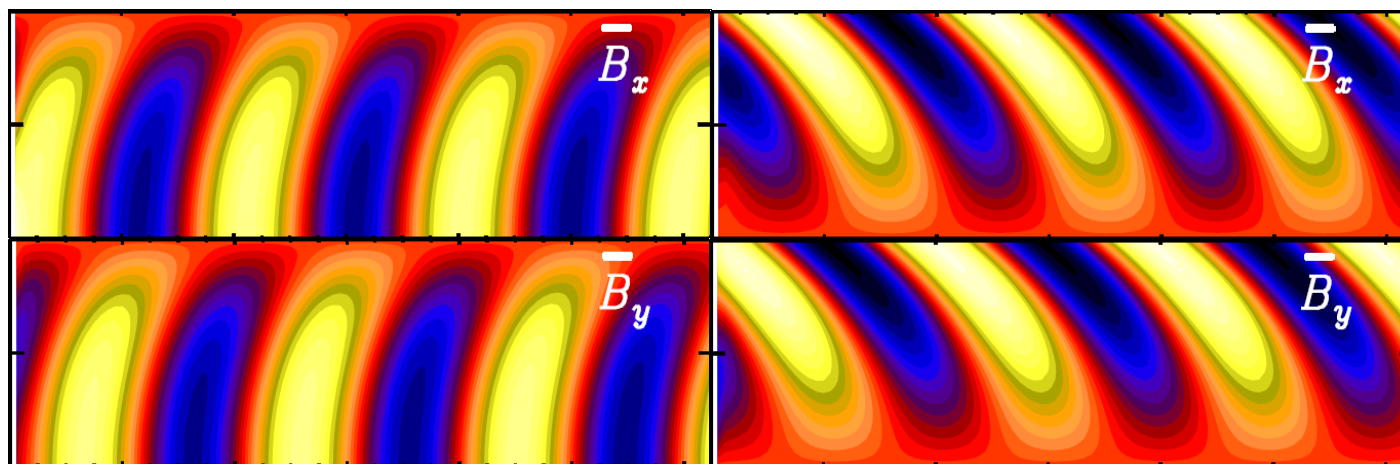


# Magnetic helicity fluxes in dynamically quenched dynamamos

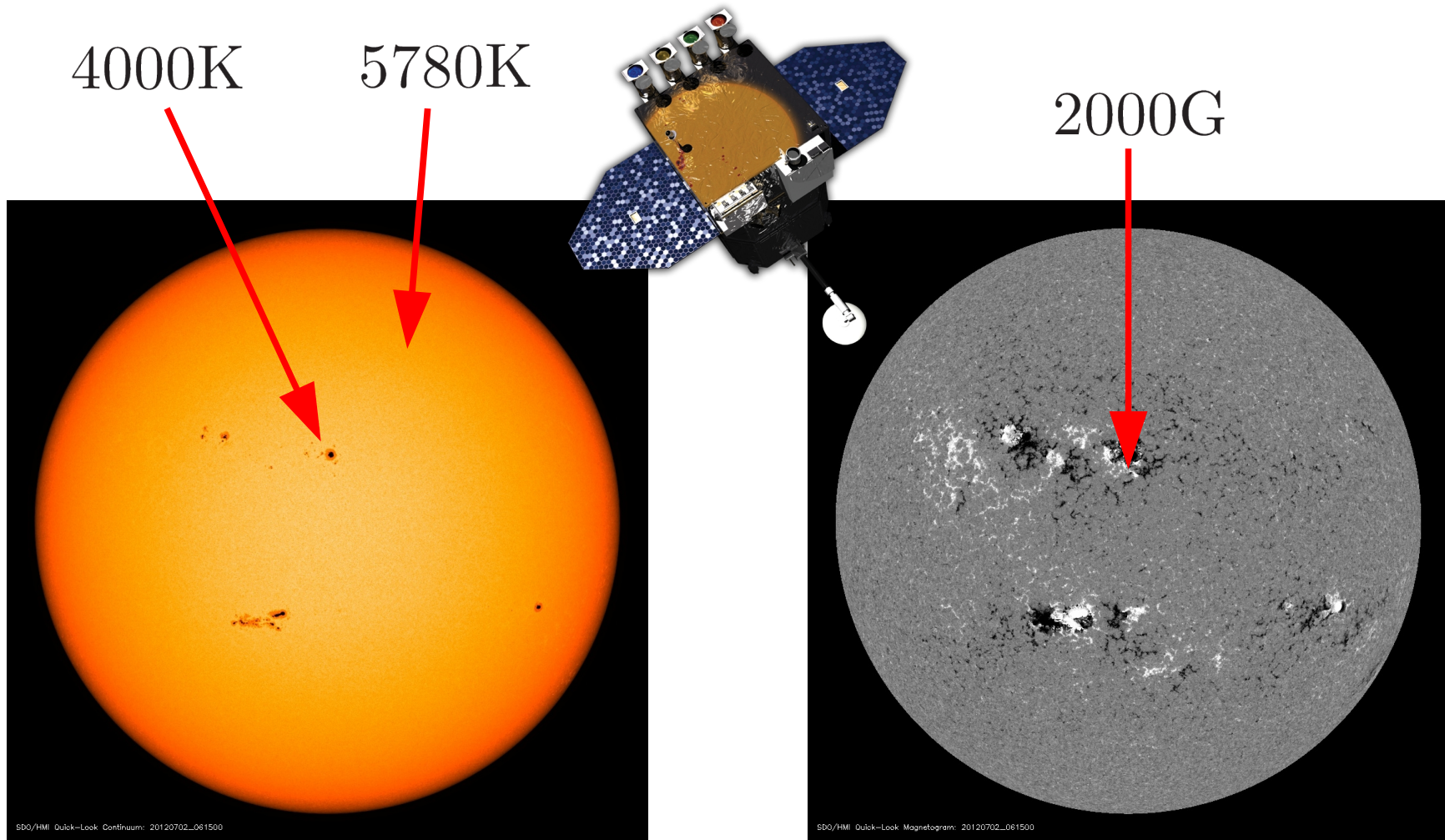
Simon Candelaresi



# Outline

- Observations of sunspots and magnetic fields.
- Dynamo mechanism.
- Mean-field model.
- Alpha-effect and alpha-quenching.
- Magnetic helicity fluxes.

# Solar Dynamics Observatory (SDO)

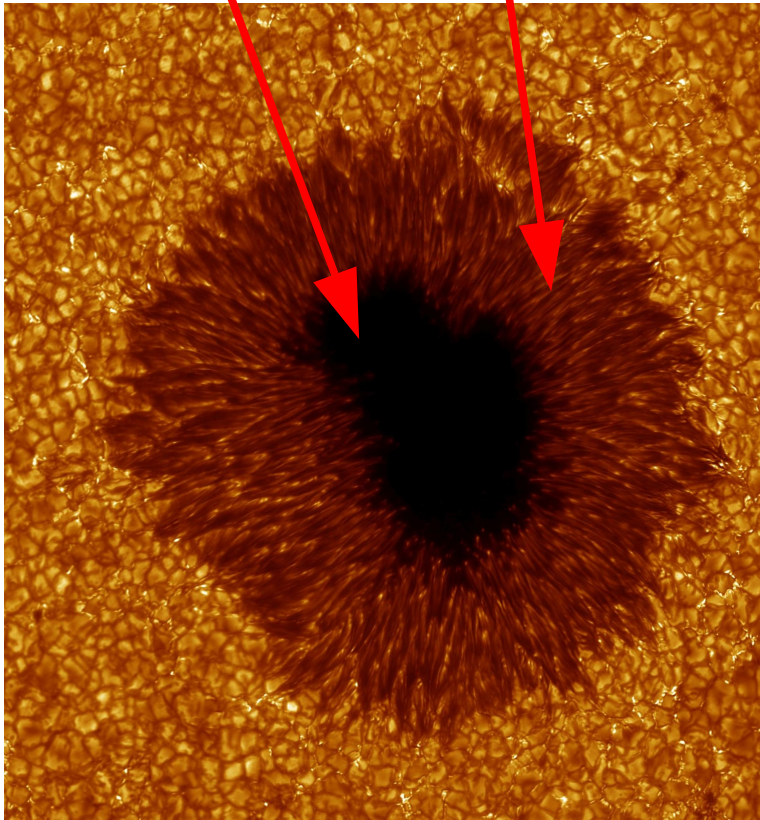


2<sup>nd</sup> July 2012, Intensity

2<sup>nd</sup> July 2012, Magnetogram

# Swedish Solar Telescope (SST)

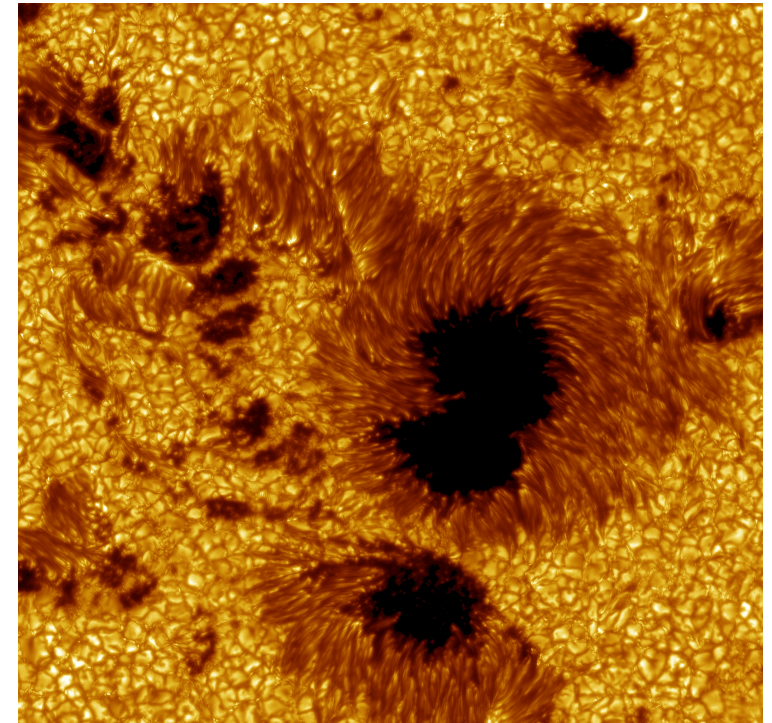
umbra      penumbra



430.5 nm (G-band), 3<sup>rd</sup> July 2003,  
(Dan Kiselman, Mats Löfdahl, 2003)

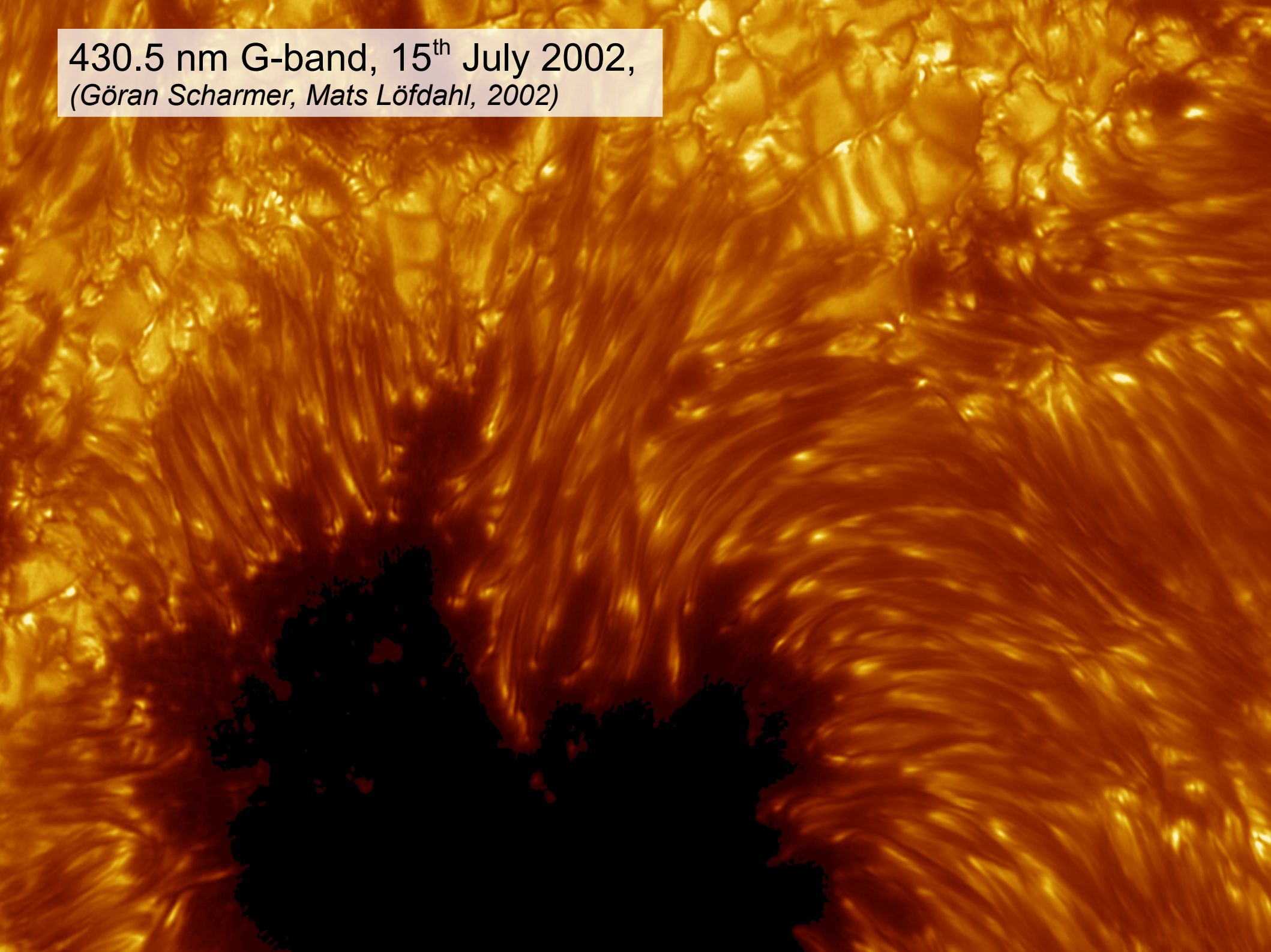


La Palma  
(Göran Scharmer)



487.7 nm, 15<sup>th</sup> July 2002,  
(Göran Scharmer, Mats Löfdahl, 2002)

430.5 nm G-band, 15<sup>th</sup> July 2002,  
(Göran Scharmer, Mats Löfdahl, 2002)

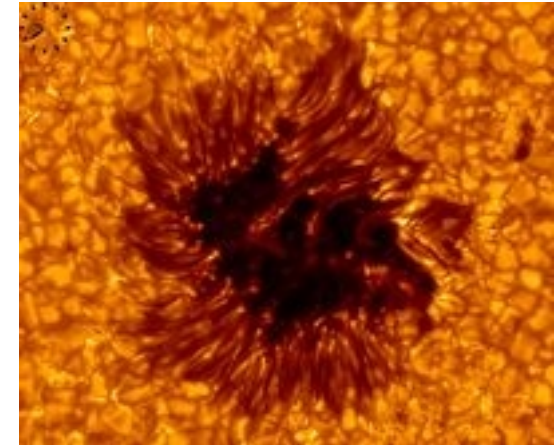
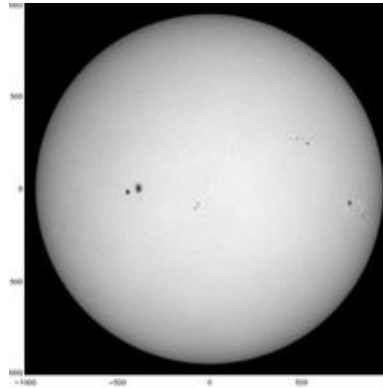


# Swedish Solar Telescope (SST)

1h quiet Sun, 656.3 nm,  
18<sup>th</sup> June 2006,  
*(Luc Rouppe van der Voort, Oslo, 2006)*

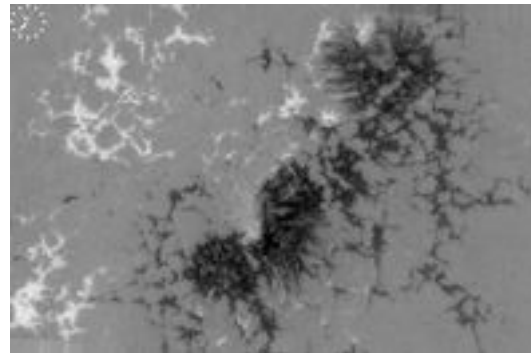


Zoom from SOHO/MDI field  
of view to SST resolution,  
August 2004,  
*(Michiel van Noort, Luc Rouppe van  
der Voort, Mats Carlsson, Oslo, 2004)*

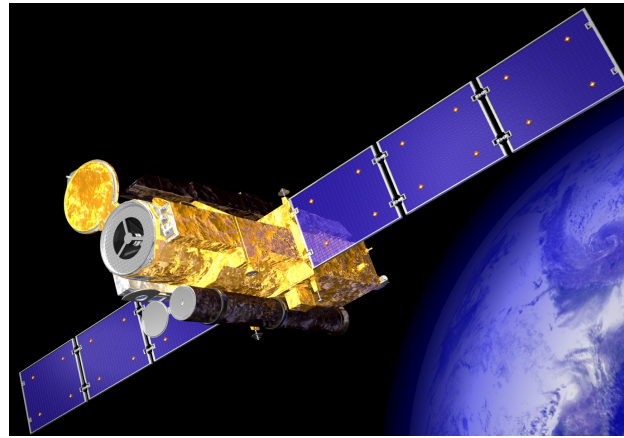


Sunspot 41 min, 430.5 nm  
G-band, 20<sup>th</sup> August 2004,  
*(Michiel van Noort and Luc Rouppe  
van der Voort, Oslo, 2004)*

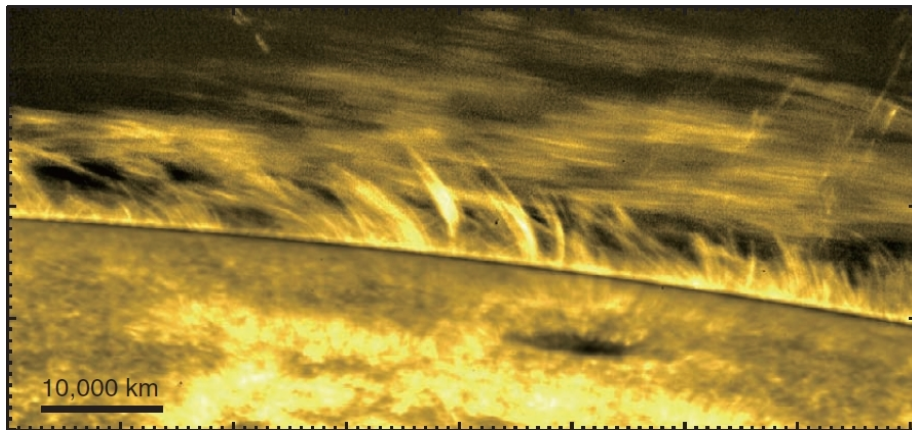
Sunspot group magnetogram,  
21<sup>st</sup> August 2004,  
*(Michiel van Noort and Luc Rouppe van  
der Voort, Oslo, 2004)*



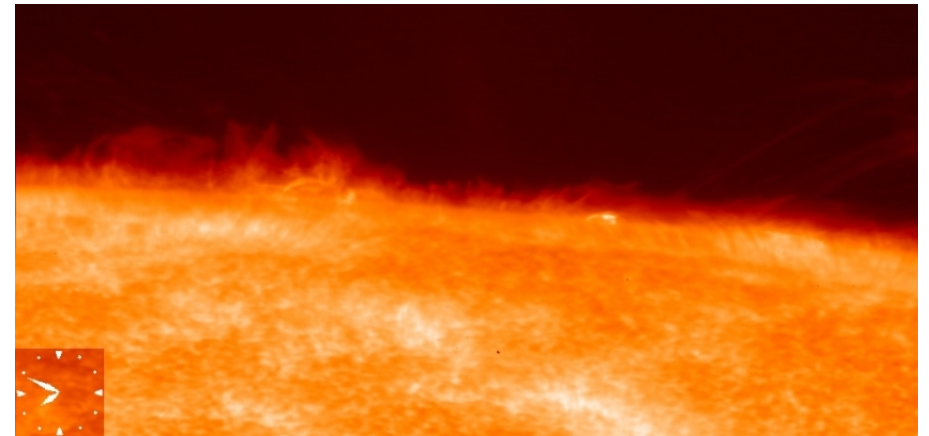
# Hinode ひので (Solar-B)



(JAXA)



Solar prominence,  
9<sup>th</sup> November 2006,  
(Okamoto, T.J. et al., 2007)

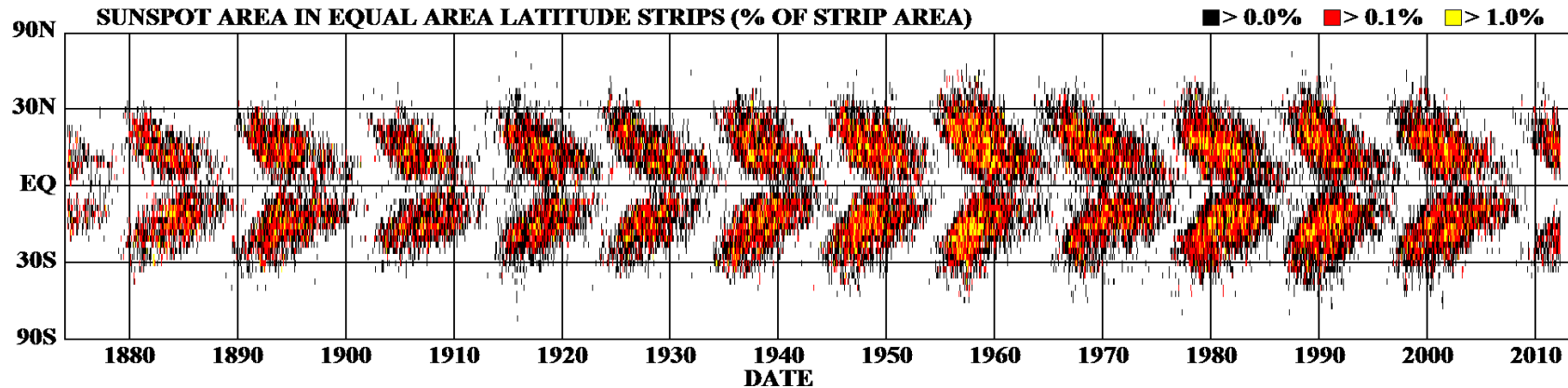


Eruption observed in Ca II H  
(397nm) above a Sun spot,  
<http://solarb.msfc.nasa.gov/news/movies.html>

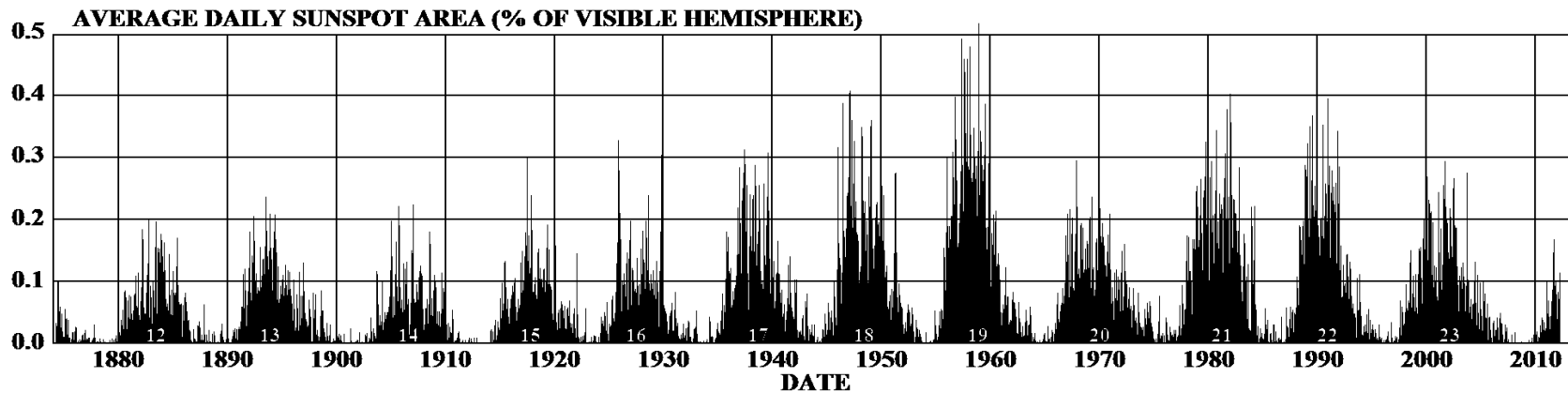
# Solar Magnetic Field

11 year cycle

## DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



蝶  
形  
图



<http://solarscience.msfc.nasa.gov/>

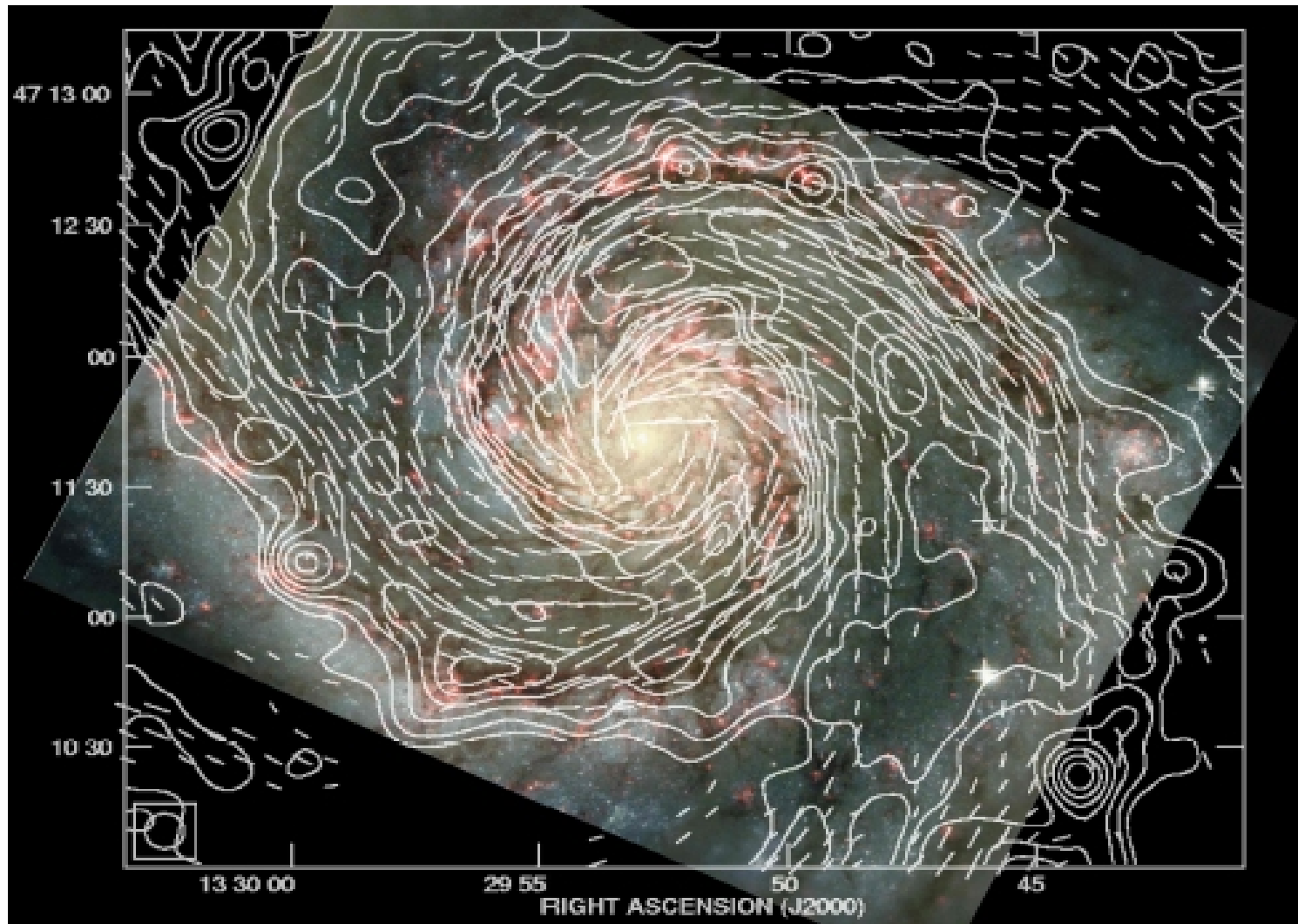
HATHAWAY/NASA/MSFC 2012/06

 dynamo working

(Hathaway/NASA)



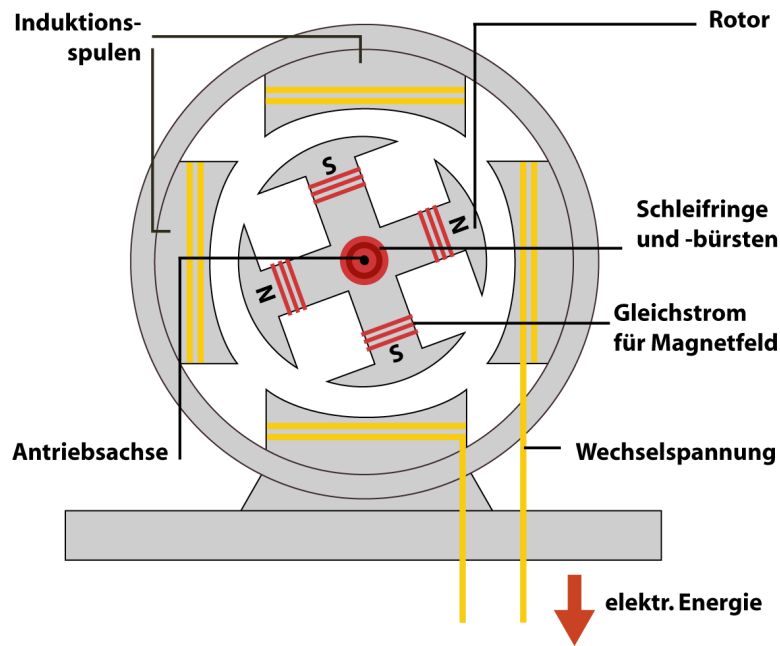
# Galactic Magnetic Fields



Galaxy M51, radio + optical  
(Fletcher et al. 2011)

# Dynamo Effect

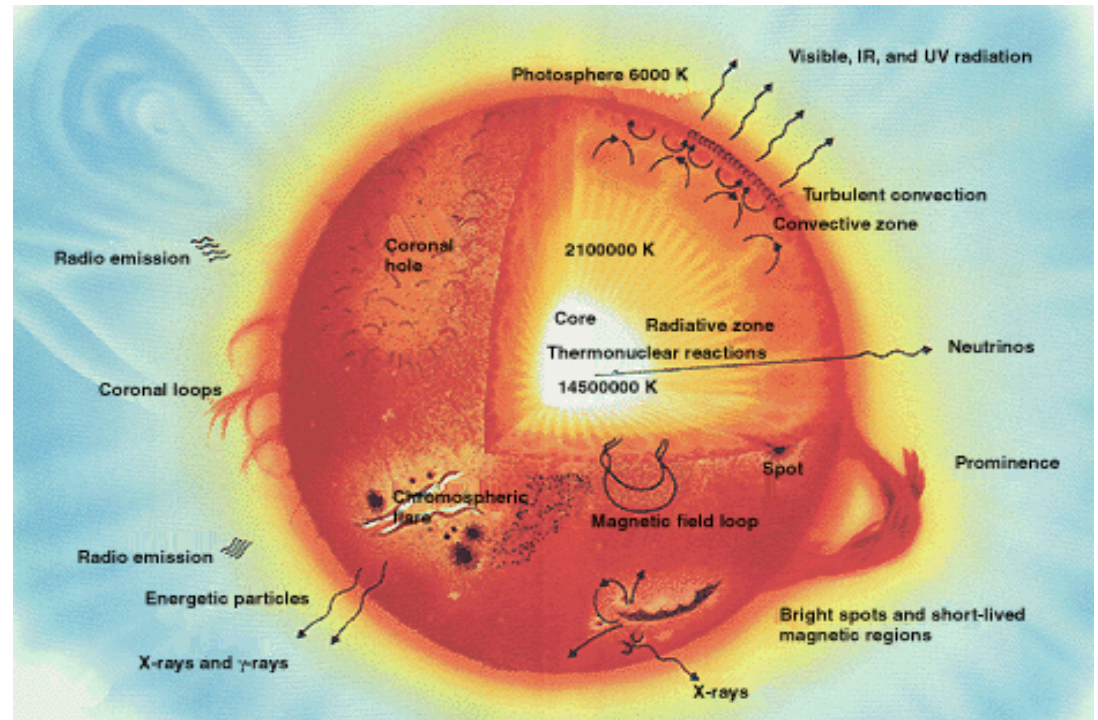
kinetic motion → induction  
→ electric energy



electric power generator  
(Wikipedia, user: Kuntoff, 2005)

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

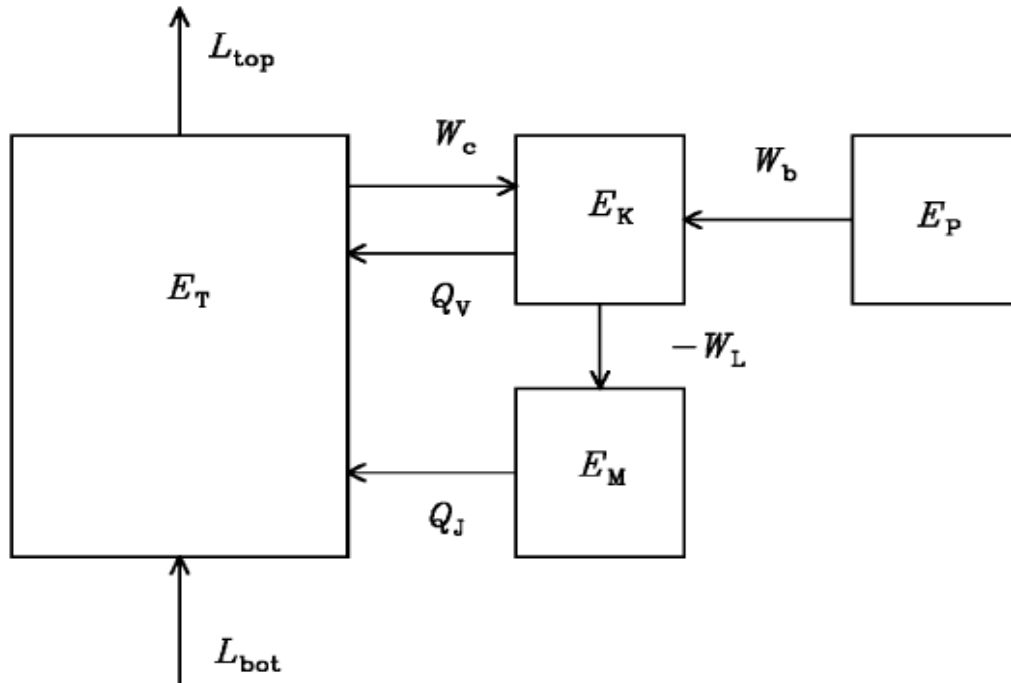
turbulent motion → induction  
→ magnetic energy



Solar model  
(NASA)

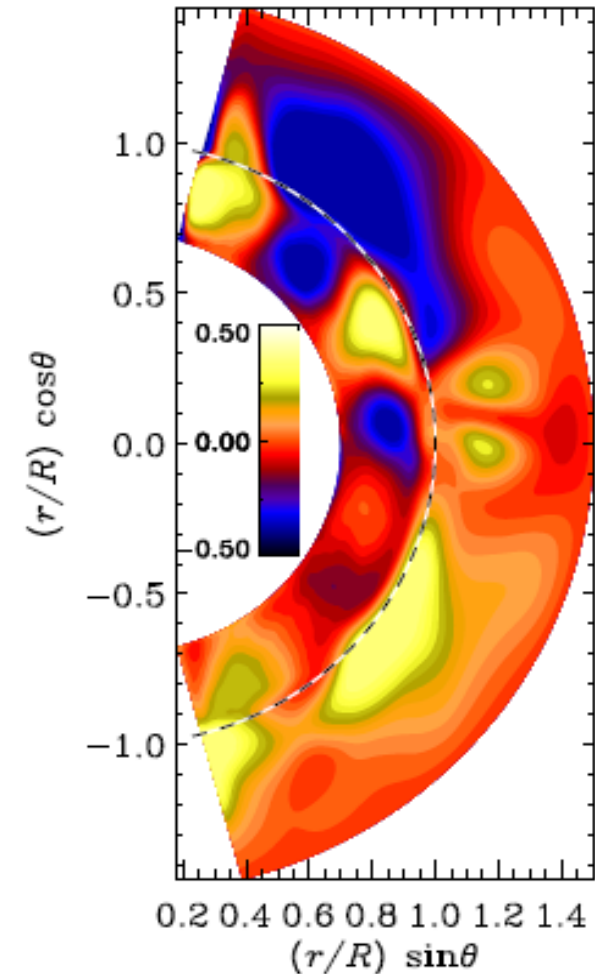
$$\partial_t \mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

# Turbulent Dynamo Schematics



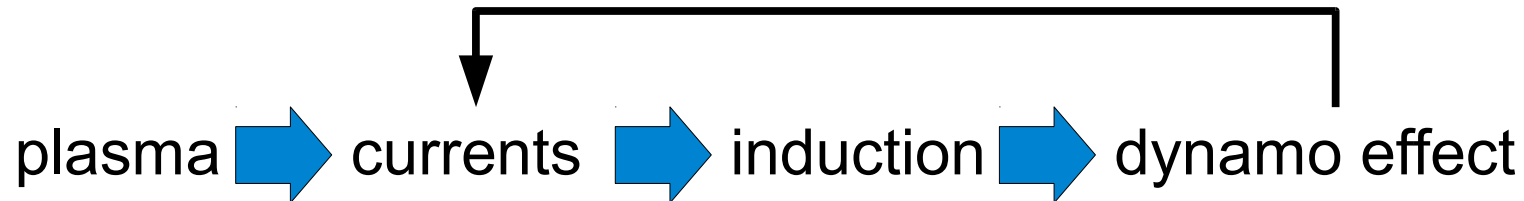
Energy budget for a dynamo.  
(Brandenburg et al., 1996)

$E_T, E_K, E_M, E_P =$   
thermal, kinetic, magnetic and  
potential energy



$\langle \overline{B}_\phi \rangle_t$  for a convection  
driven dynamo.  
(Warnecke et al., 2012)

# Dynamo Mechanism



Equations of **magnetohydrodynamics** (MHD):

Induction equation: 
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J})$$

Momentum equation: 
$$\frac{D\mathbf{U}}{Dt} = -c_S^2 \nabla \ln \rho + \mathbf{J} \times \mathbf{B} / \rho + \mathbf{F}_{\text{visc}}$$

Continuity equation: 
$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}$$

Advective derivative: 
$$D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$$

# Mean-Field Formalism

Mean-field decomposition:  $B = \bar{B} + b$

Reynolds rules:  $\overline{B_1 + B_2} = \bar{B}_1 + \bar{B}_2, \quad \overline{\bar{B}} = \bar{B}, \quad \bar{b} = 0$

$$\overline{\partial_\mu B} = \partial_\mu \bar{B}, \quad \mu = 0, 1, 2, 3$$

Mean-field induction equations:

$$\partial_t \bar{B} = \eta \nabla^2 \bar{B} + \nabla \times (\bar{U} \times \bar{B} + \bar{\mathcal{E}})$$

$$\partial_t b = \nabla \times (\bar{U} \times b + G) + \nabla \times (u \times \bar{B}) + \eta \nabla^2 b$$

Electromotive force (emf):  $\bar{\mathcal{E}} = \overline{u \times b}$

$$G = u \times b - \overline{u \times b}$$

need closure  express  $\bar{\mathcal{E}}$  in terms of the mean fields:

$$\bar{\mathcal{E}} = \bar{\mathcal{E}}(\bar{U}, \bar{B}, \dots)$$

# Electromotive Force

The EMF is assumed to be linear and homogeneous in  $\overline{B}$ .

$$\begin{aligned} \Rightarrow \mathcal{E}_i(x, t) &= \mathcal{E}_i^{(0)}(x, t) \\ &+ \int \int_{\alpha} K_{ij}(x, x', t, t') \overline{B}_j(x - x', t - t') d^3x' dt' \end{aligned}$$

Taylor expansion:

$$\overline{B}_j(x', t) = \overline{B}_j(x, t) + (x'_k - x_k) \frac{\partial \overline{B}_j(x, t)}{\partial x_k} + \dots$$

Assume local and instantaneous dependence of  $\overline{\mathcal{E}}$  on  $\overline{B}$ .

$$\Rightarrow \overline{\mathcal{E}}_i = \alpha_{ij} \overline{B}_j + b_{ijk} \frac{\partial \overline{B}_j}{\partial x_k} + \dots$$

For a turbulent system without preferred direction, i.e.  $\mathbf{U} = 0$ :

$$\overline{\mathcal{E}} = \alpha \overline{B} - \eta_t \nabla \times \overline{B}$$

$$\partial_t \overline{B} = \alpha \nabla \times \overline{B} + \eta_T \nabla^2 \overline{B}$$

# Nonlinear Alpha-Effect

Helical forcing:  $\alpha = \alpha_K = -\tau \overline{\boldsymbol{\omega} \cdot \mathbf{u}} / 3$

Back reaction of  $\overline{\mathbf{B}}$  on  $\alpha$

➡ Correction:  $\alpha$  should depend on  $\overline{\mathbf{B}}$

$$\alpha = \alpha_K \left(1 - \overline{\mathbf{B}}^2 / B_{\text{eq}}^2\right) \quad (\overline{\mathbf{B}}^2 \ll B_{\text{eq}}^2) \quad (\text{Roberts 1975})$$

Algebraic (conventional) quenching:

$$\alpha = \frac{\alpha_K}{1 + \overline{\mathbf{B}}^2 / B_{\text{eq}}^2} \quad (\text{Ivanova 1977})$$

Catastrophic quenching (fit):

$$\alpha = \frac{\alpha_K}{1 + R_m \overline{\mathbf{B}}^2 / B_{\text{eq}}^2}$$

(Vainshtein 1992)

$$\text{Sun:} \quad R_m = 10^9$$

$$\text{Galaxies:} \quad R_m = 10^{18}$$

# Alpha-Effect

$\alpha$  effect:  $\alpha = \alpha_K + \alpha_M$  (magnetic helicity conservation)

$$\alpha_K = -\tau \overline{\boldsymbol{\omega} \cdot \mathbf{u}} / 3$$

$$\alpha_M = \tau \overline{\mathbf{j} \cdot \mathbf{b}} / (3\bar{\rho}) = \tau k^2 \overline{\mathbf{a} \cdot \mathbf{b}} / (3\bar{\rho}) = \bar{h}_f$$

(Pouquet et al. 1976)

helically driven dynamo  $\bar{h}_{K,f} = \overline{\boldsymbol{\omega} \cdot \mathbf{u}}$

➔ production of magnetic helicity  $\bar{h}_{M,f} = \overline{\mathbf{a} \cdot \mathbf{b}}$

➔ total magnetic helicity conservation  $\bar{h}_{M,m} = \overline{\mathbf{A} \cdot \mathbf{B}}$

$\overline{\mathbf{a} \cdot \mathbf{b}}$  works against dynamo:  $E_M \propto 1/\text{Re}_M$   $\text{Re}_M = \frac{UL}{\eta}$

Sun:  $\text{Re}_M = 10^9$  galaxies:  $\text{Re}_M = 10^{18}$



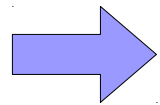
# Dynamical Alpha-Quenching

Magnetic helicity evolution:

$$\text{mean: } \frac{\partial \bar{h}_m}{\partial t} = 2\bar{\mathcal{E}} \cdot \bar{\mathbf{B}} - 2\eta\mu_0\bar{\mathbf{J}} \cdot \bar{\mathbf{B}} - \nabla \cdot \bar{\mathbf{F}}_m$$

$$\text{fluctuating: } \frac{\partial \bar{h}_f}{\partial t} = -2\bar{\mathcal{E}} \cdot \bar{\mathbf{B}} - 2\eta\mu_0\bar{\mathbf{j}} \cdot \bar{\mathbf{b}} - \nabla \cdot \bar{\mathbf{F}}_f$$

$$\alpha_M = \bar{h}_f$$



$$\frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left( \frac{\bar{\mathcal{E}} \cdot \bar{\mathbf{B}}}{B_{\text{eq}}^2} + \frac{\alpha_M}{R_m} \right) - \frac{\partial}{\partial z} \bar{\mathcal{F}}_\alpha$$

# Magnetic Helicity Fluxes

$$\text{I) } \frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left( \frac{\overline{\boldsymbol{\varepsilon}} \cdot \overline{\mathbf{B}}}{B_{\text{eq}}^2} + \frac{\alpha_M}{R_m} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

$$\text{II) } \partial_t \overline{\mathbf{B}} = \alpha \nabla \times \overline{\mathbf{B}} + \eta_T \nabla^2 \overline{\mathbf{B}}$$

$$\text{III) } \overline{\boldsymbol{\varepsilon}} = \alpha \overline{\mathbf{B}} - \eta_t \nabla \times \overline{\mathbf{B}}$$

1D mean-field in z

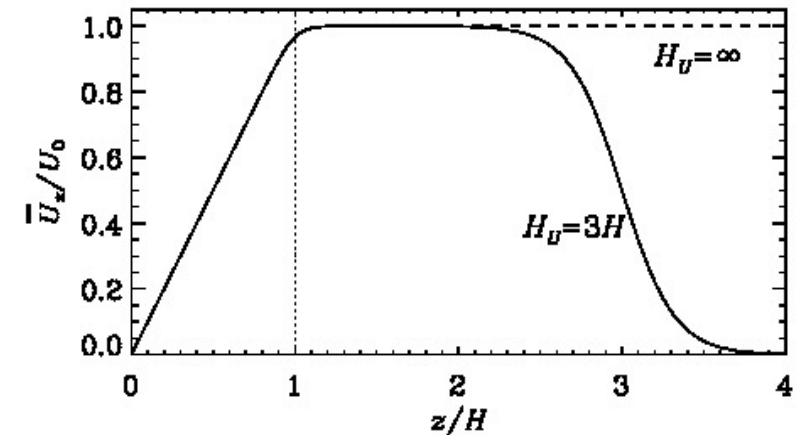
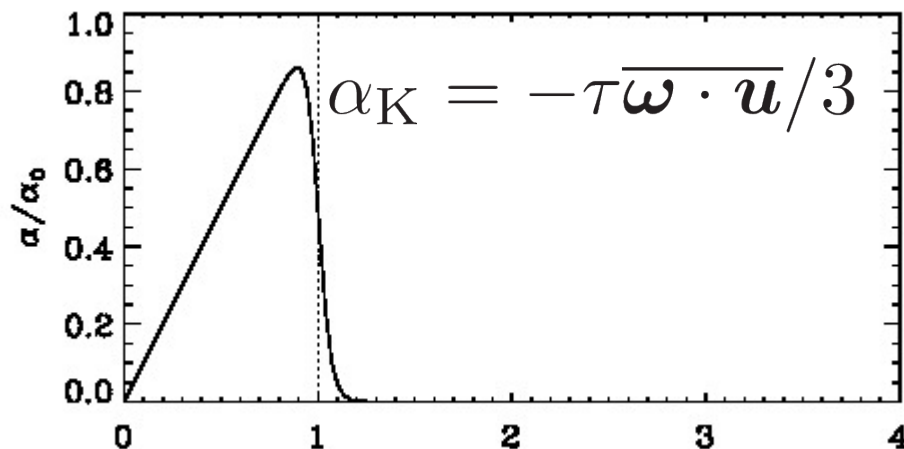
$\alpha$  diffusion

$$\kappa_\alpha \frac{\partial \alpha_M}{\partial z}$$

advective:

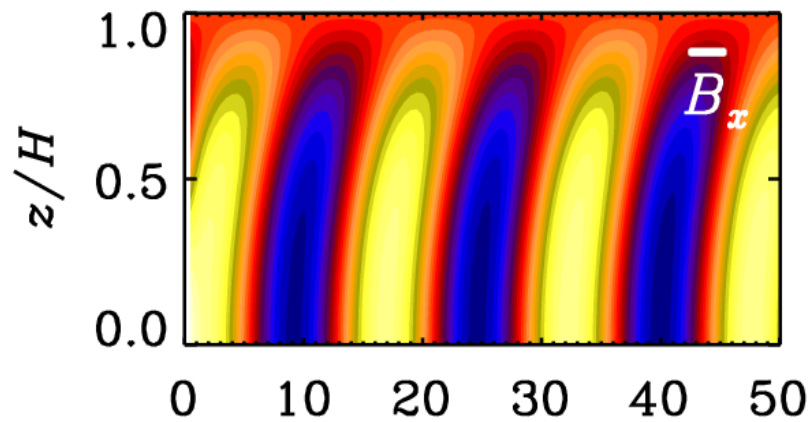
$$\alpha_M \overline{U}$$

Helical forcing profile:

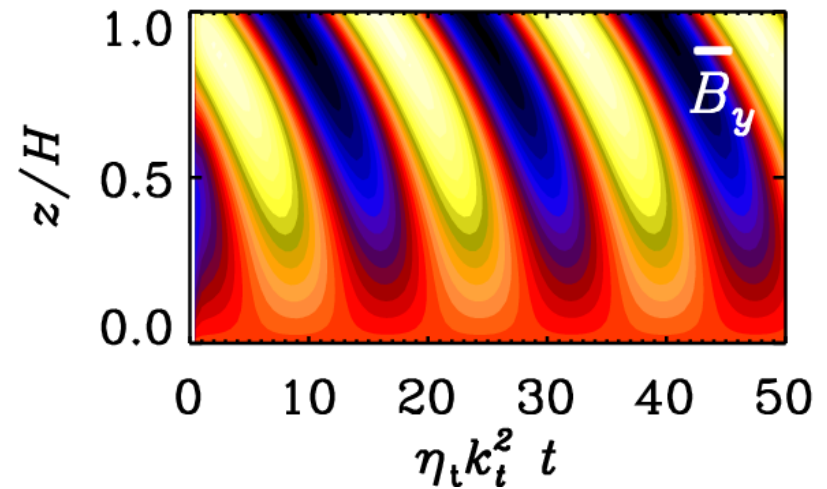
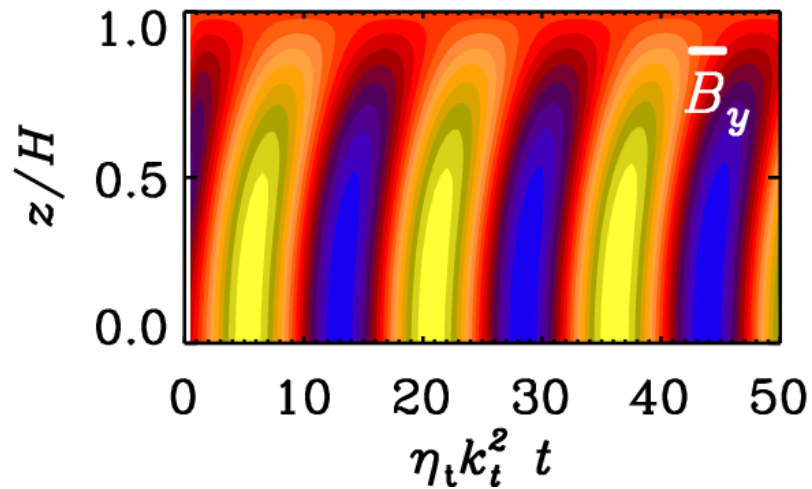
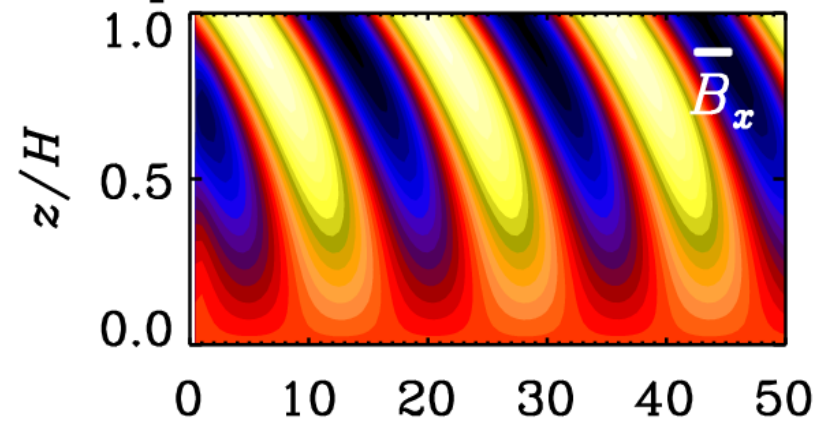


# Dynamo Waves

vertical field condition, S

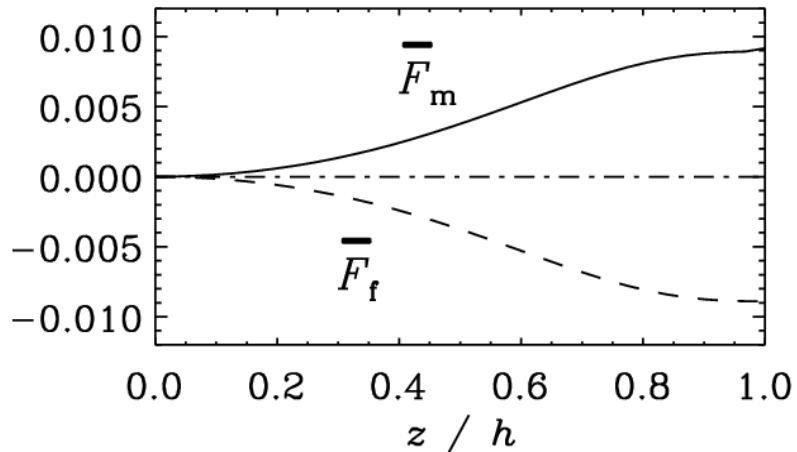


perfect conductor, A



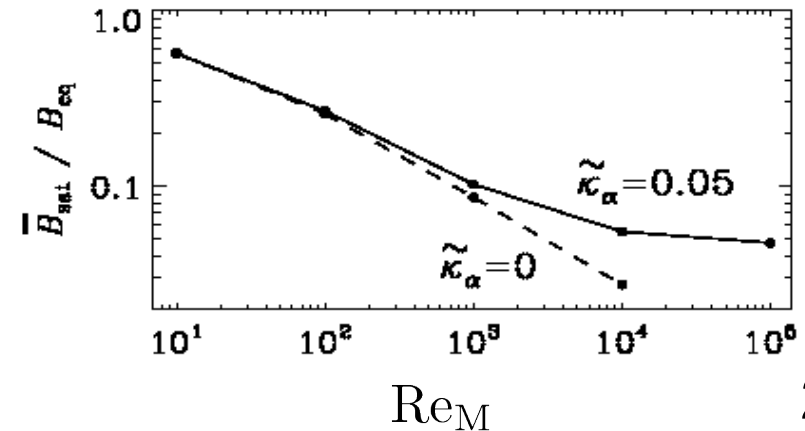
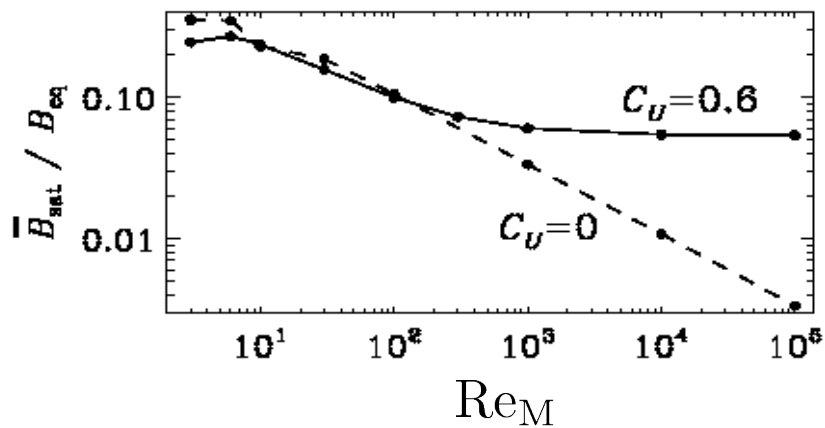
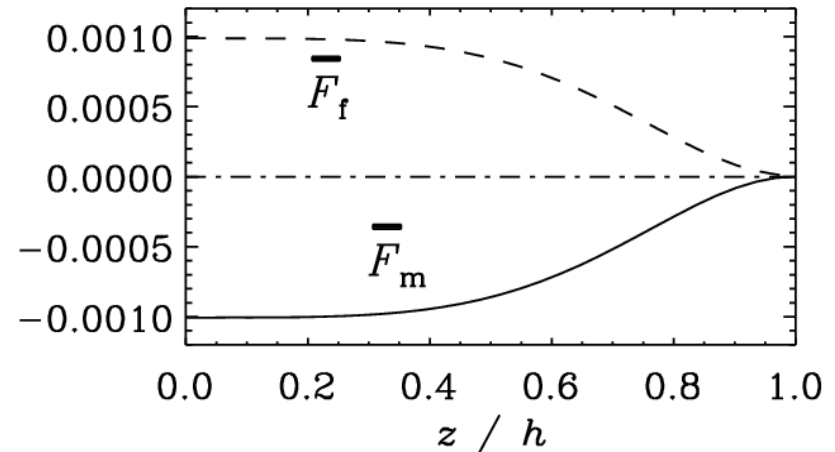
# Magnetic Felicity Fluxes

open boundary  
symmetric  
wind



vs.

closed boundary  
antisymmetric  
 $\kappa_\alpha$



# Conclusions

- Helical turbulence can drive large-scale dynamo action.
- Convective motions in plasma drive dynamos.
- Dynamical alpha-quenching as more self-consistent model.

- Advective magnetic helicity fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

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# Appendix

Viscous force:  $\mathbf{F}_{\text{visc}} = \rho^{-1} \nabla \cdot 2\nu\rho\mathbf{S}$

Strain tensor:  $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij} \nabla \cdot \mathbf{U}$

Sound speed:  $c_S = \sqrt{\gamma \frac{p}{\rho}}$