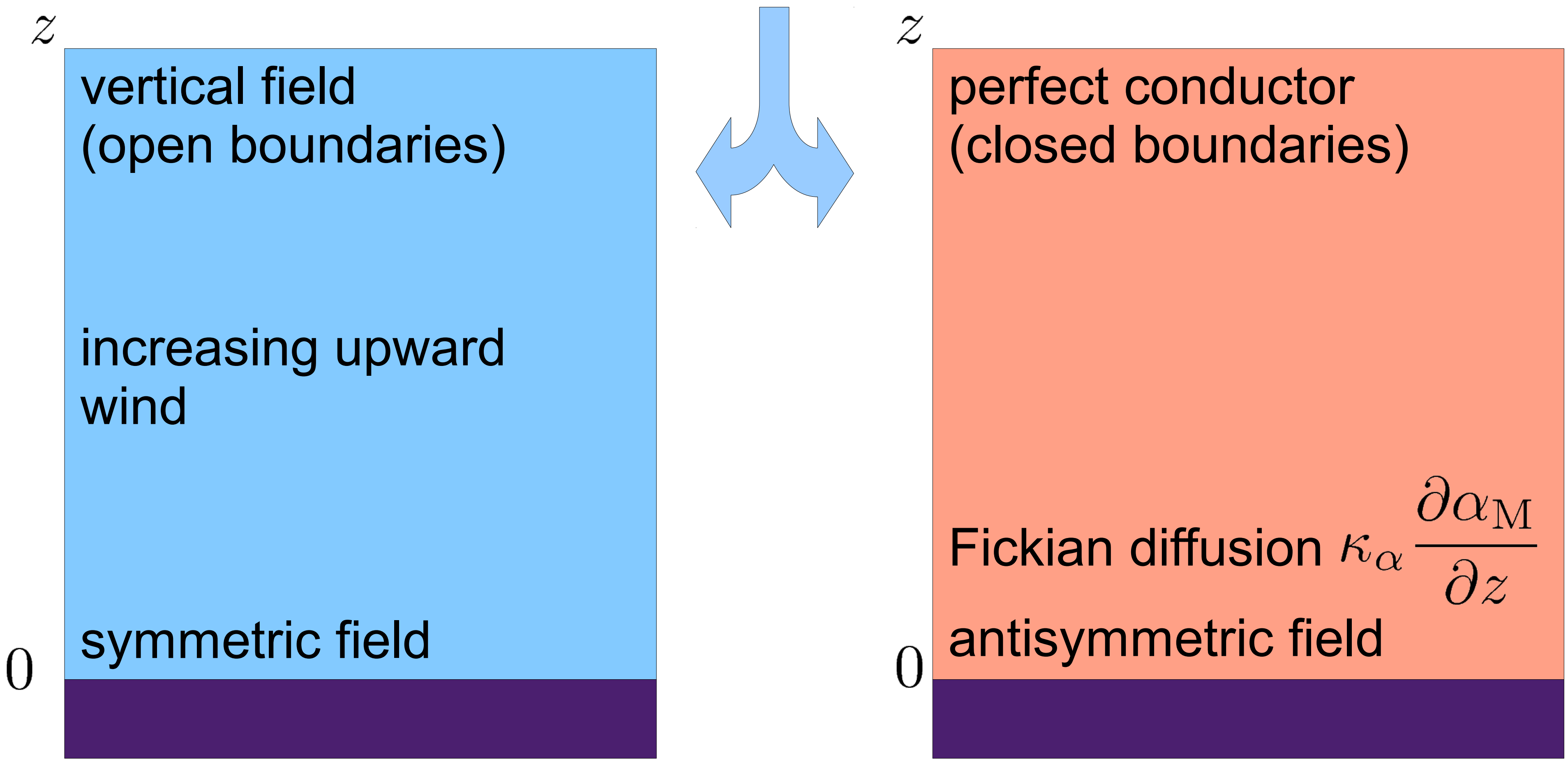
$$\frac{\partial \overline{h}_\mathrm{m}}{\partial t} = 2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_\mathrm{m}$$

$$\frac{\partial \overline{h}_\mathrm{f}}{\partial t} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{j} \cdot \boldsymbol{b}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_\mathrm{f}$$



Magnetic helicity fluxes

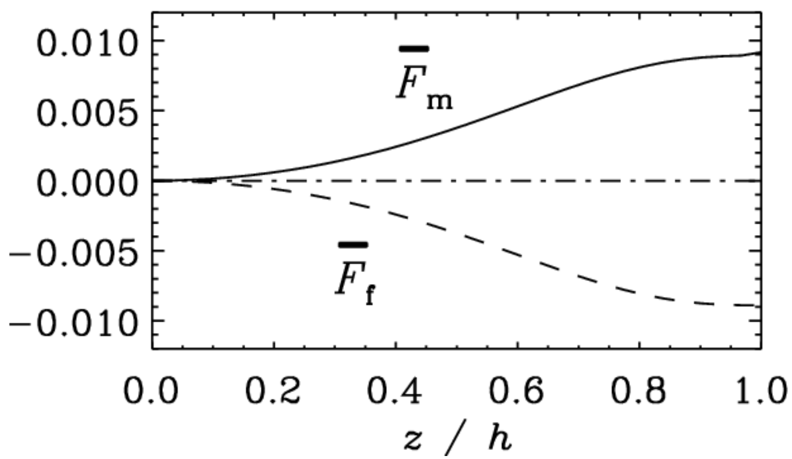
Solve equations for one hemisphere.
Impose (anti)symmetric field at the equator.

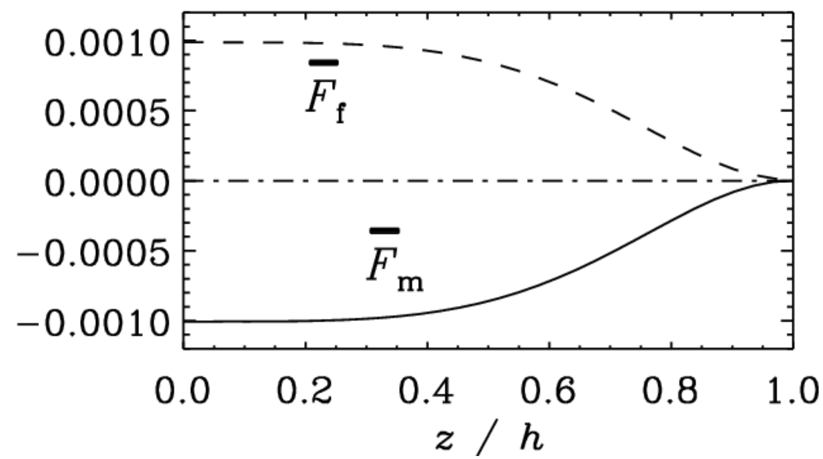


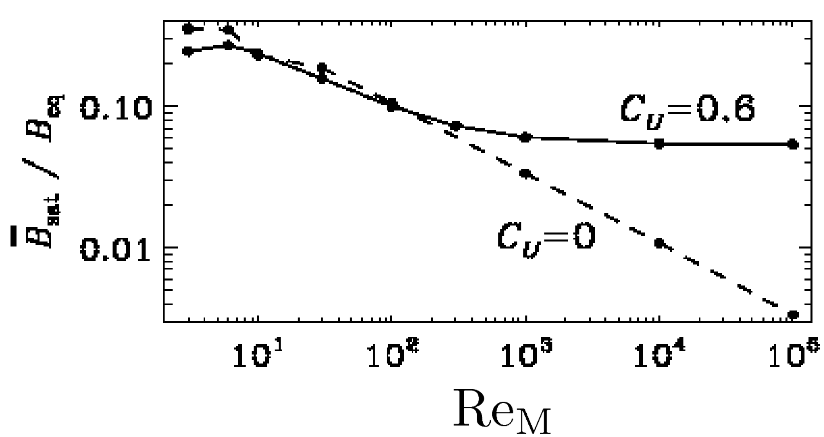
$$\mathrm{Re}_\mathrm{M} = \frac{U_\mathrm{rms} L}{\eta}$$

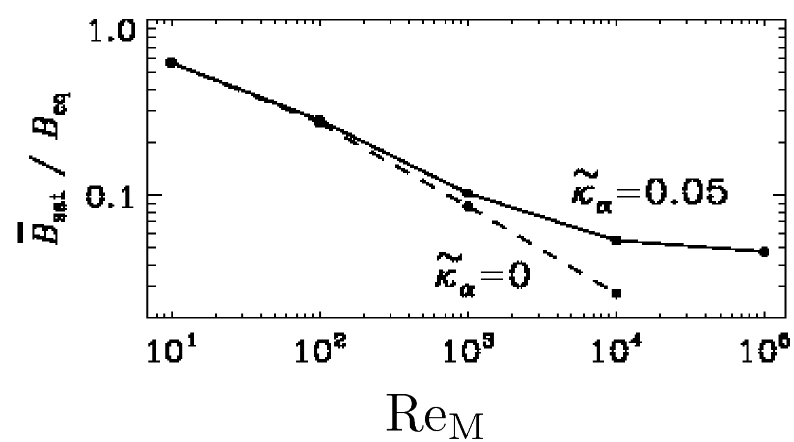
Magnetic helicity fluxes

open boundary symmetric wind	vs.	closed boundary antisymmetric κ_α







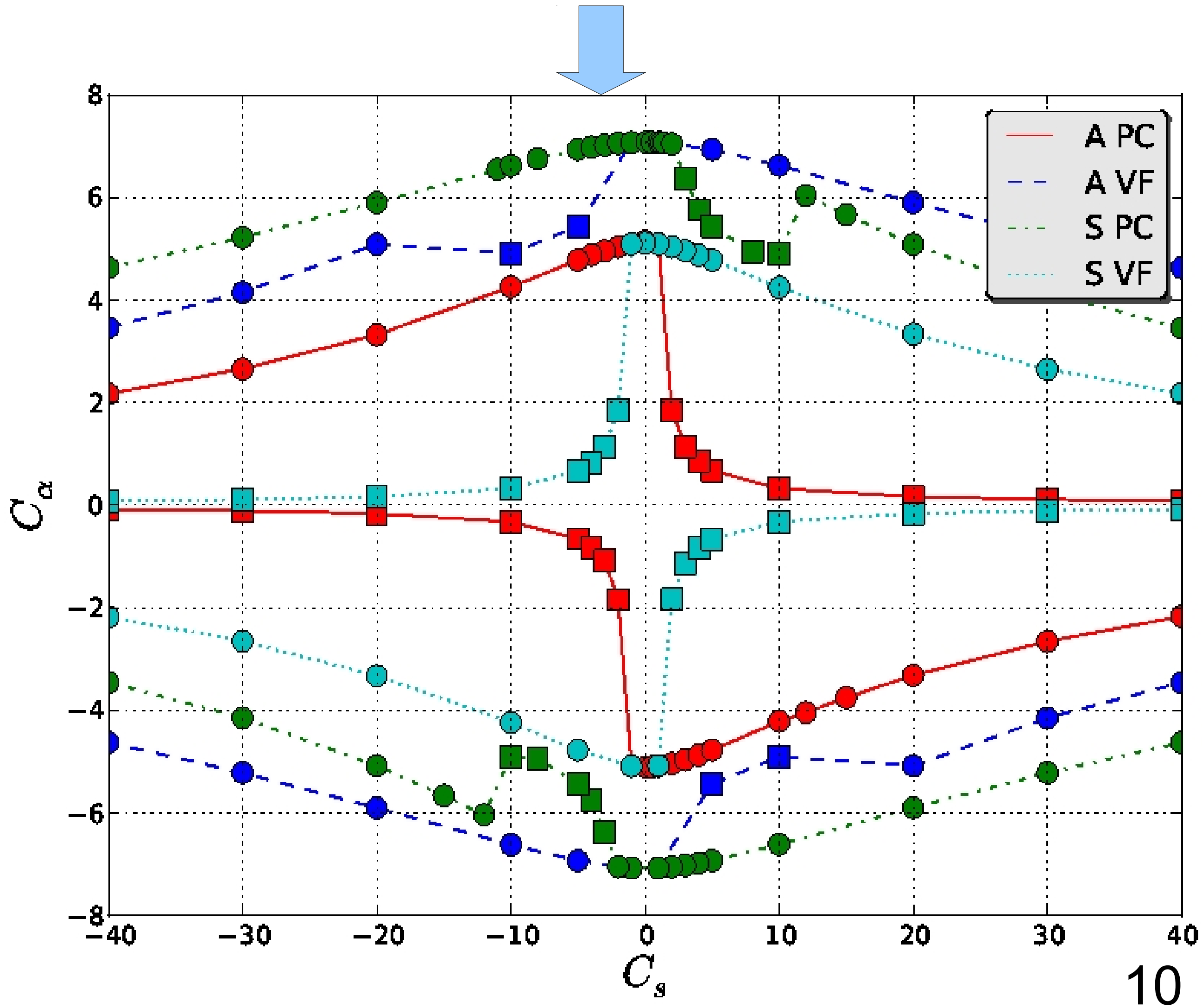


Adding shear

Critical values for the forcing and the shearing amplitude

Shearing velocity field:

$$\overline{\mathbf{U}} = \begin{pmatrix} 0 \\ Sz \\ 0 \end{pmatrix}$$

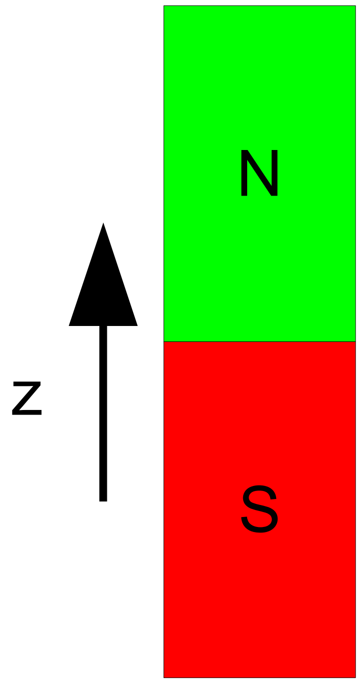


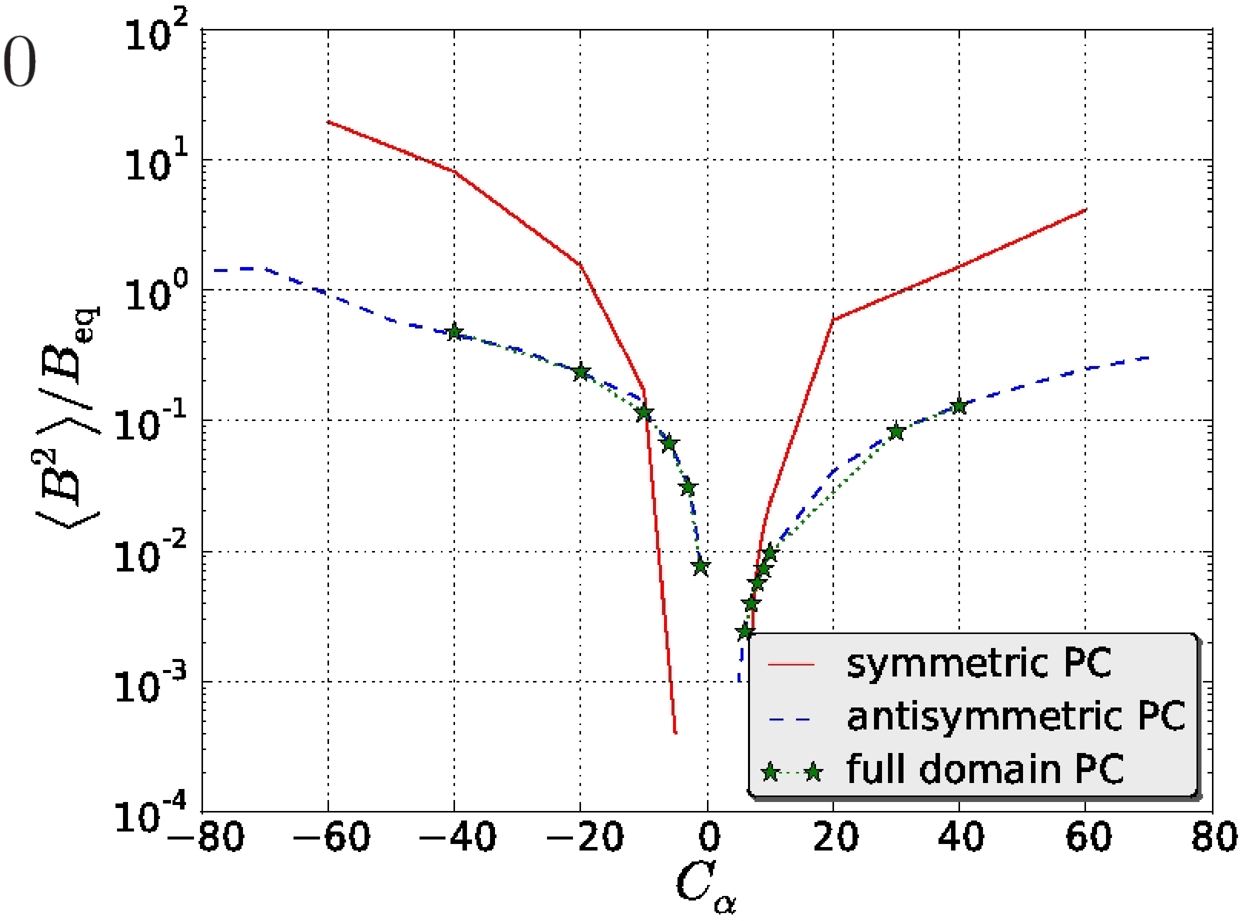
Full domain

Imposed parity in the hemispheric model is artificial.

Include both hemispheres.

$$C_{\mathrm{S}} = -10$$



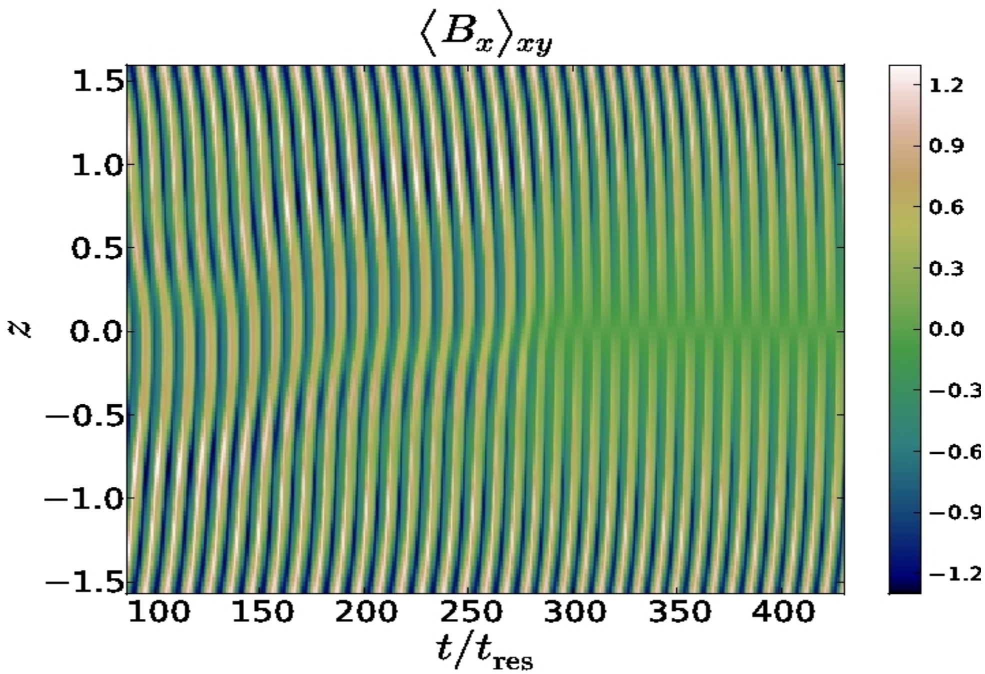


Preferred antisymmetric mode?

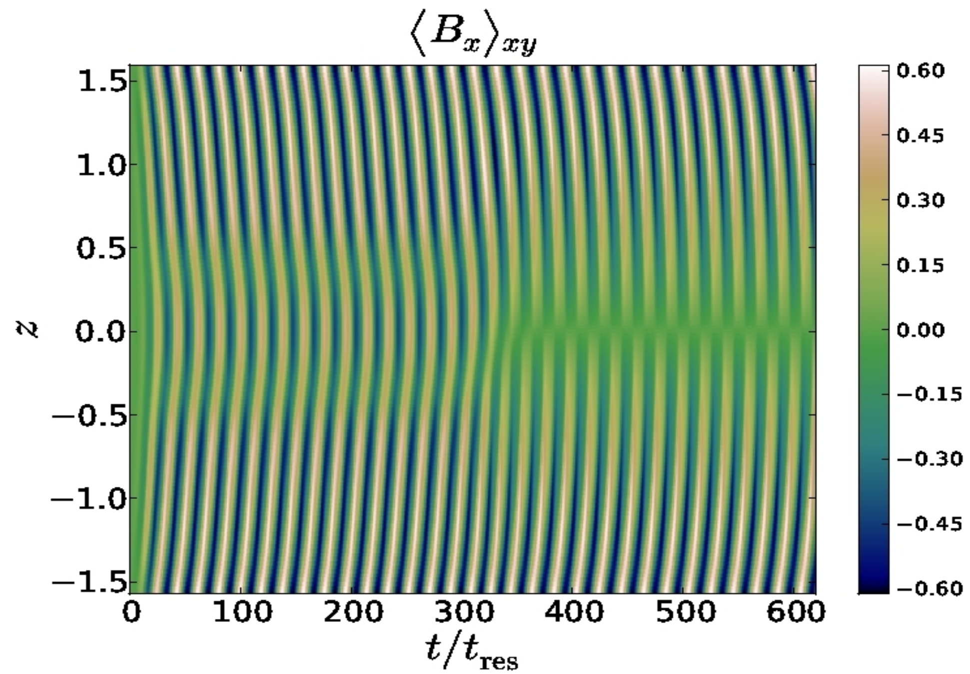
Parity change

Look at the parity of the magnetic field $\overline{B}_y$

Random initial field



Symmetric initial field

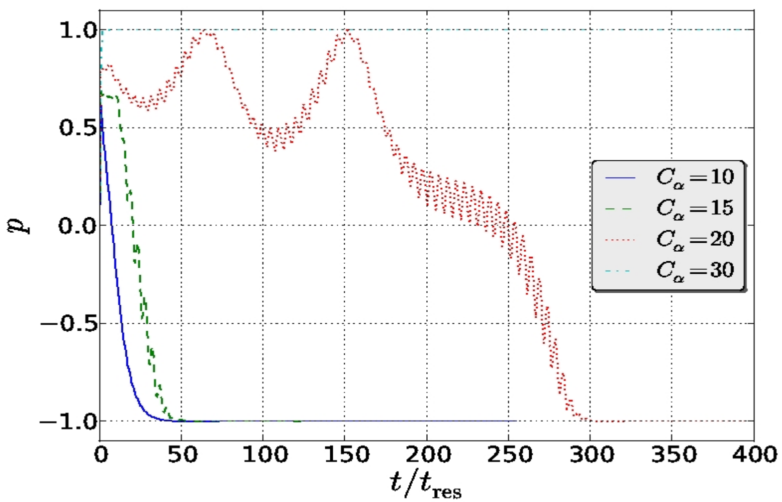


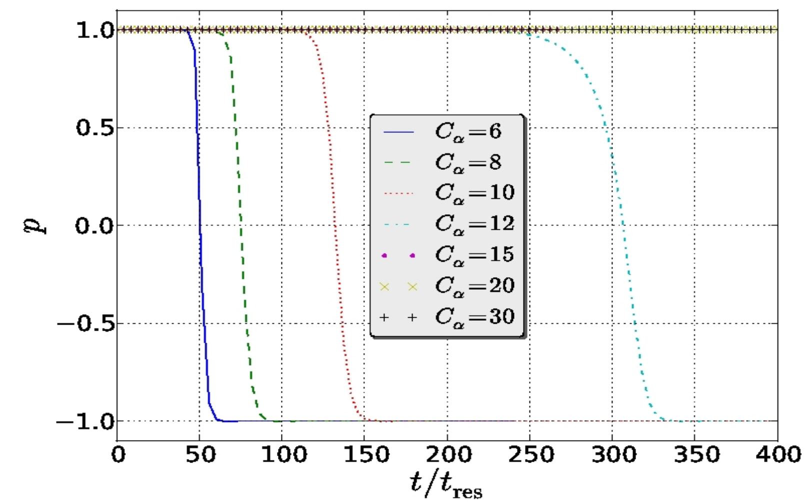
The antisymmetric solution seems to be the preferred one.

Parity change

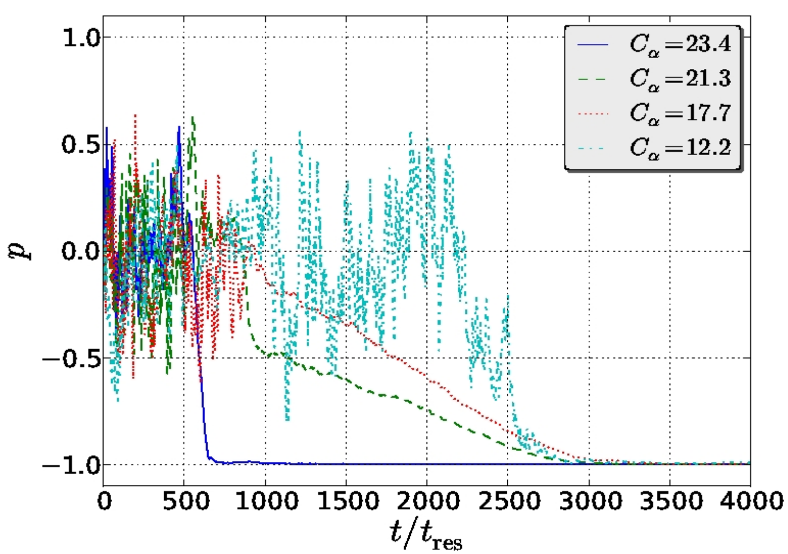
Random initial field | Symmetric initial field

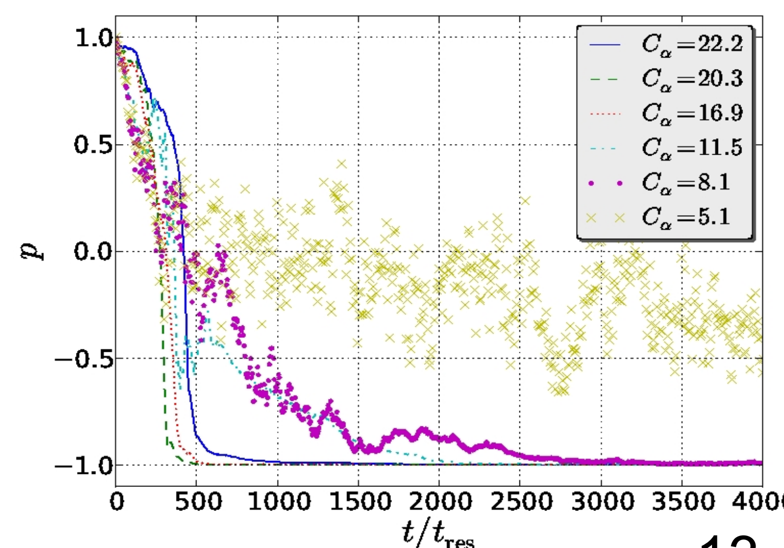
MF





DNS





Conclusions

- Advective magnetic helicity α_f fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

- Symmetric mode is unstable.
- The antisymmetric mode seems to be the preferred one.
- Check the growth rate of the modes.

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Appendix

Viscous force: $\boldsymbol{F}_{\mathrm{visc}} = \rho^{-1}\boldsymbol{\nabla}\cdot 2\nu\rho\mathbf{S}$

Strain tensor: $\mathsf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{U}$

Sound speed: $c_{\mathrm{S}} = \sqrt{\gamma\frac{p}{\rho}}$