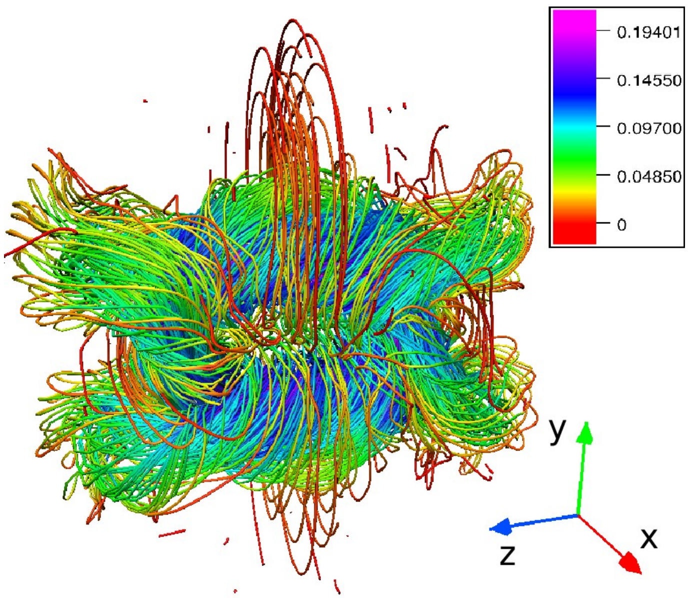
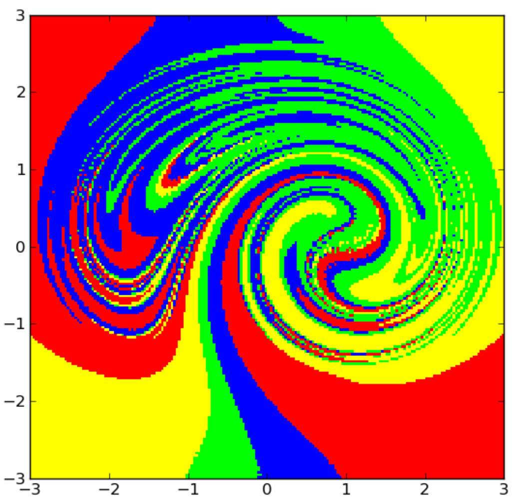
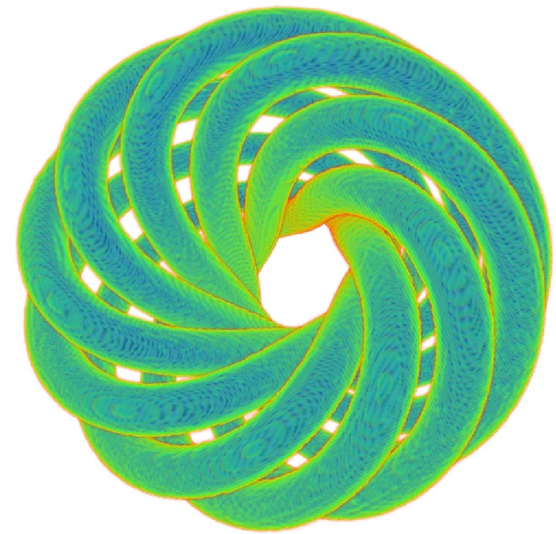
Magnetic Helicity in Astrophysics

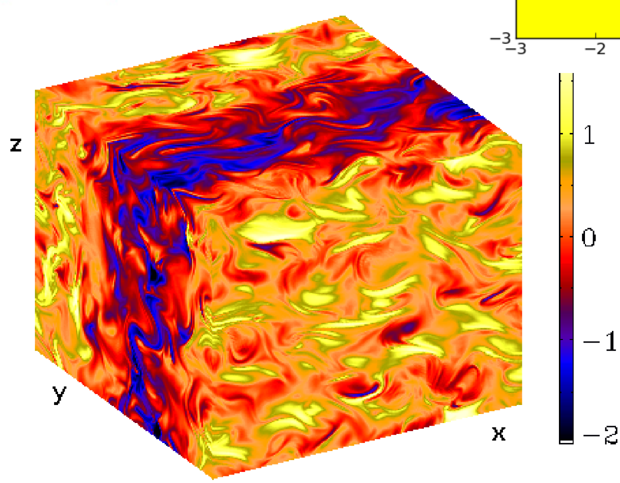
Stockholm University

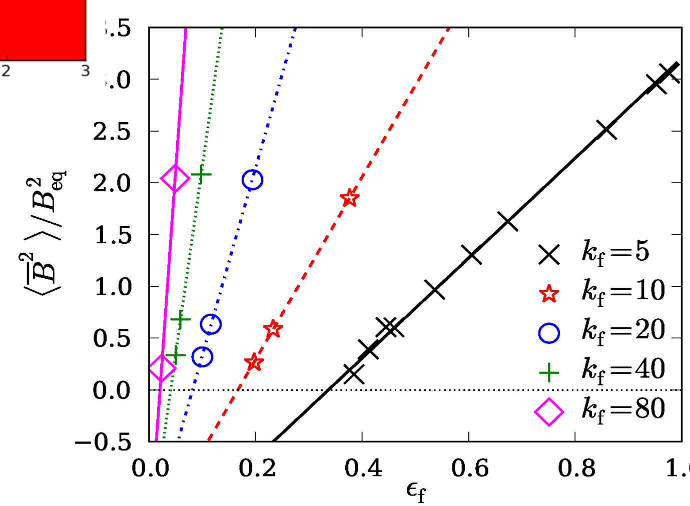


Simon Candelaresi



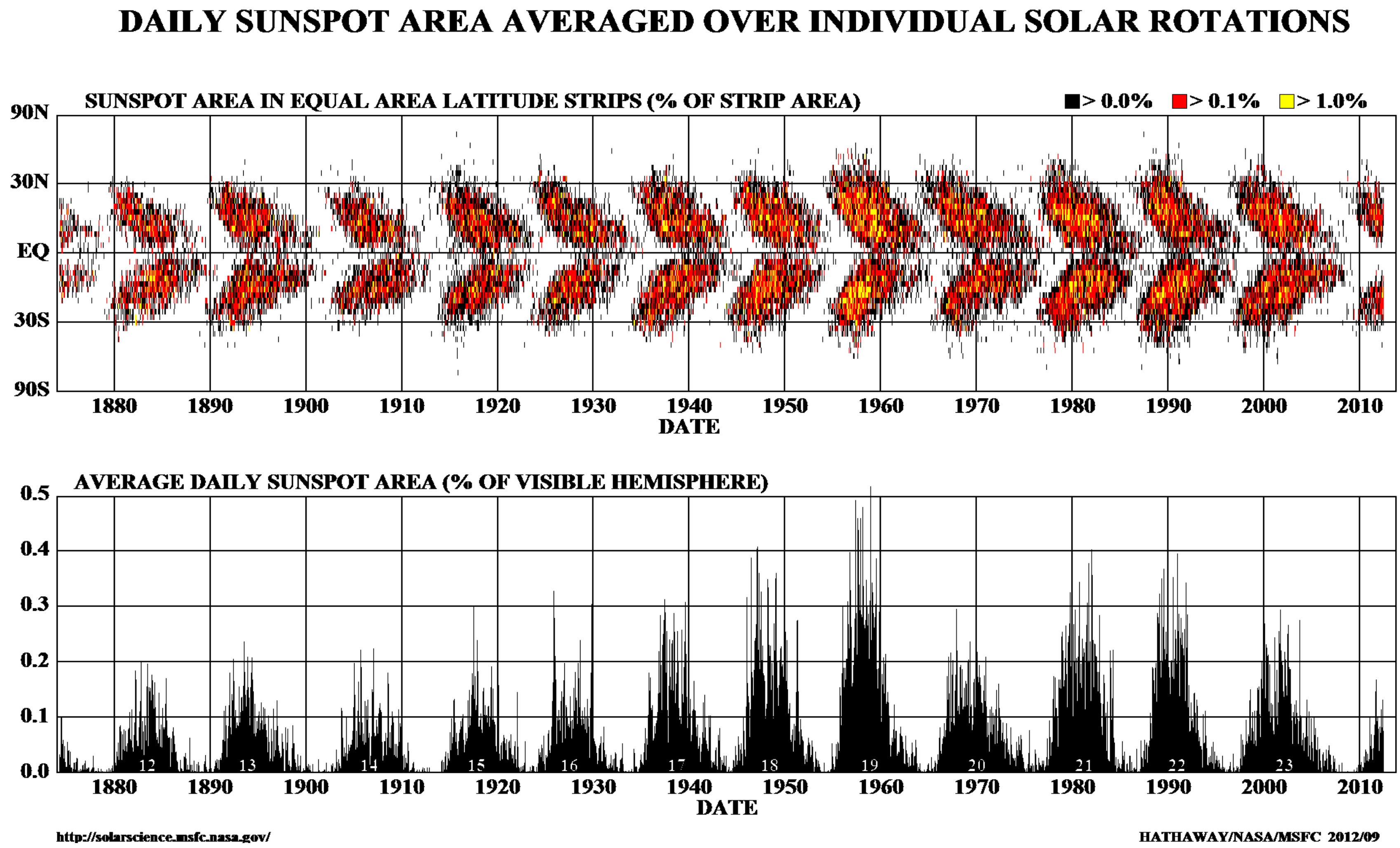






Observations of Magnetic Fields

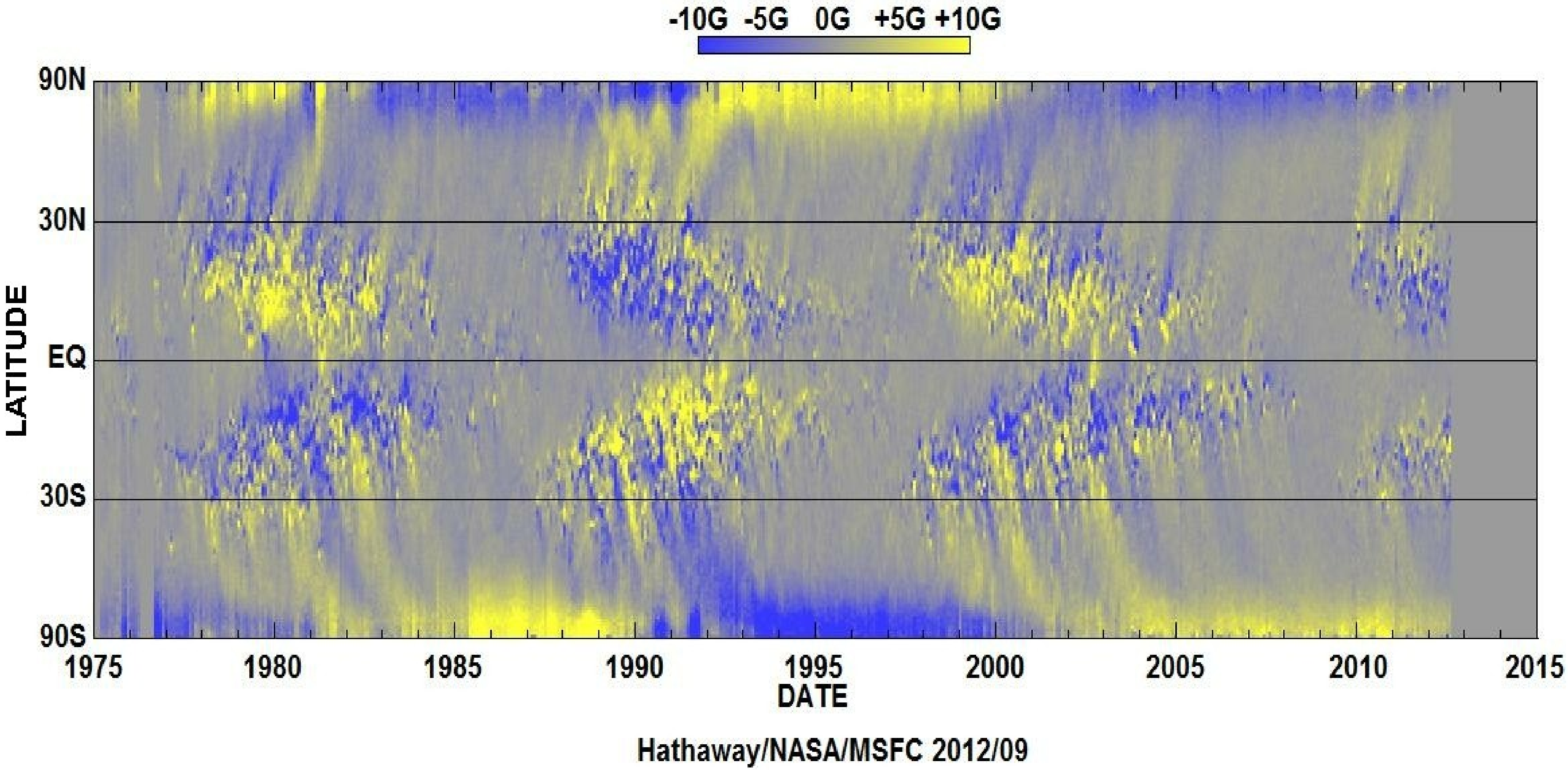
11 years sunspot cycle



(NASA 2012)

Observations of Magnetic Fields

22 years magnetic cycle

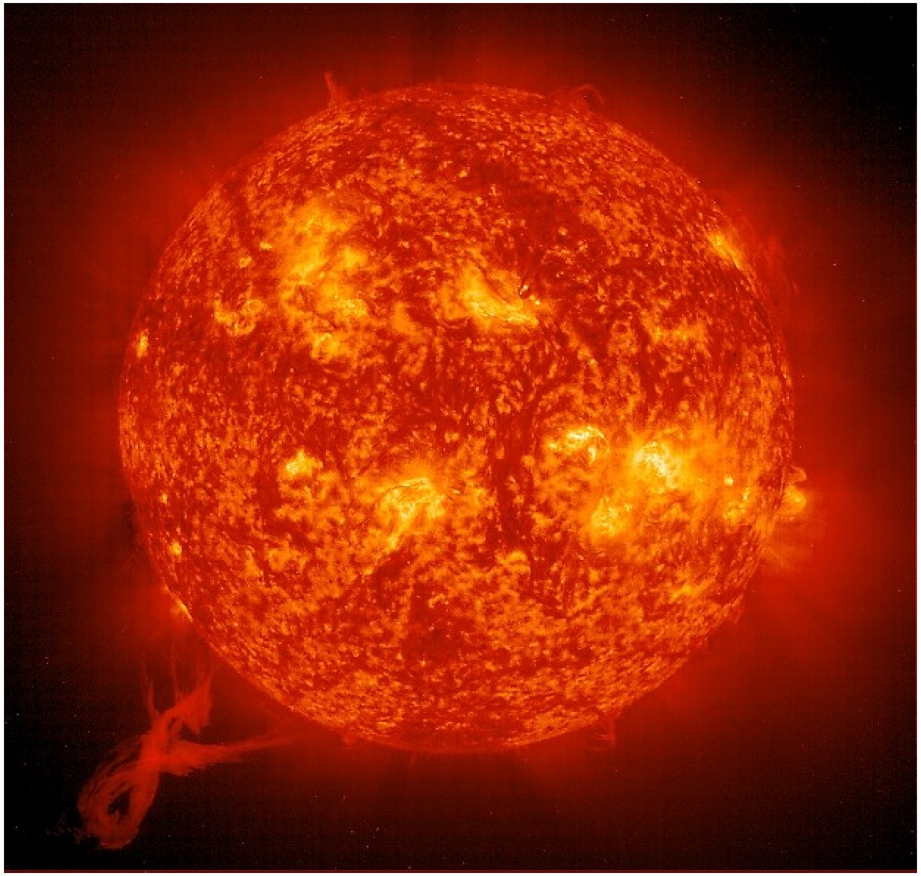


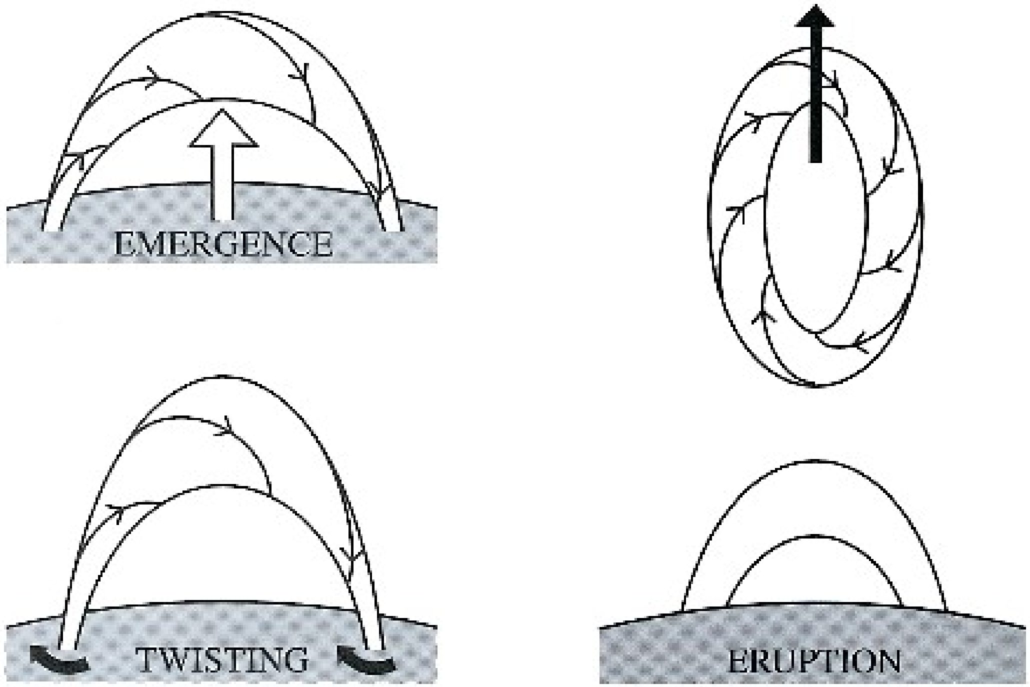
→ dynamo working

(NASA 2012)

Observations of Magnetic Fields

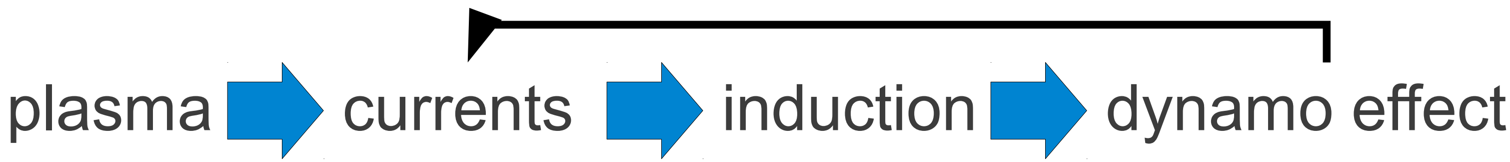
twisted magnetic fields





- Twisted fields are more likely to erupt *(Canfield et al. 1999)*.

Dynamo Mechanism



Equations of **magnetohydrodynamics** (MHD):

Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Momentum equation: $$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

Continuity equation: $$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Advective derivative: $$\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U} \cdot \boldsymbol{\nabla}$$

Mean-Field Formalism

small-scale turbulent motion ⟷ large-scale magnetic fields

separation of scales

Mean-field decomposition: $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$

e.g.: $\overline{\boldsymbol{B}} = \frac{1}{L_x L_y} \int \boldsymbol{B} \, \mathrm{d}x \, \mathrm{d}y$

Mean-field induction equation:

$$\partial_t \overline{\boldsymbol{B}} = \eta \nabla^2 \overline{\boldsymbol{B}} + \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}})$$

Electromotive force (EMF): $\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$

need closure → express $\overline{\boldsymbol{\mathcal{E}}}$ in terms of the mean fields:

$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{\mathcal{E}}}(\overline{\boldsymbol{U}}, \overline{\boldsymbol{B}}, ...)$$

Mofatt, H. K., 1978, Krause and Raedler, 1980

EMF and Nonlinear Alpha-Effect

For a turbulent system without preferred direction, i.e. $\boldsymbol{U} = 0$:

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \nabla \times \overline{\boldsymbol{B}}$$

$$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \alpha \overline{\boldsymbol{B}} + \eta_{\rm T} \boldsymbol{\nabla}^2 \overline{\boldsymbol{B}}$$

α effect: $\alpha = \alpha_{\rm K} + \alpha_{\rm M}$ (magnetic helicity conservation)

$$\alpha_{\rm K} = -\tau \overline{\boldsymbol{\omega} \cdot \boldsymbol{u}}/3$$

$$\alpha_{\rm M} = \tau \overline{\boldsymbol{j} \cdot \boldsymbol{b}}/(3\overline{\rho}) \approx \tau k^2 \overline{\boldsymbol{a} \cdot \boldsymbol{b}}/(3\overline{\rho}) = \overline{h}_{\rm f}$$

(Pouquet et al. 1976)

Magnetic helicity densitiy: $h = \boldsymbol{A} \cdot \boldsymbol{B}$

$\overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^9$ galaxies: ${\rm Re}_{\rm M} = 10^{18}$

→ catastrophic alpha quenching

Magnetic Helicity Conservation

without fluxes (closed system): $\lim_{\eta\to 0} \frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle = 0$

linear stability: $\lambda = \alpha k - \eta_\mathrm{T} k^2 = (C_\alpha - 1)\eta_\mathrm{T} k^2$

large scale: $k = k_1 = 1$

$\epsilon_\mathrm{f} =$ fracitonal helicity

$$C_\alpha = \epsilon_\mathrm{f}\frac{k_\mathrm{f}}{k_1}$$

$$C_\alpha^\mathrm{crit} = 1$$

$$\epsilon_\mathrm{f}^\mathrm{crit} = \left(\frac{k_\mathrm{f}}{k_1}\right)^{-1}$$

Simulations:

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_\mathrm{S}^2\boldsymbol{\nabla}\ln\rho + \boldsymbol{J}\times\boldsymbol{B}/\rho + \boldsymbol{F}_\mathrm{visc} + \boldsymbol{f}$$

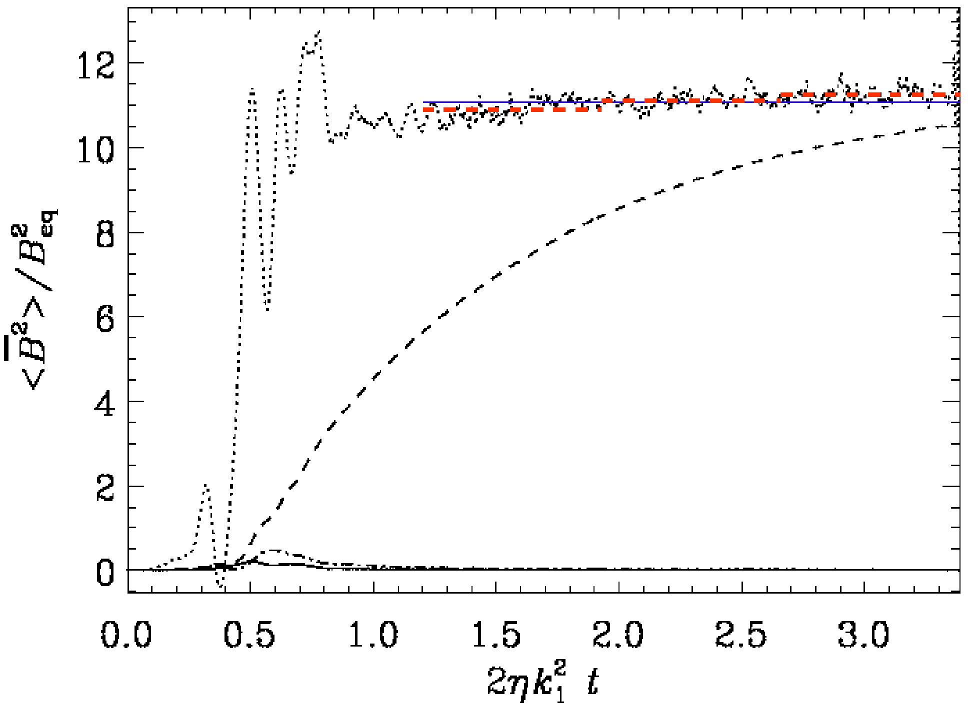
helical forcing

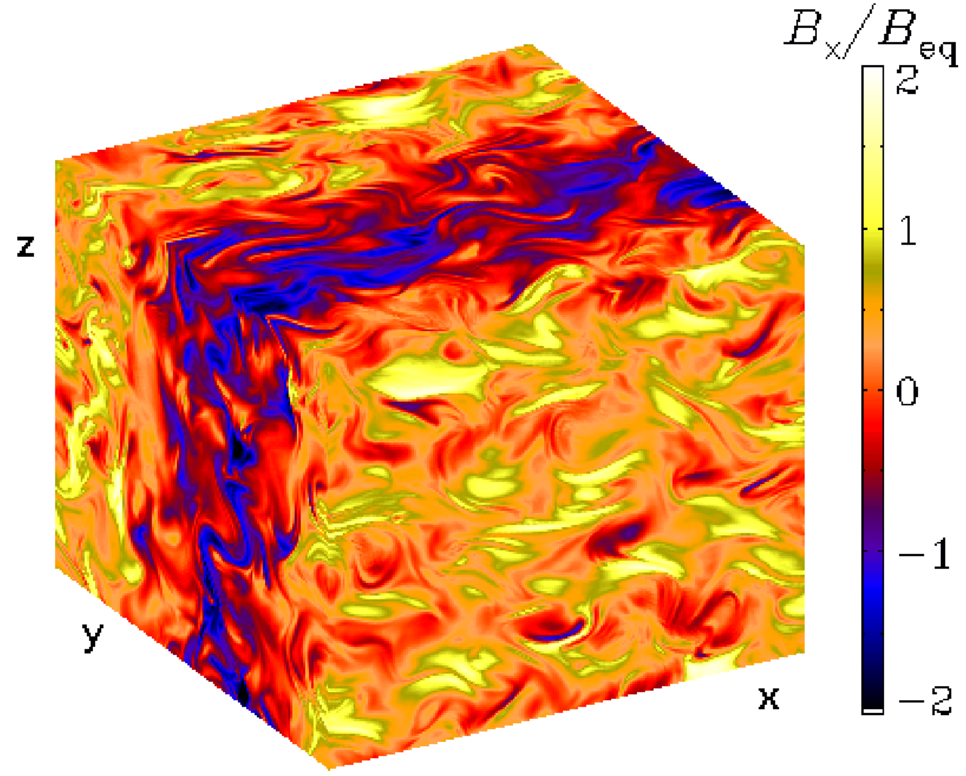
triply periodic box (helicity conservation)

parameters: ϵ_f and k_f

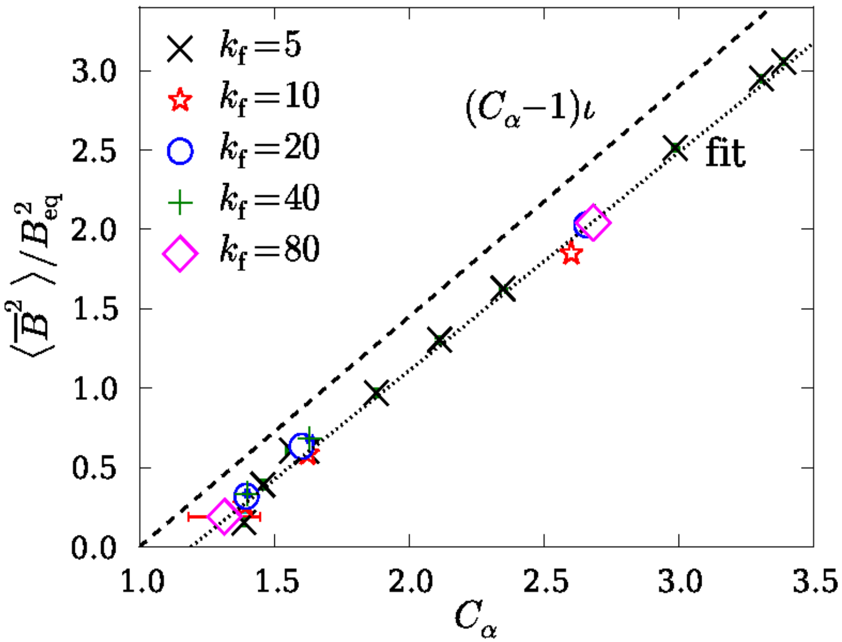
Magnetic Helicity Conservation

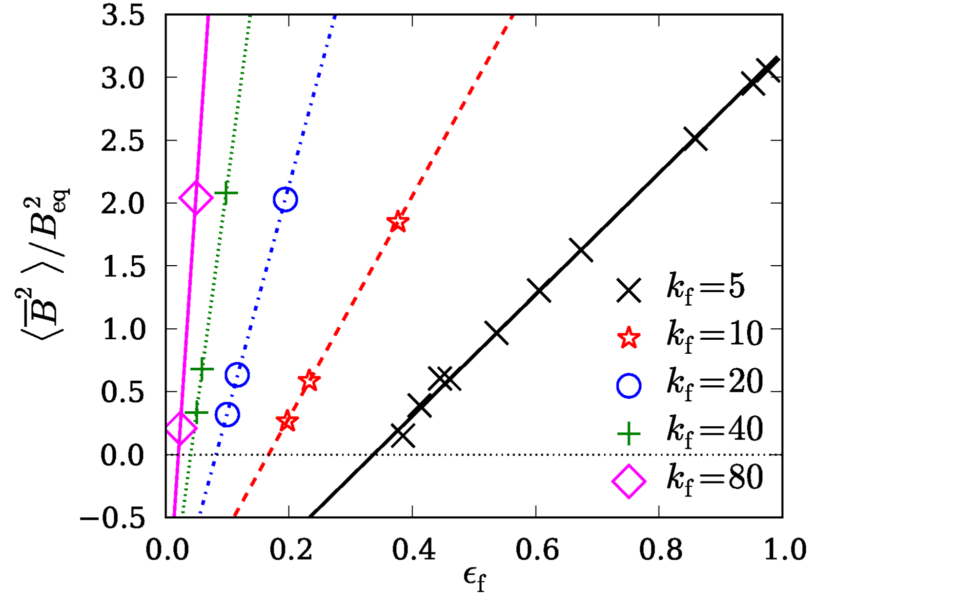
Slow saturation of the mean magnetic field.

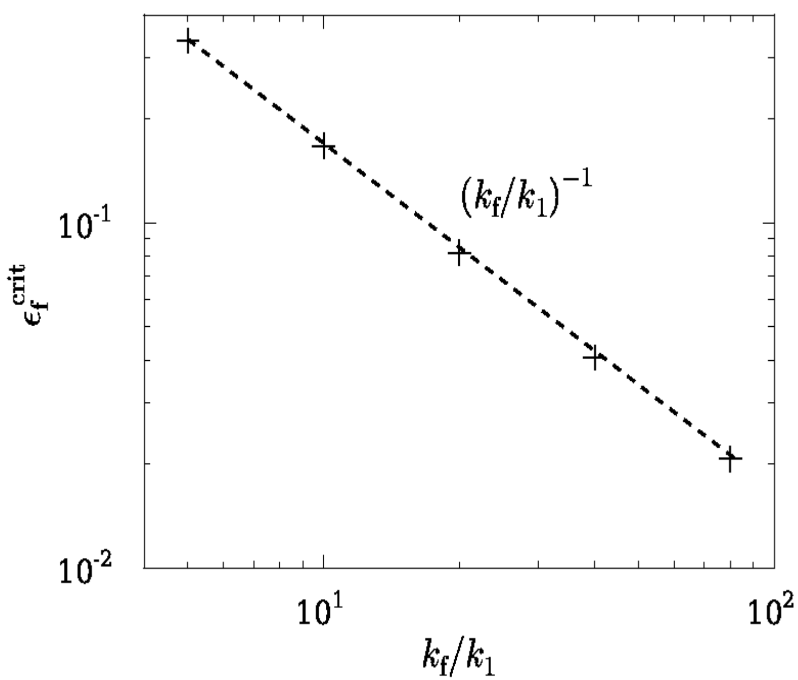




Magnetic Helicity Conservation







$$C_\alpha^{\mathrm{crit}} = 1$$

$$\epsilon_f^{\mathrm{crit}} = \left(\frac{k_f}{k_1}\right)^{-1}$$

Mean-field predictions are confirmed quantitatively.

Magnetic Helicity Fluxes

I) $$\frac{\partial \alpha_{\rm M}}{\partial t} = -2\eta_{\rm t} k_{\rm f}^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm M}}{R_{\rm m}} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

II) $$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \alpha \overline{\boldsymbol{B}} + \eta_{\rm T} \boldsymbol{\nabla}^2 \overline{\boldsymbol{B}}$$

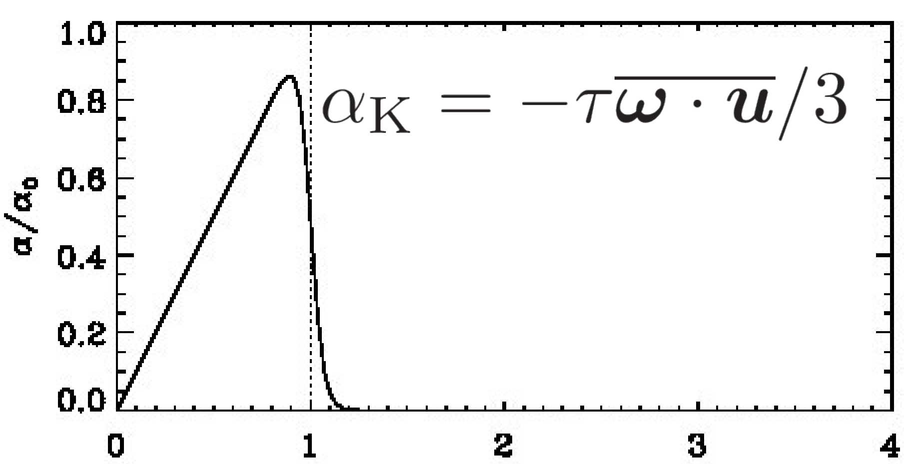
III) $$\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \nabla \times \overline{\boldsymbol{B}}$$

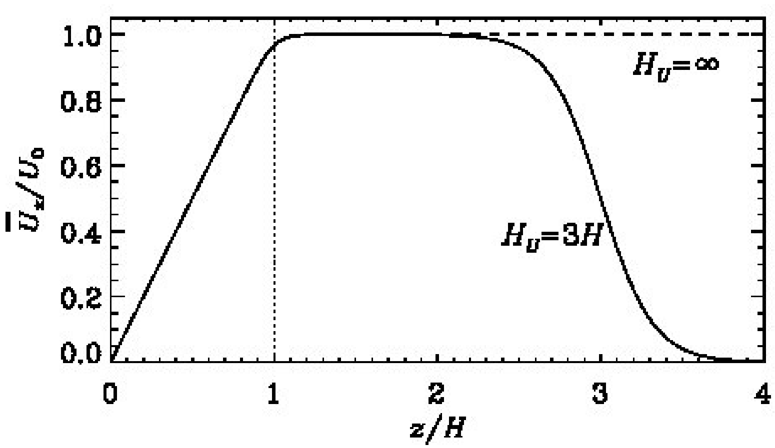
1D mean-field in z

α diffusion $\kappa_\alpha \frac{\partial \alpha_{\rm M}}{\partial z}$

advective: $\alpha_{\rm M} \overline{\boldsymbol{U}}$

Helical forcing profile:



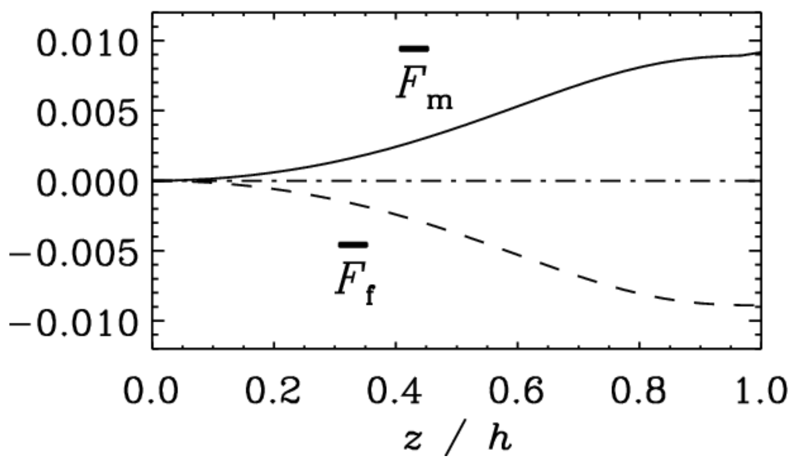


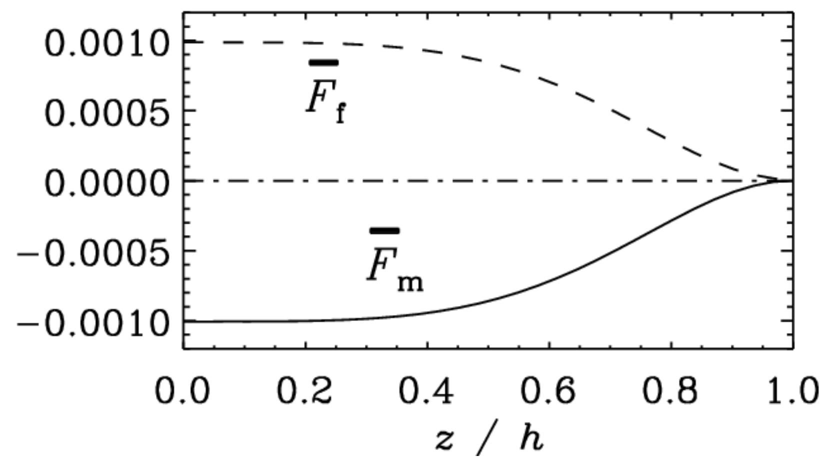
Magnetic Helicity Fluxes

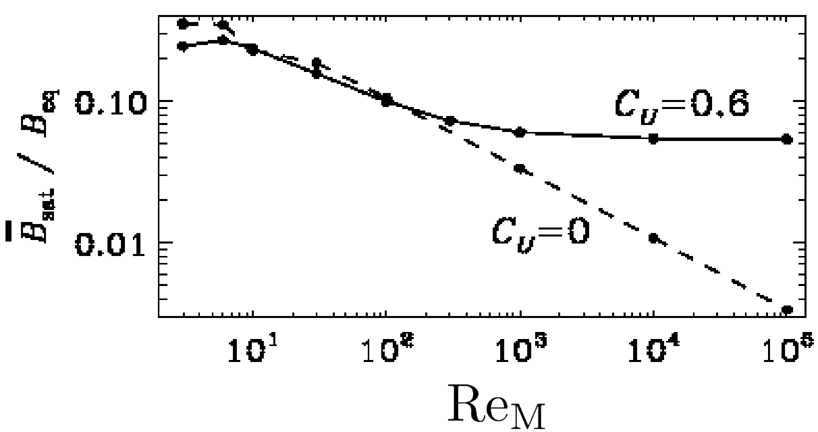
open boundary
symmetric
wind

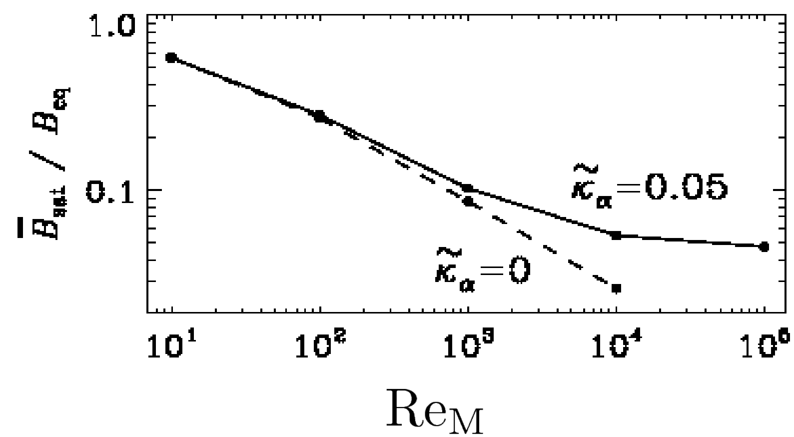
vs.

closed boundary
antisymmetric
κ_α







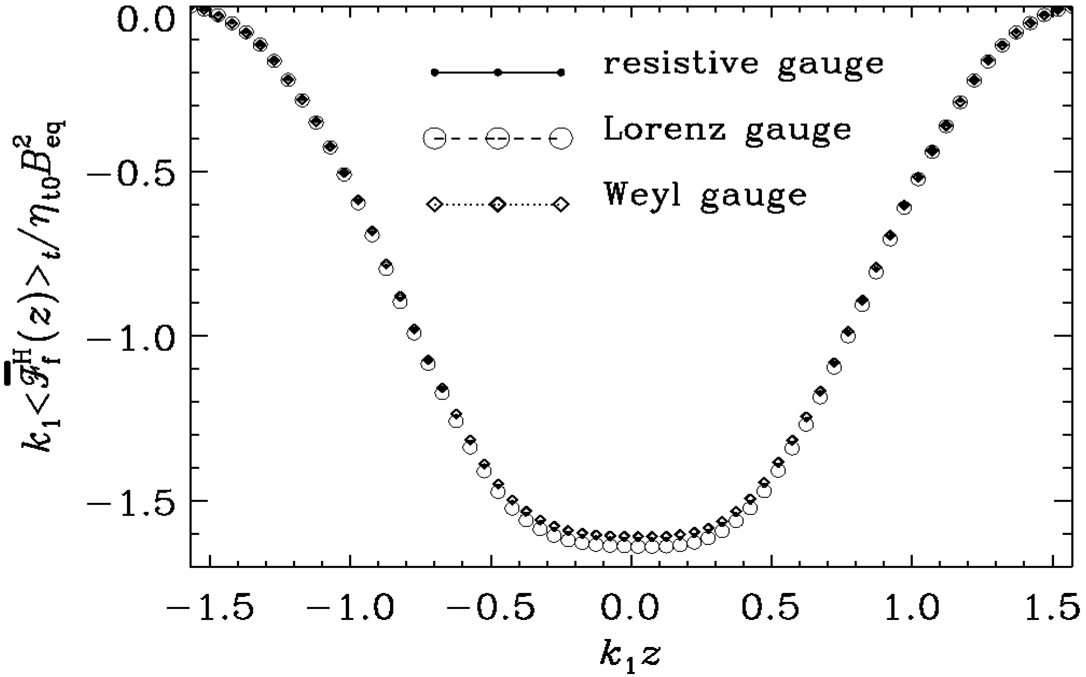


Gauge Issues

Gauge transformation: $\boldsymbol{A} \to \boldsymbol{A} + \boldsymbol{\nabla}\Lambda$

$$h'_{\rm m} = h_{\rm m} + \boldsymbol{\nabla}\Lambda \cdot \boldsymbol{B}$$

- resistive gauge
- pseudo-Lorenz gauge
- Weyl gauge



- helical forcing analog. MF
- periodic boundaries
- 128X128x256 box

Time averaged magnetic helicity fluxes do not depend on the gauge.

Its importance for dynamos is saved.

Advective Gauges

induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

uncurl

resistive gauge

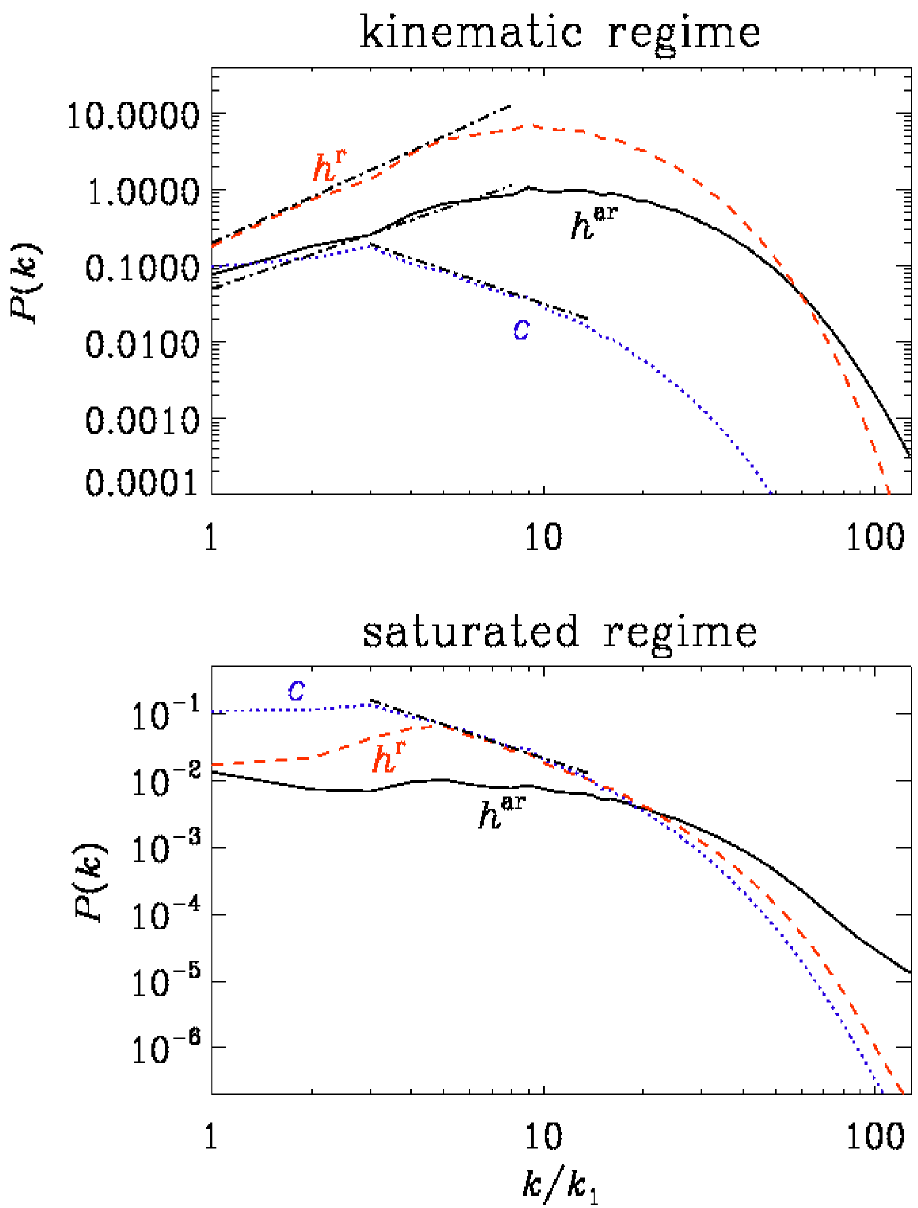
$$\frac{\partial \boldsymbol{A}^{\mathrm{r}}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A}^{\mathrm{r}}$$

advecto-resistive gauge

$$\frac{\partial \boldsymbol{A}^{\mathrm{ar}}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J} - \boldsymbol{\nabla}(\boldsymbol{U} \cdot \boldsymbol{A}^{\mathrm{ar}} - \eta \boldsymbol{\nabla} \cdot \boldsymbol{A}^{\mathrm{ar}})$$

- measure helicity transport
- spatial distribution of the magnetic helicity

Advective Gauges



passive scalar: $\frac{\mathrm{D}C}{\mathrm{D}t} = -\kappa \nabla^2 C$

In the kinematic regime h^{ar} behaves like a passive scalar.

h^{ar} has strong high-k tail

↓

efficient turbulent cascade in the advecto-resistive gauge

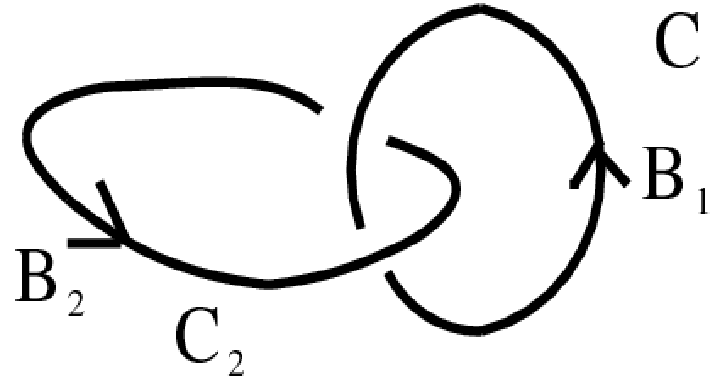
Magnetic Helicity

Measure for the topology:

$$H_{\mathrm{M}} = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 2n\phi_1\phi_2$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B} \qquad \phi_i = \int_{S_i} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$$

$n =$ number of mutual linking



Conservation of magnetic helicity:

$$\lim_{\eta \to 0} \frac{\partial}{\partial t} \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0 \qquad \eta = \text{magnetic resistivity}$$

Realizability condition:

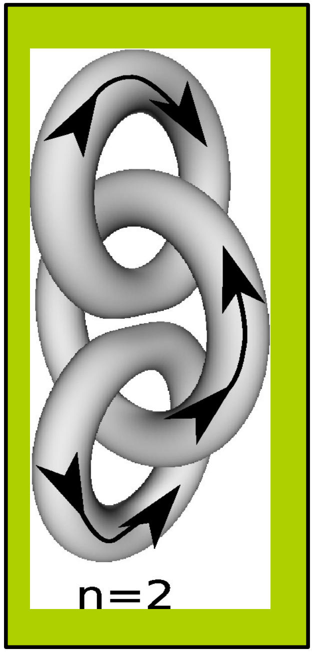
$$E_{\mathrm{m}}(k) \geq k|H(k)|/2\mu_0$$

Magnetic energy is bound from below by magnetic helicity.

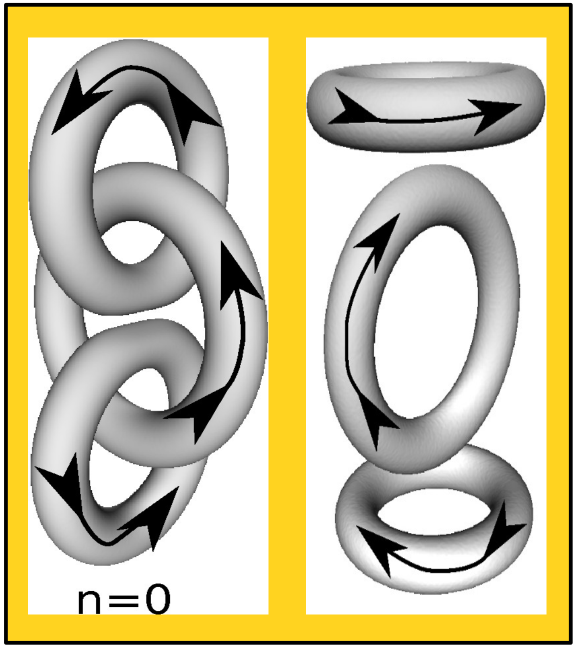
Interlocked Flux Rings

actual linking vs. magnetic helicity

$H_{\mathrm{M}} \neq 0$



$H_{\mathrm{M}} = 0$



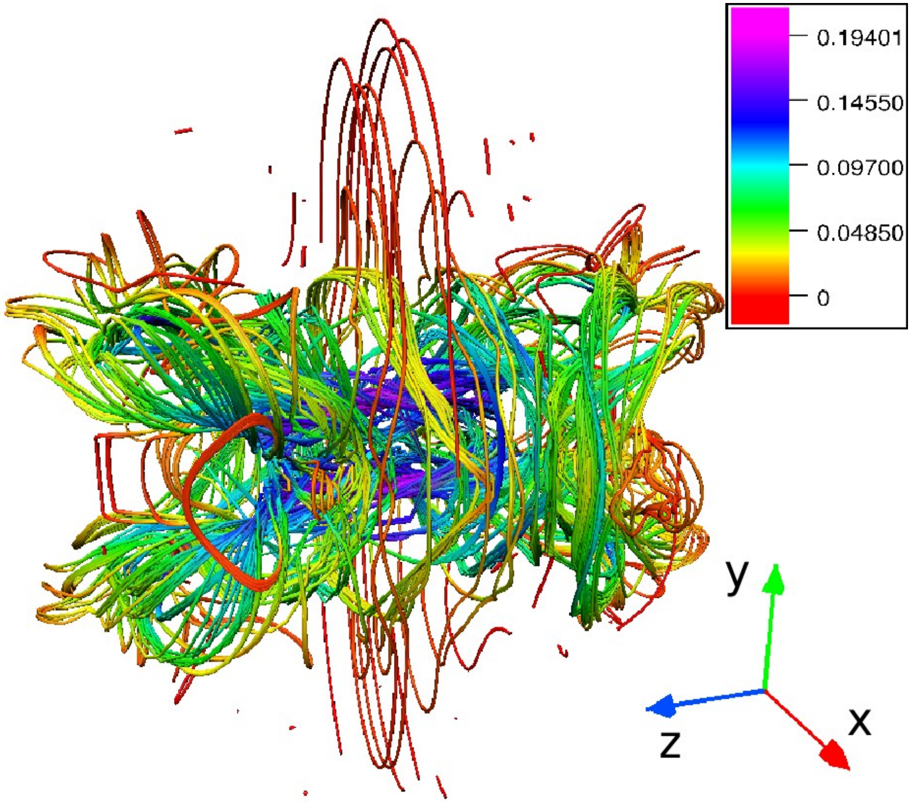
- initial condition: flux tubes
- isothermal compressible gas
- viscous medium
- periodic boundaries

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A} \qquad \frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

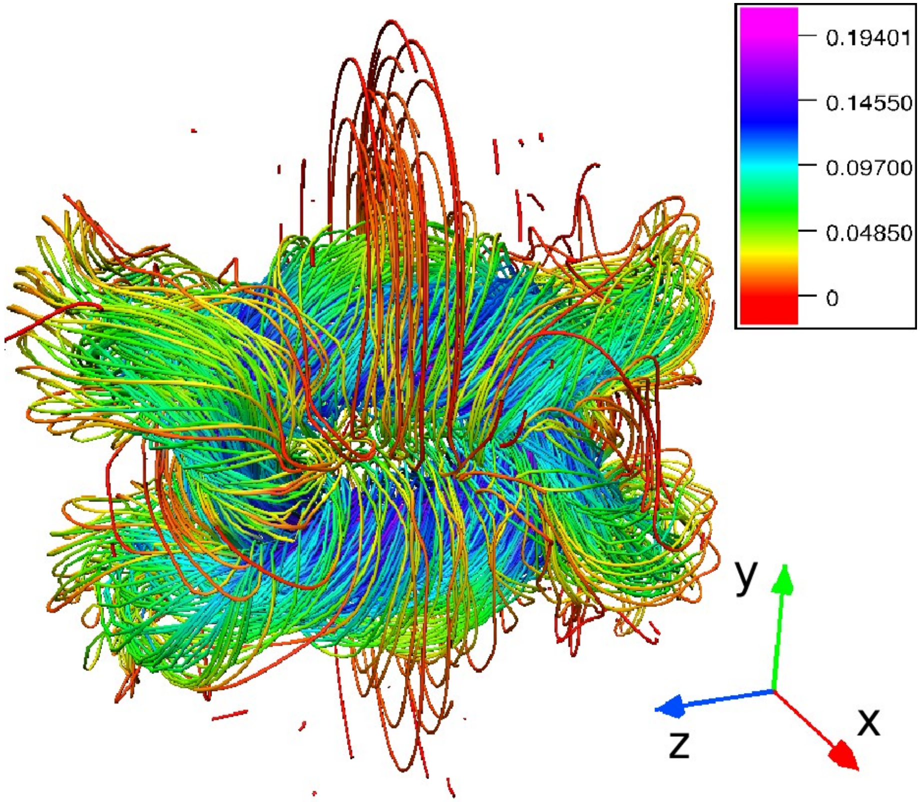
$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

Interlocked Flux Rings

$\tau = 4$

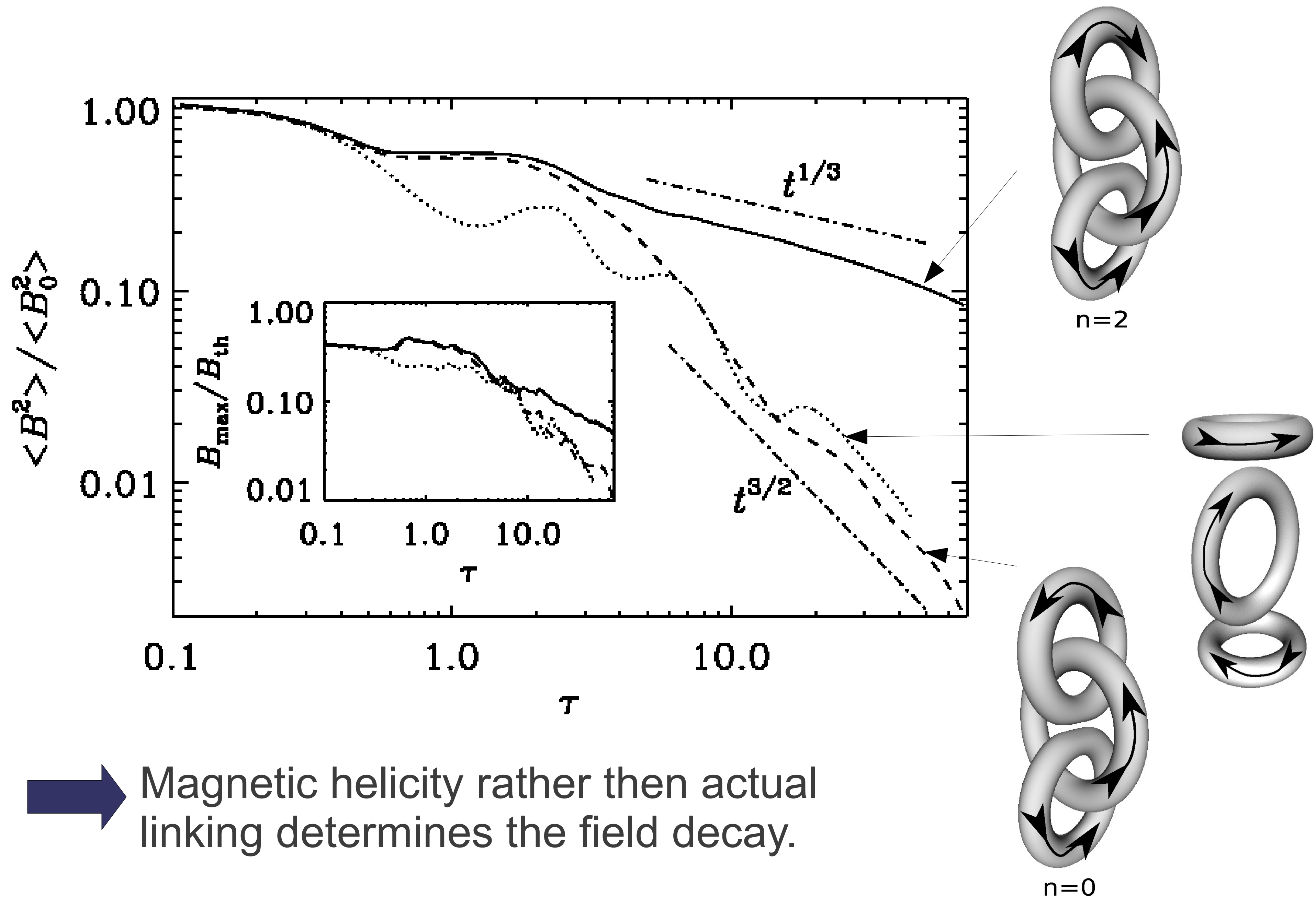


$H_{\mathrm{M}} = 0$



$H_{\mathrm{M}} \neq 0$

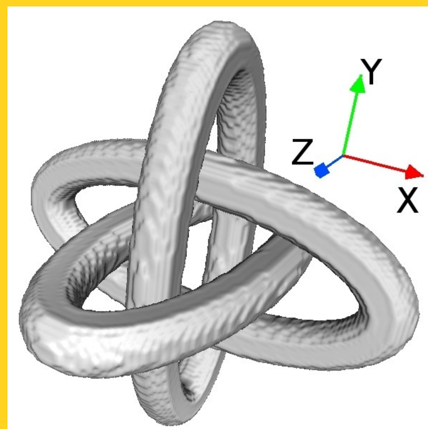
Interlocked Flux Rings



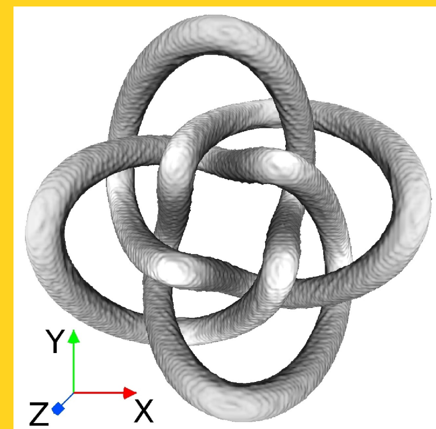
➡ Magnetic helicity rather then actual linking determines the field decay.

IUCAA Knot and Borromean Rings

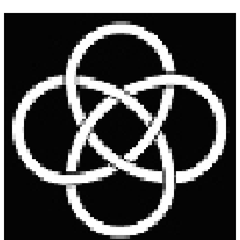
- Is magnetic helicity sufficient?
- Higher order invariants?



Borromean rings



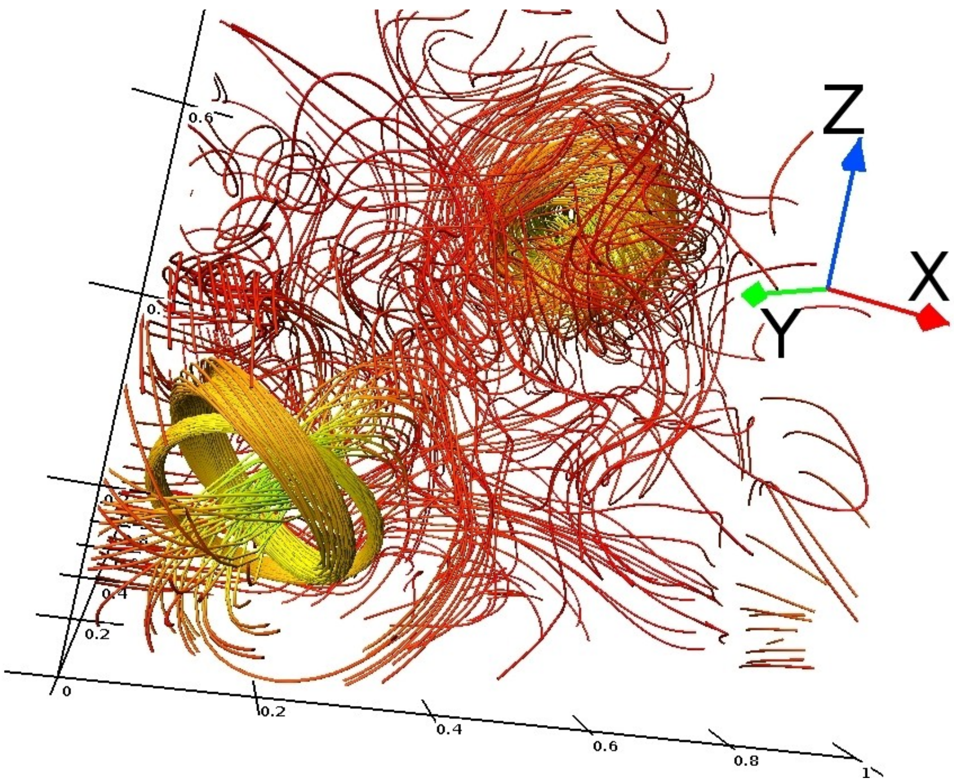
IUCAA knot



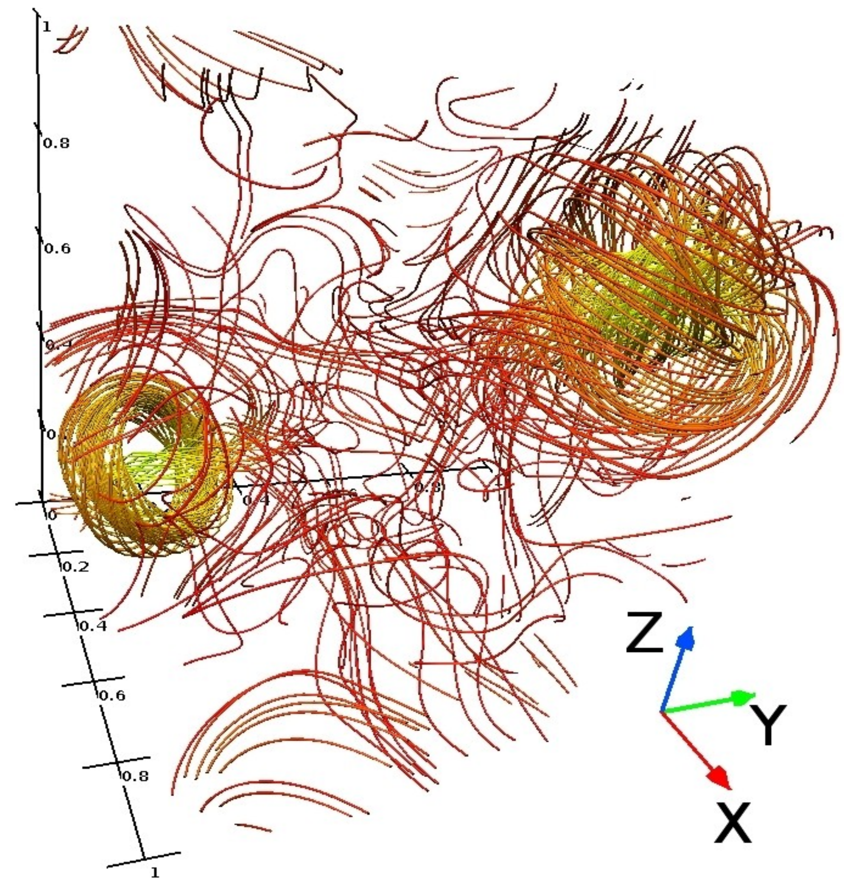
$$H_{\mathrm{M}} = 0$$

IUCAA = The Inter-University Centre for Astronomy and Astrophysics, Pune, India

Reconnection Characteristics



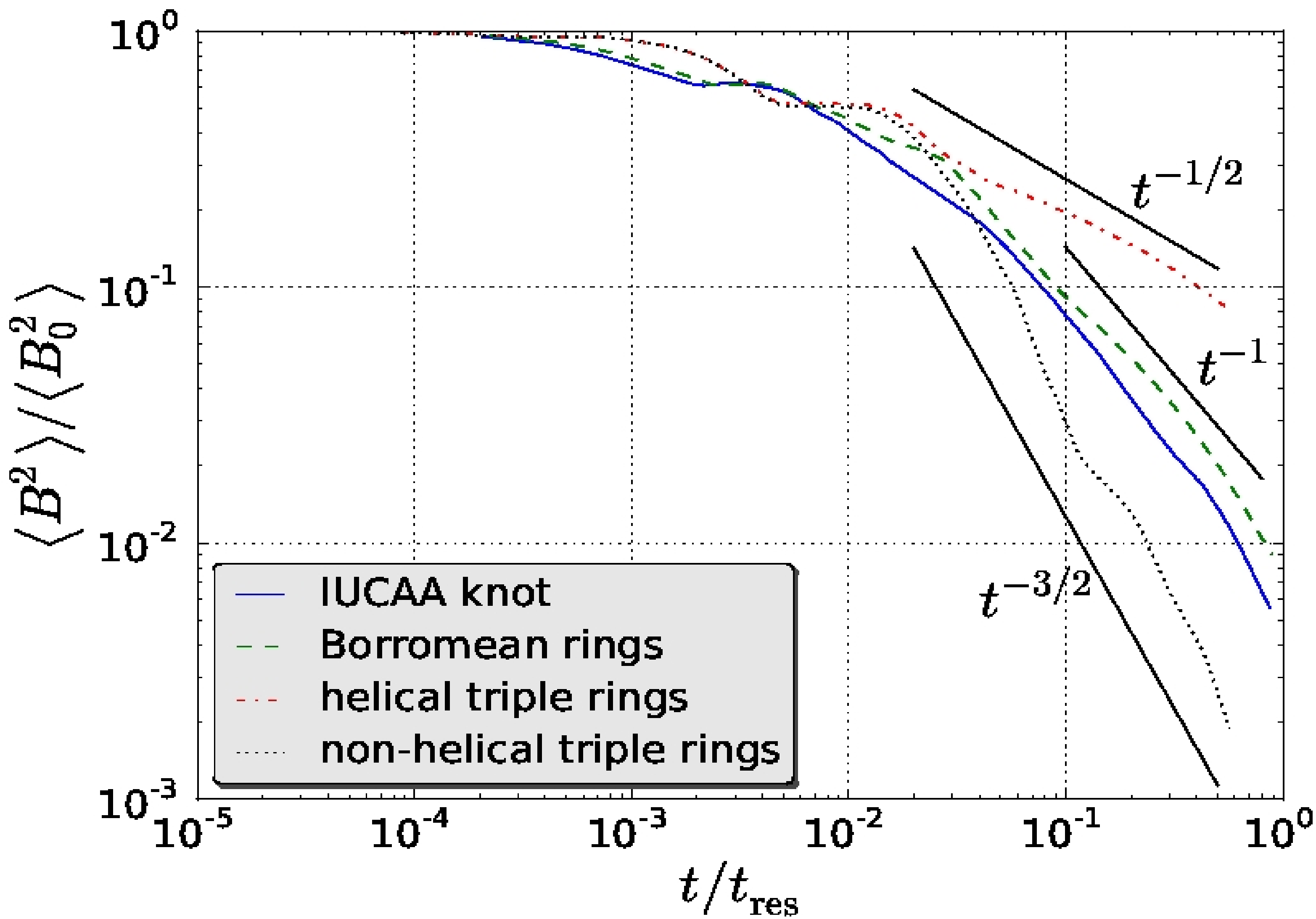
$t = 70$



$t = 78$

3 rings → Twisted ring + interlocked rings → 2 twisted rings

Magnetic Energy Decay



Higher order invariants?

Summary

- MF predictions confirmed in magnetic helicity conservation.
- Magnetic helicity fluxes alleviate catastrophic alpha quenching.
- Gauge problem solved for statistical averages.
- Adveco-resistive gauge transports helicity to small scales.
- Braiding increases stability through the *realizability condition*.
- Turbulent magnetic field decay is restricted by magnetic helicity.

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