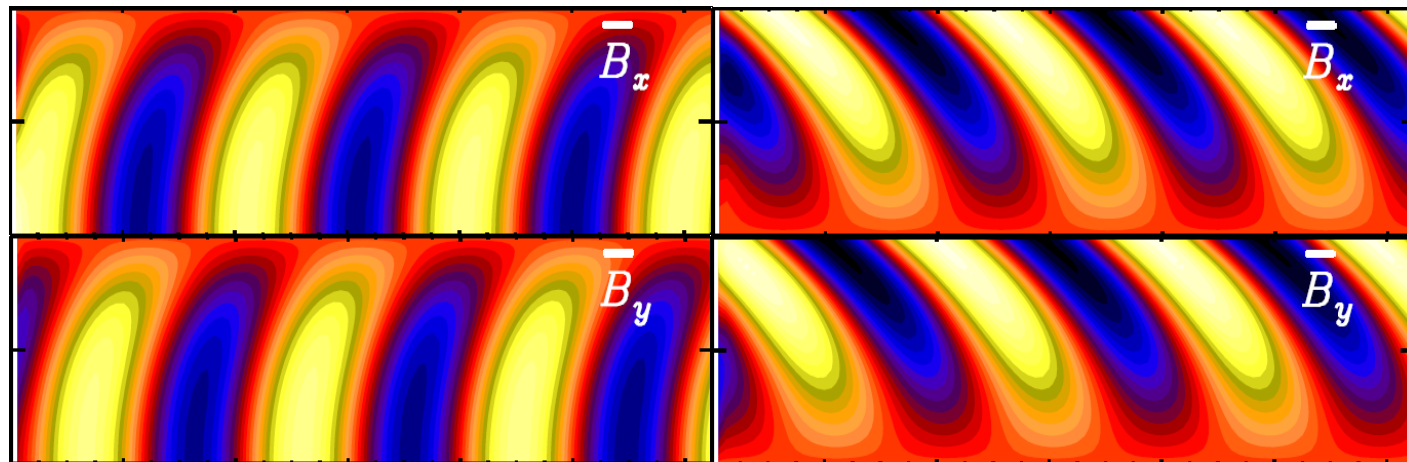
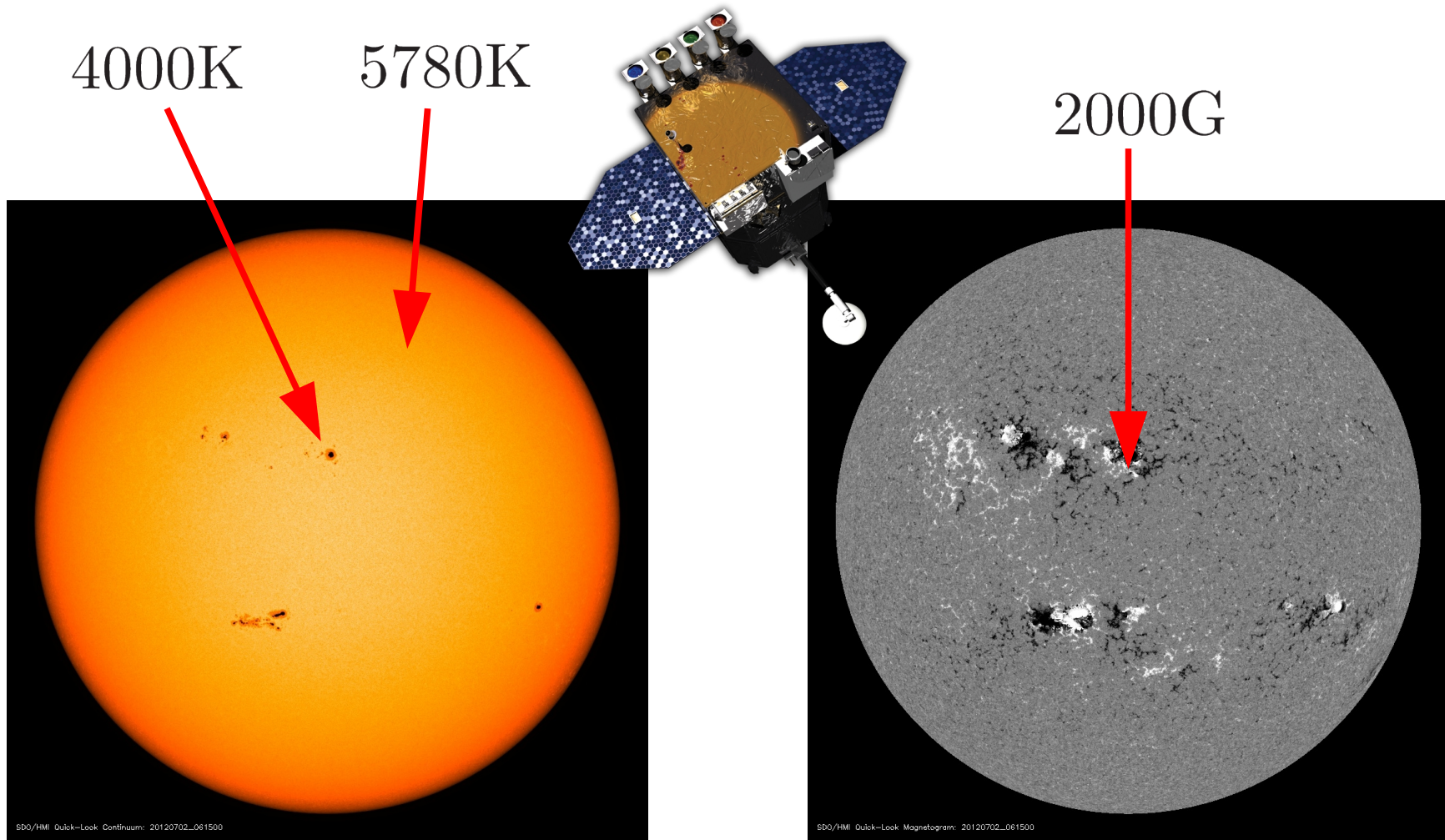


Magnetic helicity fluxes in dynamically quenched dynamamos

Simon Candelaresi



Solar Dynamics Observatory (SDO)

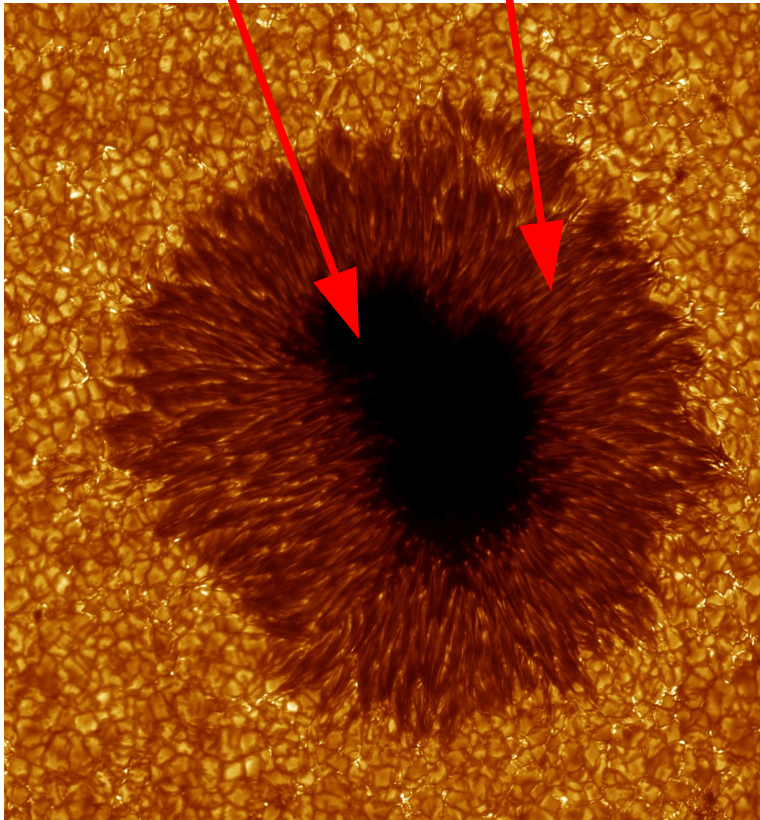


2nd July 2012, Intensity

2nd July 2012, Magnetogram

Swedish Solar Telescope (SST)

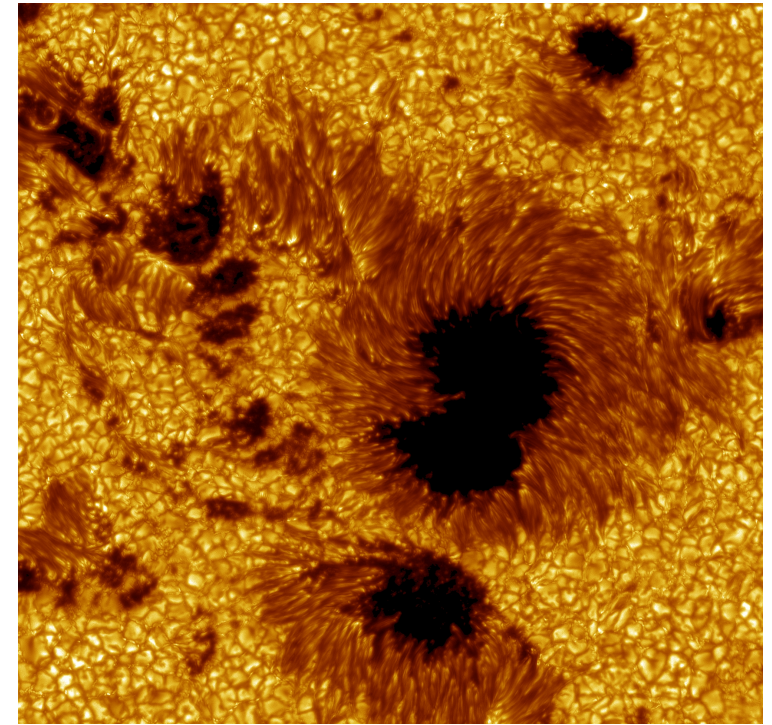
umbra penumbra



430.5 nm (G-band), 3rd July 2003,
(Dan Kiselman, Mats Löfdahl, 2003)

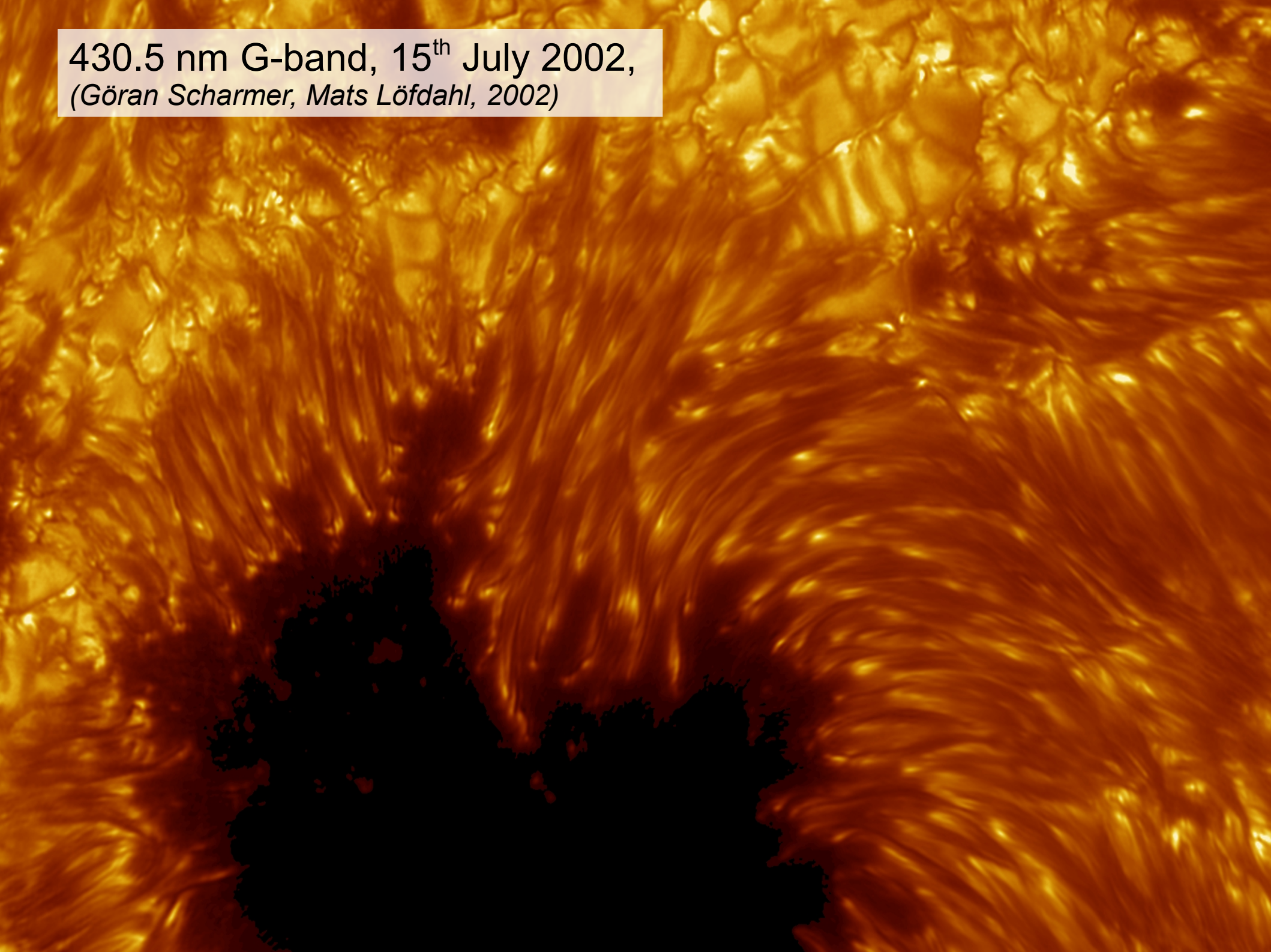


La Palma
(Göran Scharmer)



487.7 nm, 15th July 2002,
(Göran Scharmer, Mats Löfdahl, 2002)

430.5 nm G-band, 15th July 2002,
(Göran Scharmer, Mats Löfdahl, 2002)

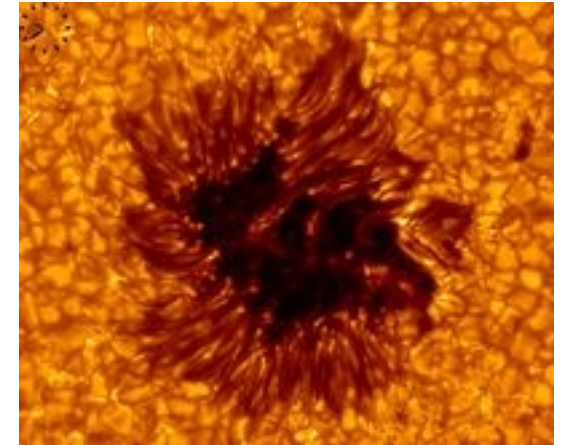
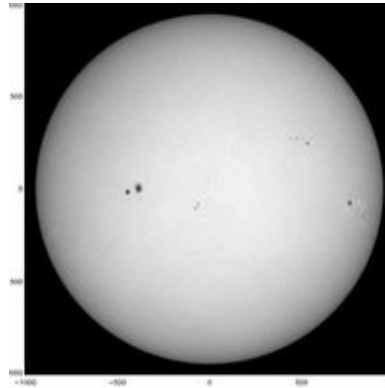


Swedish Solar Telescope (SST)

1h quiet Sun, 656.3 nm,
18th June 2006,
(Luc Rouppe van der Voort, Oslo, 2006)

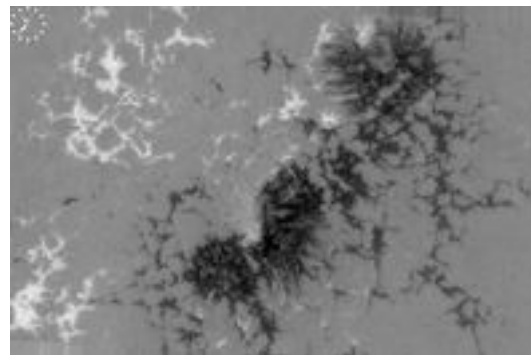


Zoom from SOHO/MDI field
of view to SST resolution,
August 2004,
*(Michiel van Noort, Luc Rouppe van
der Voort, Mats Carlsson, Oslo, 2004)*

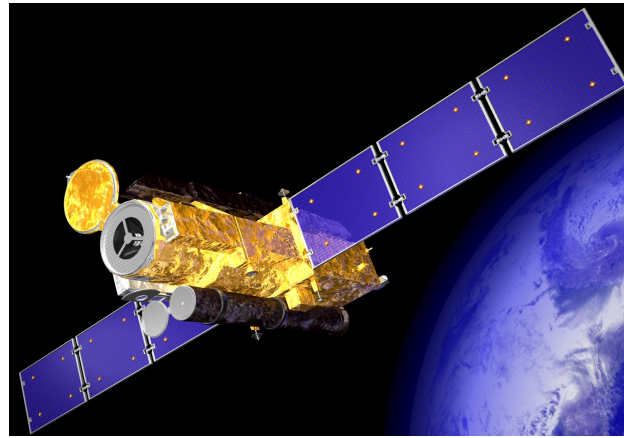


Sunspot 41 min, 430.5 nm
G-band, 20th August 2004,
*(Michiel van Noort and Luc Rouppe
van der Voort, Oslo, 2004)*

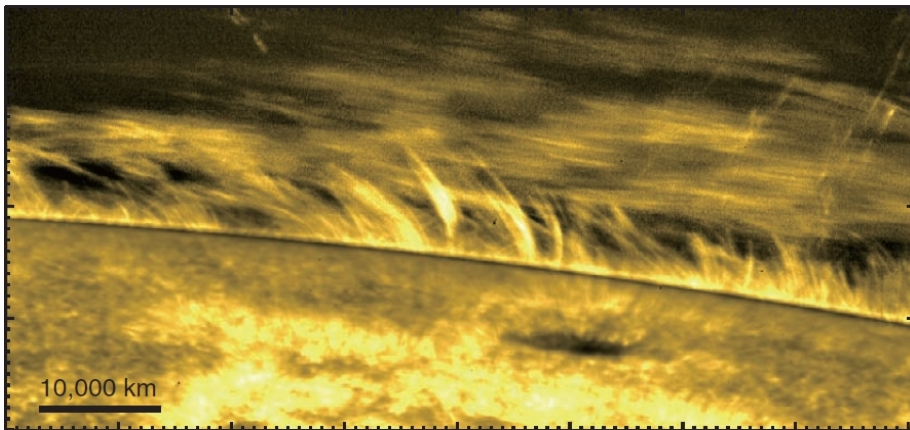
Sunspot group magnetogram,
21st August 2004,
*(Michiel van Noort and Luc Rouppe van
der Voort, Oslo, 2004)*



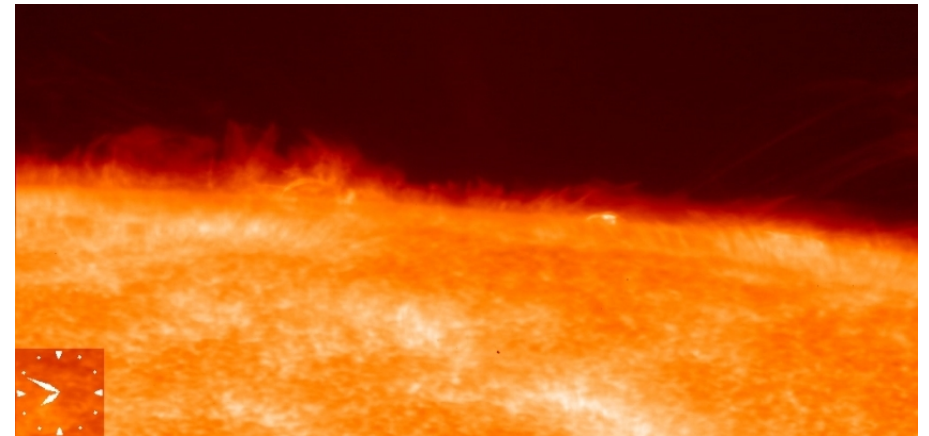
Hinode ひので (Solar-B)



(JAXA)



Solar prominence,
9th November 2006,
(Okamoto, T.J. et al., 2007)

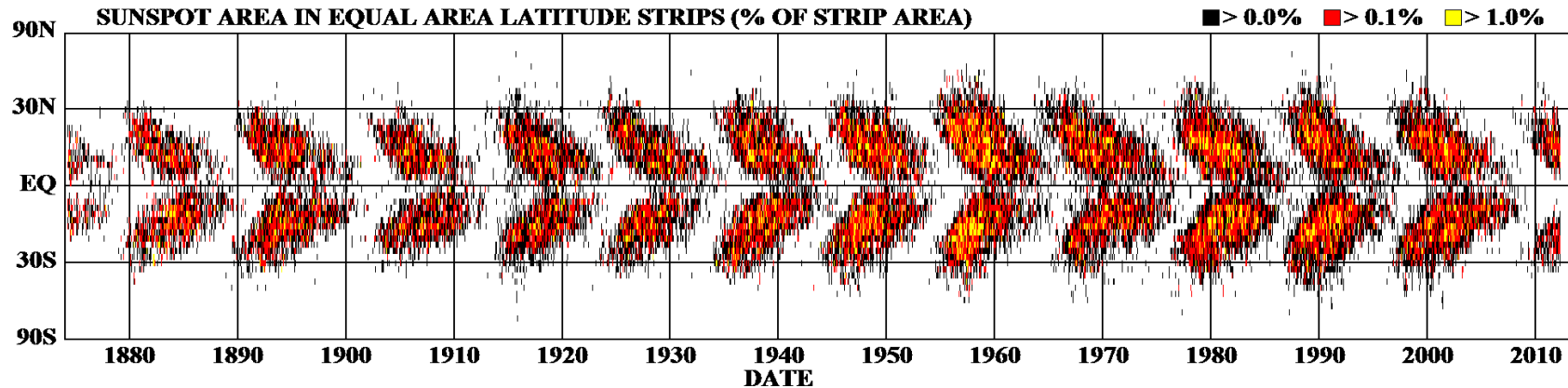


Eruption observed in Ca II H
(397nm) above a Sun spot,
<http://solarb.msfc.nasa.gov/news/movies.html>

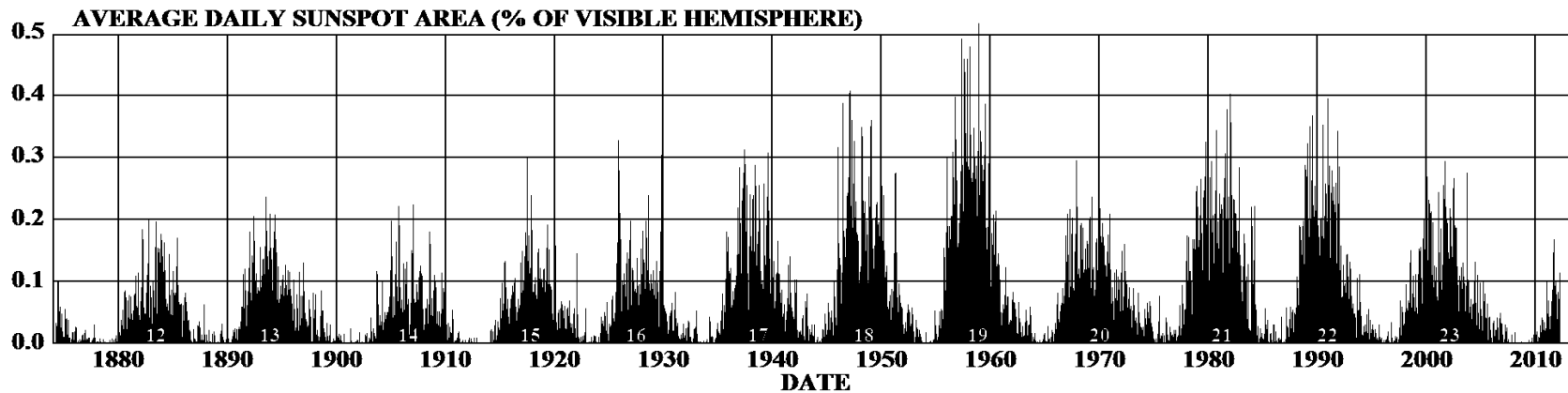
Solar Magnetic Field

11 year cycle

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



蝶
形
图



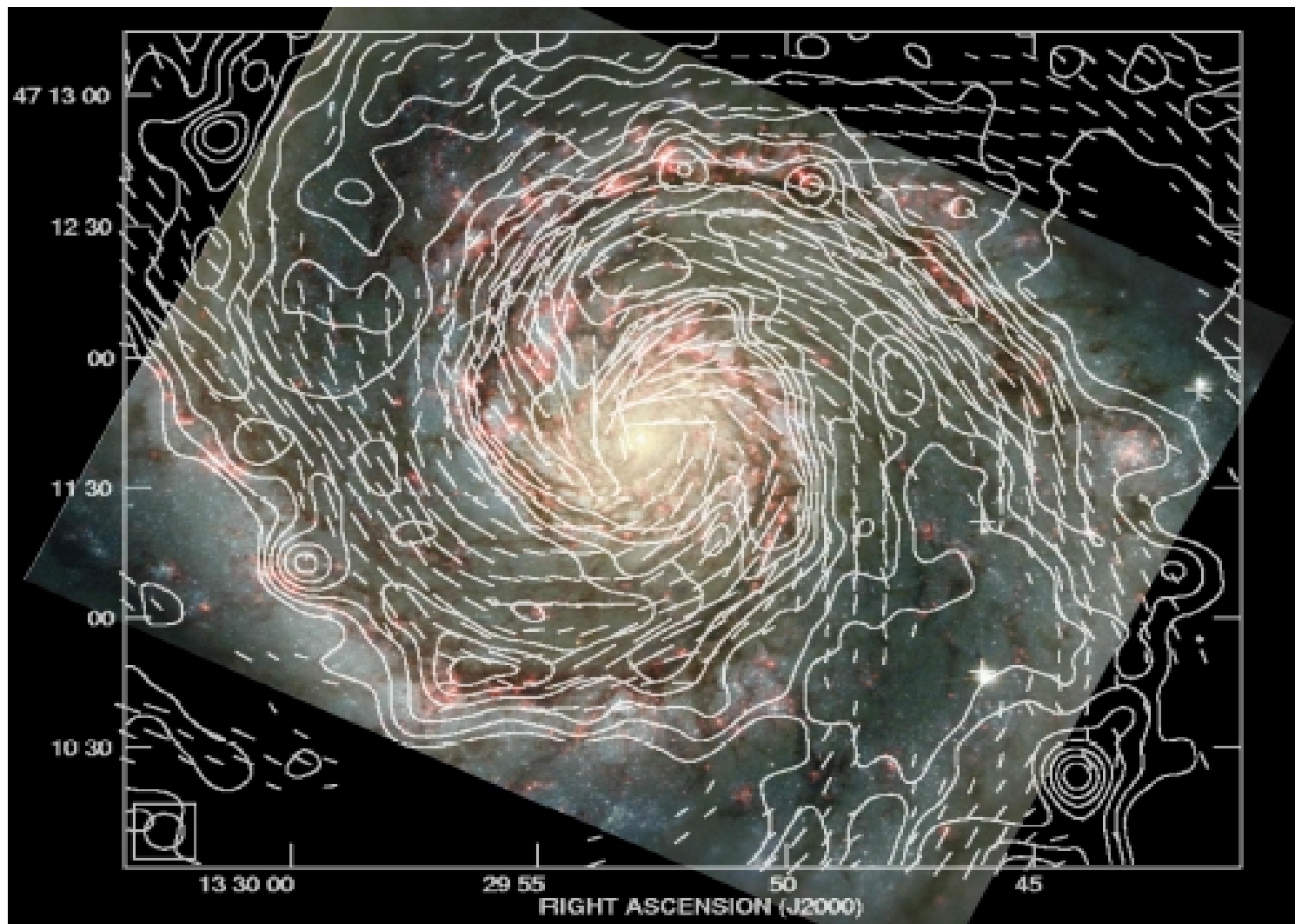
<http://solarscience.msfc.nasa.gov/>

HATHAWAY/NASA/MSFC 2012/06

 dynamo working

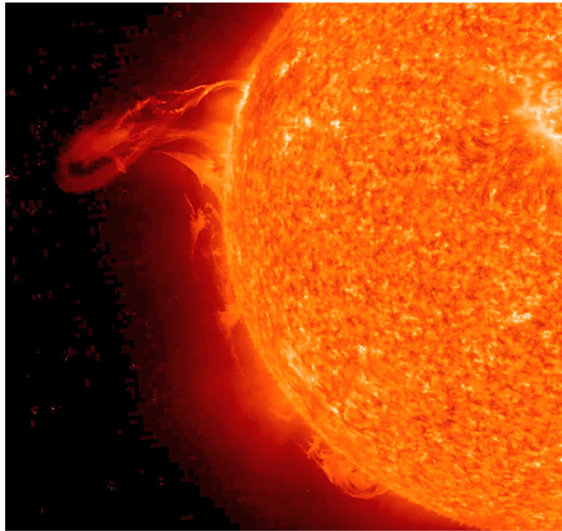
(Hathaway/NASA)

Galactic Magnetic Fields

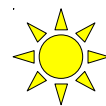
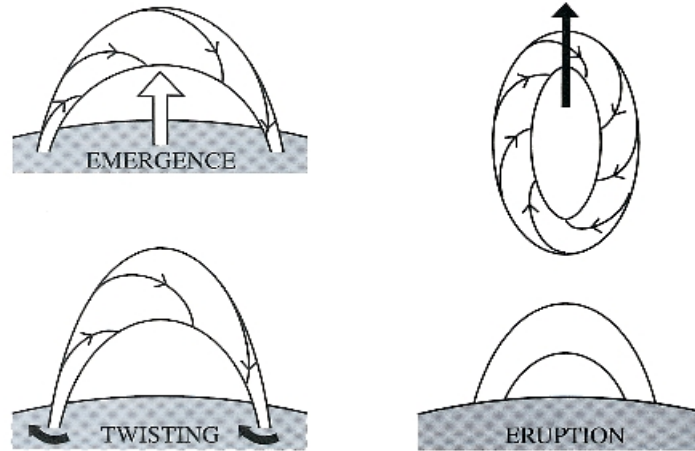


Galaxy M51, radio + optical
(Fletcher et al. 2011)

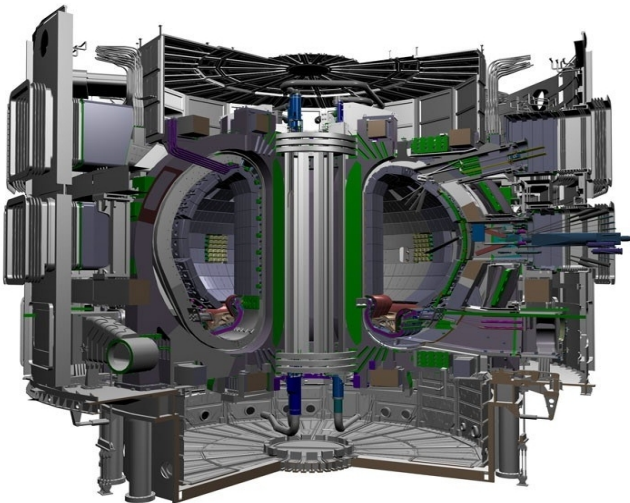
Twisted Magnetic Fields



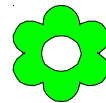
SOHO, 7th May 2010



Twisted fields are more likely to erupt,
(*Canfield et al. 1999*)



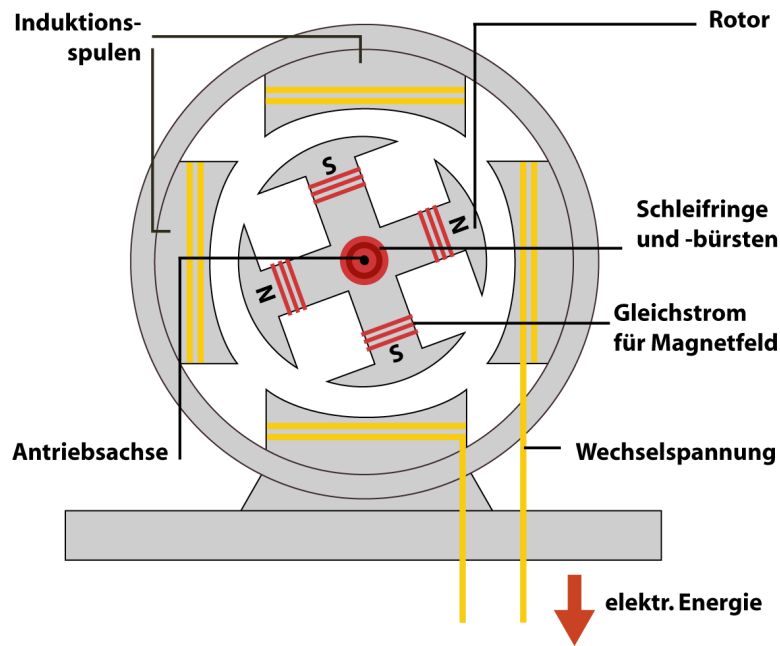
ITER



Twist increases the stability of
magnetic fields in tokamaks.

Dynamo Effect

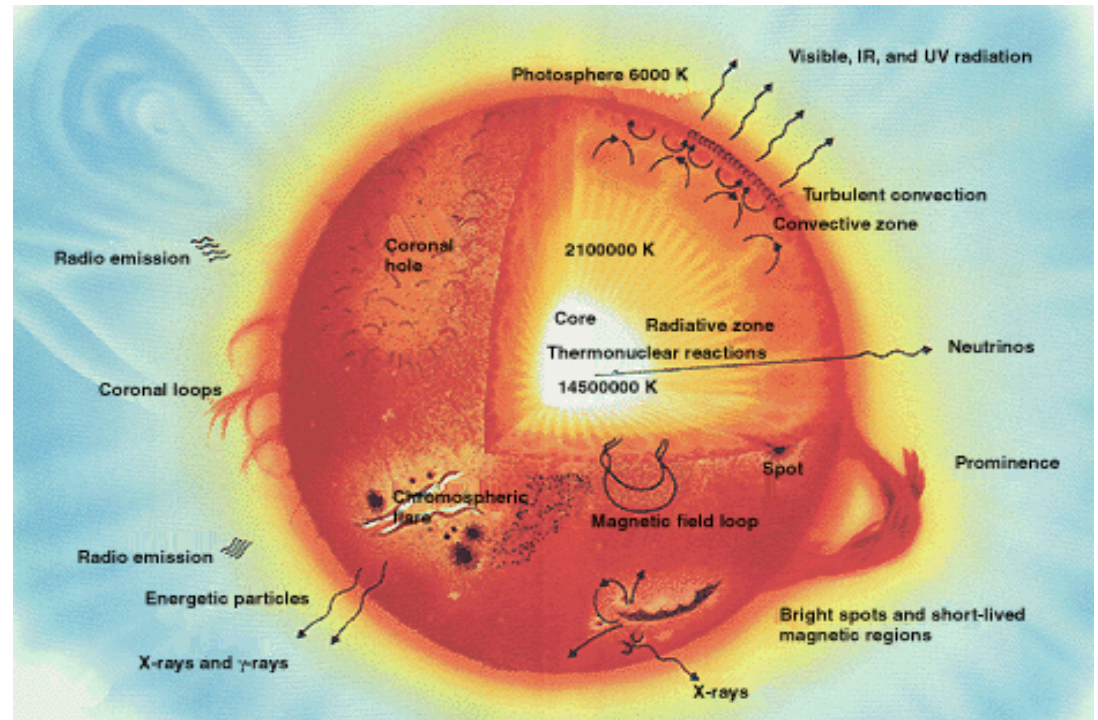
kinetic motion → induction
→ electric energy



electric power generator
(Wikipedia, user: Kuntoff, 2005)

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

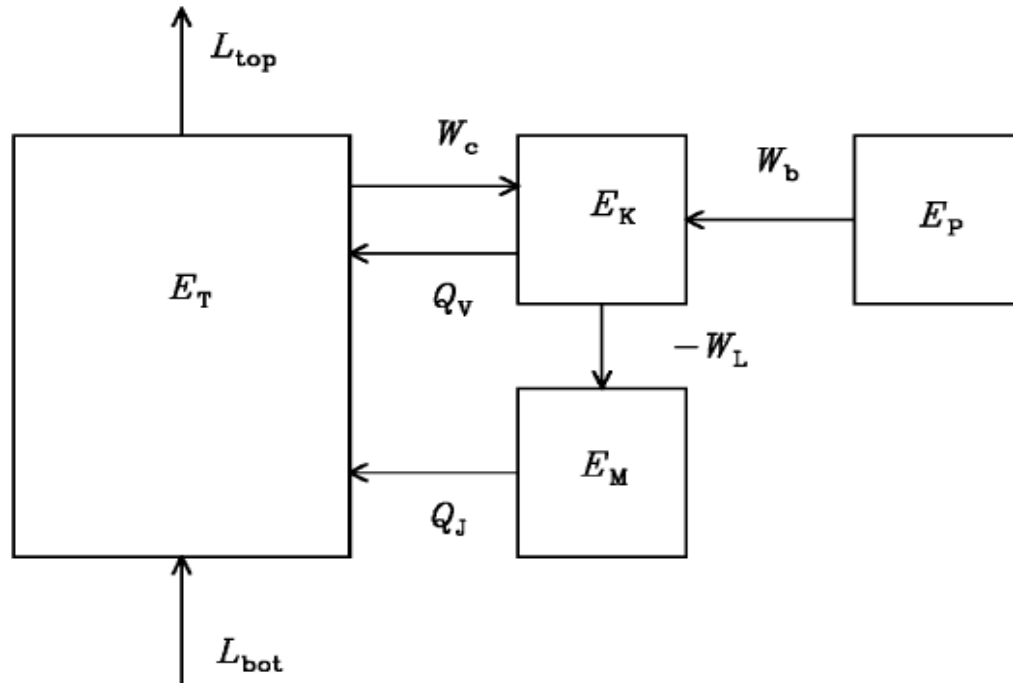
turbulent motion → induction
→ magnetic energy



Solar model
(NASA)

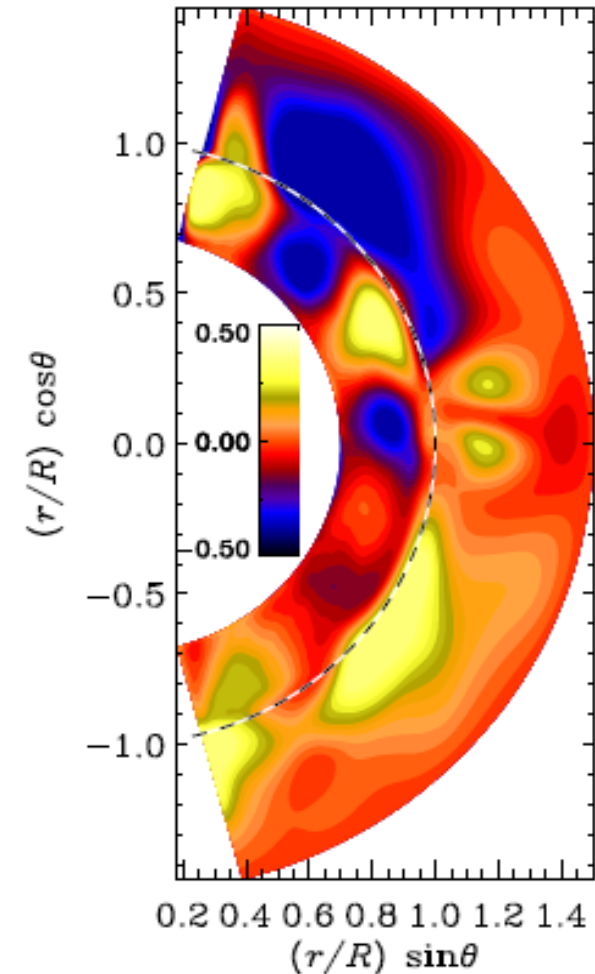
$$\partial_t \mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}_{10}$$

Turbulent Dynamo Schematics



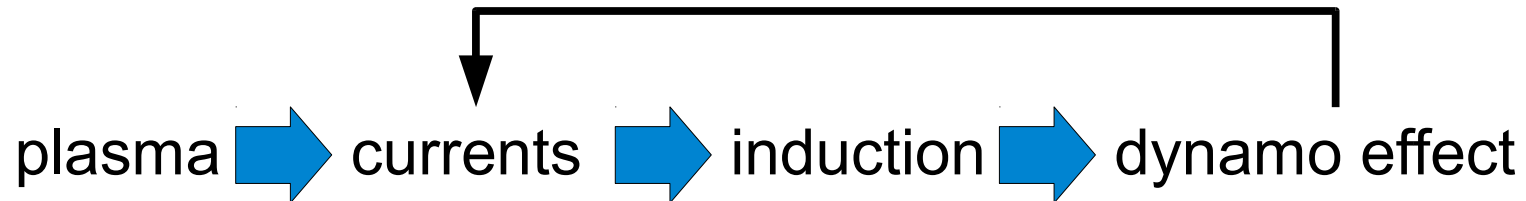
Energy budget for a dynamo.
(Brandenburg et al., 1996)

$E_T, E_K, E_M, E_P =$
thermal, kinetic, magnetic and
potential energy



$\langle \overline{B}_\phi \rangle_t$ for a convection
driven dynamo.
(Warnecke et al., 2012)

Dynamo Mechanism



Equations of **magnetohydrodynamics** (MHD):

Induction equation:
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mathbf{J})$$

Momentum equation:
$$\frac{D\mathbf{U}}{Dt} = -c_S^2 \nabla \ln \rho + \mathbf{J} \times \mathbf{B} / \rho + \mathbf{F}_{\text{visc}}$$

Continuity equation:
$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}$$

Advective derivative:
$$D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$$

Mean-Field Formalism

Mean-field decomposition: $B = \overline{B} + b$

Reynolds rules: $\overline{B_1 + B_2} = \overline{B_1} + \overline{B_2}$, $\overline{\overline{B}} = \overline{B}$, $\overline{b} = 0$

$$\overline{\partial_\mu B} = \partial_\mu \overline{B}, \quad \mu = 0, 1, 2, 3$$

Mean-field induction equations:

$$\partial_t \overline{B} = \eta \nabla^2 \overline{B} + \nabla \times (\overline{U} \times \overline{B} + \overline{\mathcal{E}})$$

$$\partial_t b = \nabla \times (\overline{U} \times b + G) + \nabla \times (u \times \overline{B}) + \eta \nabla^2 b$$

Electromotive force (emf): $\overline{\mathcal{E}} = \overline{u \times b}$

$$G = u \times b - \overline{u \times b}$$

Electromotive Force

The EMF is assumed to be linear and homogeneous in \overline{B} .

$$\begin{aligned} \Rightarrow \mathcal{E}_i(x, t) &= \mathcal{E}_i^{(0)}(x, t) \\ &+ \int \int_{\alpha} K_{ij}(x, x', t, t') \overline{B}_j(x - x', t - t') d^3x' dt' \end{aligned}$$

Taylor expansion:

$$\overline{B}_j(x', t) = \overline{B}_j(x, t) + (x'_k - x_k) \frac{\partial \overline{B}_j(x, t)}{\partial x_k} + \dots$$

Assume local and instantaneous dependence of $\overline{\mathcal{E}}$ on \overline{B} .

$$\Rightarrow \overline{\mathcal{E}}_i = \alpha_{ij} \overline{B}_j + b_{ijk} \frac{\partial \overline{B}_j}{\partial x_k} + \dots$$

For a turbulent system without preferred direction, i.e. $U = 0$:

$$\overline{\mathcal{E}} = \alpha \overline{B} - \eta_t \nabla \times \overline{B}$$

$$\partial_t \overline{B} = \alpha \nabla \times \overline{B} + \eta_T \nabla^2 \overline{B}$$

Alpha-Quenching

Helical forcing: $\alpha = \alpha_K = -\tau \overline{\boldsymbol{\omega} \cdot \mathbf{u}} / 3$

Back reaction of \overline{B} on α

➡ Correction: α should depend on \overline{B}

$$\alpha = \alpha_K \left(1 - \overline{B}^2 / B_{\text{eq}}^2\right) \quad (\overline{B}^2 \ll B_{\text{eq}}^2) \quad (\text{Roberts 1975})$$

Algebraic (conventional) quenching:

$$\alpha = \frac{\alpha_K}{1 + \overline{B}^2 / B_{\text{eq}}^2} \quad (\text{Ivanova 1977})$$

Catastrophic quenching (fit):

$$\alpha = \frac{\alpha_K}{1 + R_m \overline{B}^2 / B_{\text{eq}}^2}$$

(Vainshtein 1992)

Sun:	$R_m = 10^9$
Galaxies:	$R_m = 10^{18}$

Alpha-Effect

α effect: $\alpha = \alpha_K + \alpha_M$

$$\alpha_K = -\tau \overline{\boldsymbol{\omega} \cdot \mathbf{u}} / 3$$

$$\alpha_M = \tau \overline{\mathbf{j} \cdot \mathbf{b}} / (3\bar{\rho}) = \tau k^2 \overline{\mathbf{a} \cdot \mathbf{b}} / (3\bar{\rho}) = \bar{h}_m$$

(Pouquet et al. 1976)

helically driven dynamo $\bar{h}_{K,f} = \overline{\boldsymbol{\omega} \cdot \mathbf{u}}$

➔ production of magnetic helicity $\bar{h}_{M,f} = \overline{\mathbf{a} \cdot \mathbf{b}}$

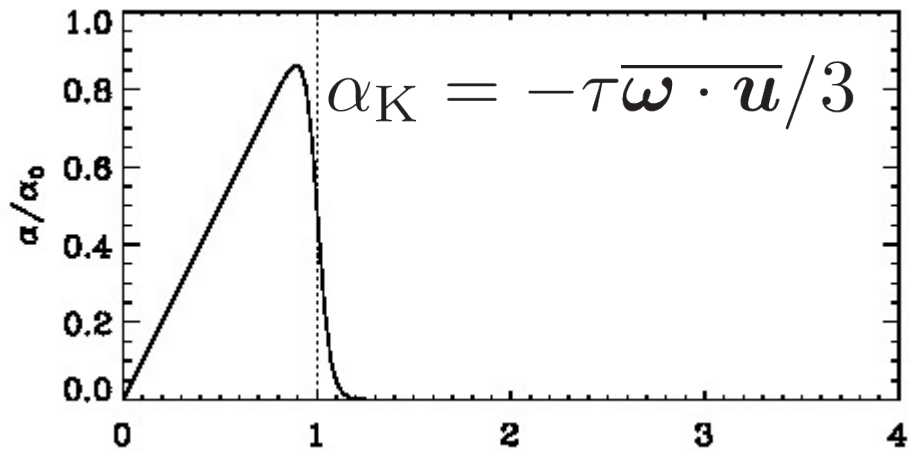
➔ total magnetic helicity conservation $\bar{h}_{M,m} = \overline{\mathbf{A} \cdot \mathbf{B}}$

$\overline{\mathbf{a} \cdot \mathbf{b}}$ works against dynamo: $E_M \propto 1/\text{Re}_M$ $\text{Re}_M = \frac{UL}{\eta}$

Sun: $\text{Re}_M = 10^9$ galaxies: $\text{Re}_M = 10^{18}$

Magnetic Helicity Fluxes

$$\frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left(\frac{\overline{\boldsymbol{\varepsilon}} \cdot \overline{\mathbf{B}}}{B_{\text{eq}}^2} + \frac{\alpha_M}{R_m} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

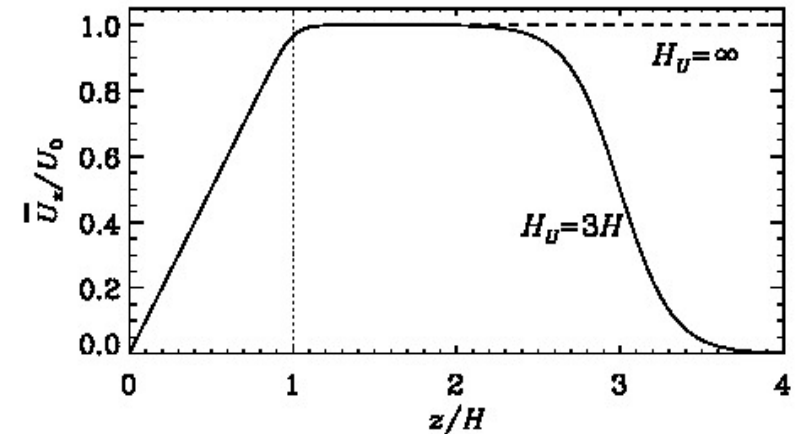


advective:
 $\alpha_M \overline{\mathbf{U}}$

α diffusion
 $k_\alpha \frac{\partial \alpha_M}{\partial z}$

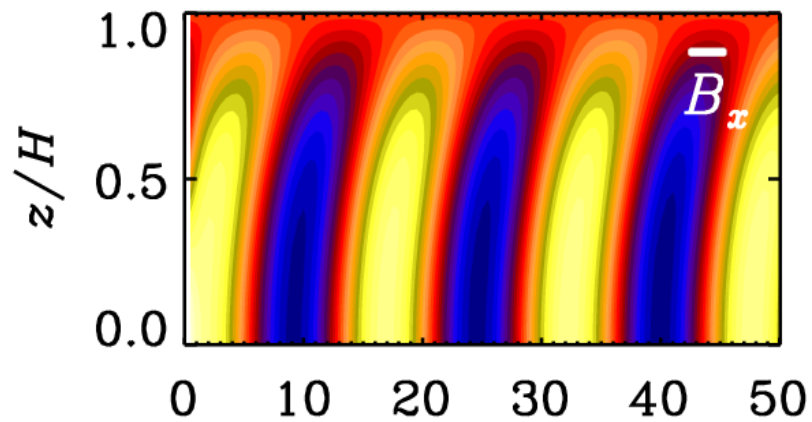
$$\frac{\partial \overline{h}_m}{\partial t} = 2\overline{\boldsymbol{\varepsilon}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \nabla \cdot \overline{\mathbf{F}}_m$$

$$\frac{\partial \overline{h}_f}{\partial t} = -2\overline{\boldsymbol{\varepsilon}} \cdot \overline{\mathbf{B}} - 2\eta\mu_0 \overline{\mathbf{j}} \cdot \overline{\mathbf{b}} - \nabla \cdot \overline{\mathbf{F}}_f$$

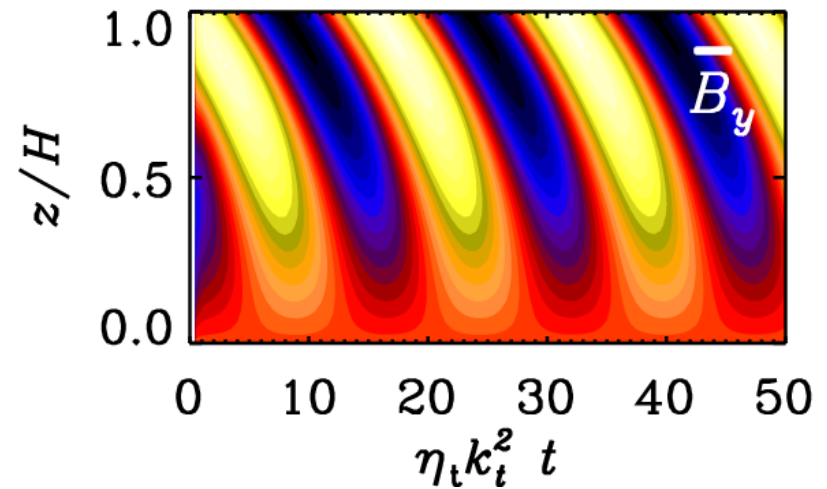
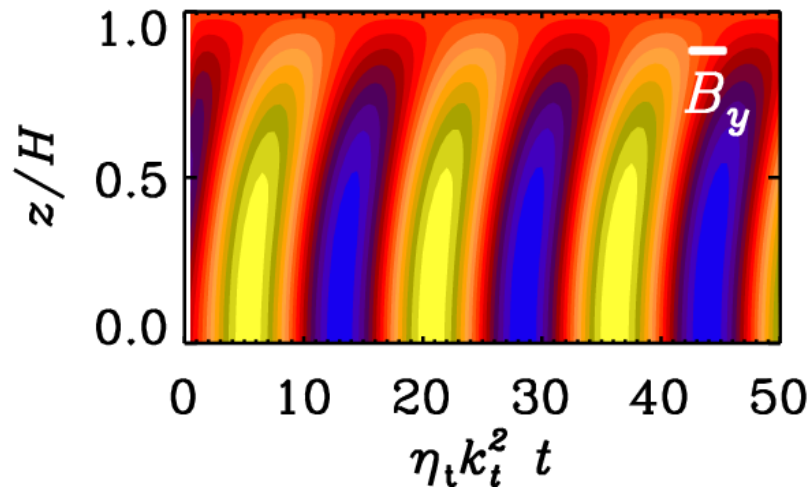
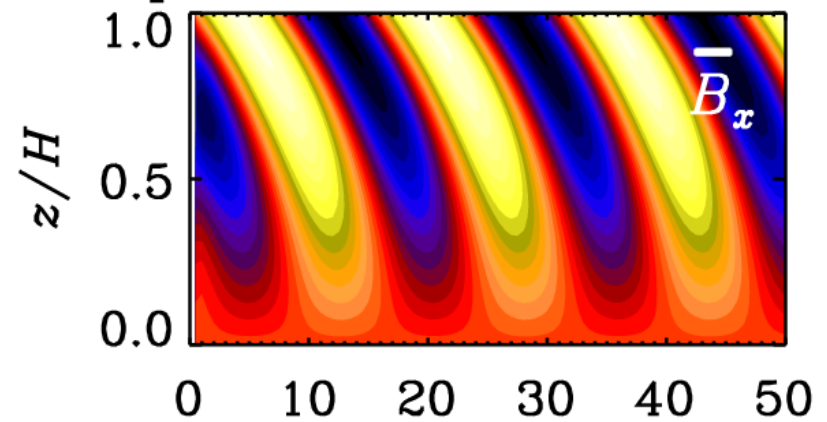


Dynamo Waves

vertical field condition, S

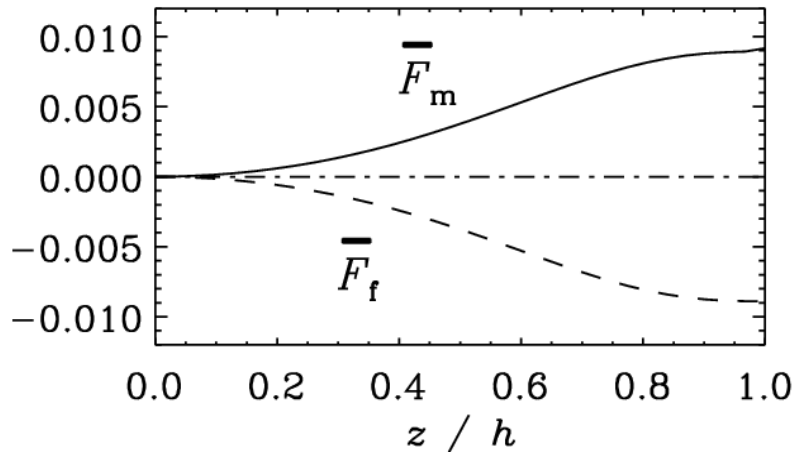


perfect conductor, A



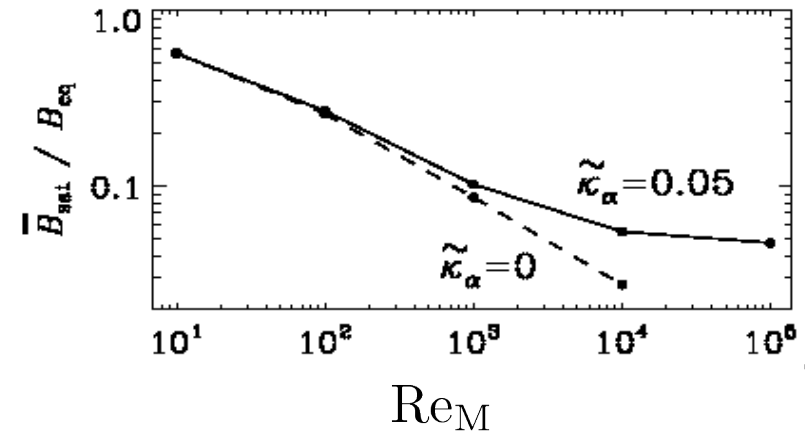
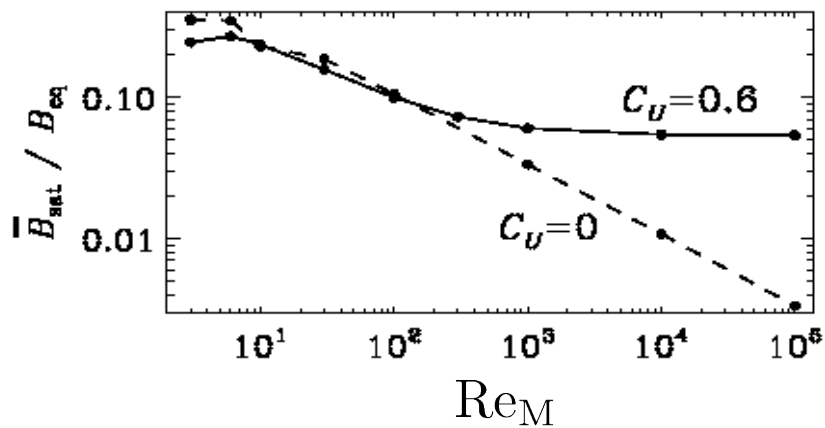
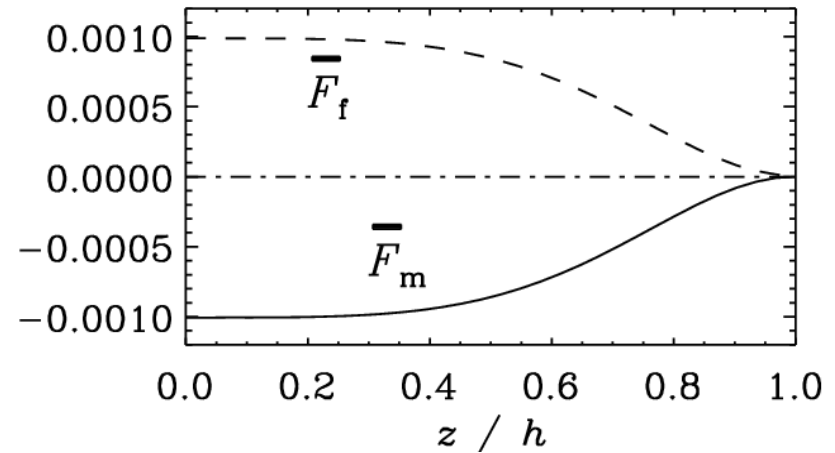
Magnetic Felicity Fluxes

open boundary
symmetric
wind



vs.

closed boundary
antisymmetric
 κ_α



Conclusions

- Helical turbulence can drive large-scale dynamo action.
- Convective motions in plasma drive dynamos.
- Dynamical alpha-quenching as more self-consistent model.

- Advective magnetic helicity fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

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Appendix

Viscous force: $\mathbf{F}_{\text{visc}} = \rho^{-1} \nabla \cdot 2\nu\rho\mathbf{S}$

Strain tensor: $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij} \nabla \cdot \mathbf{U}$

Sound speed: $c_S = \sqrt{\gamma \frac{p}{\rho}}$