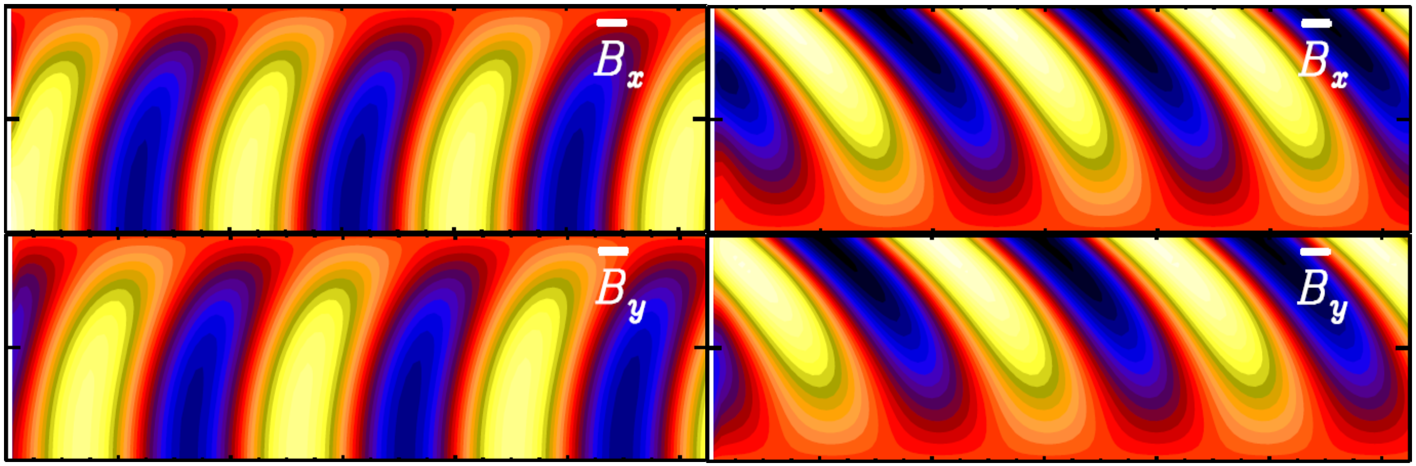
Magnetic helicity fluxes in dynamically quenched dynamos

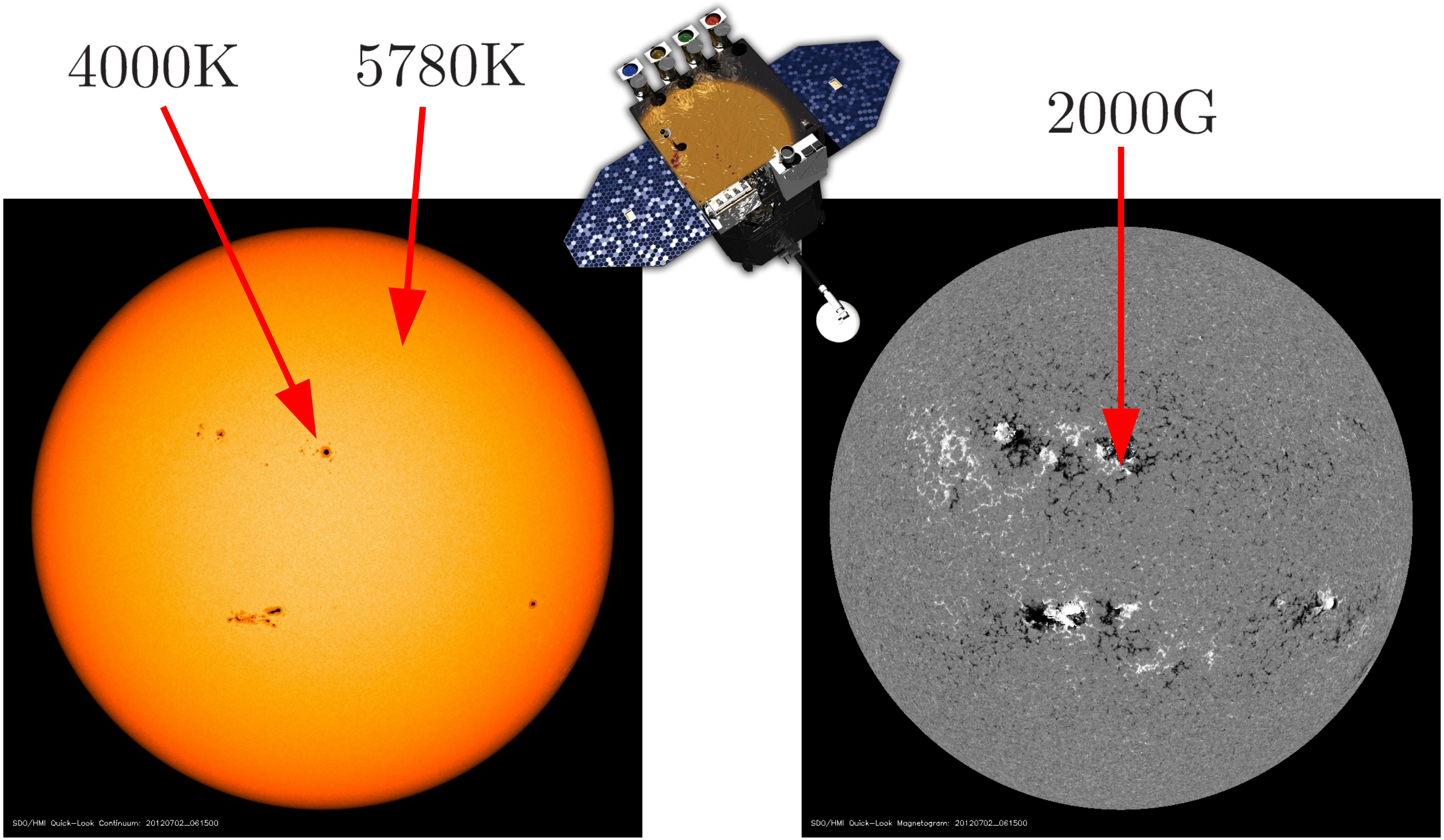
Simon Candelaresi



Outline

- Observations of sunspots and magnetic fields.
- Dynamo mechanism
- Mean-field model.
- Alpha-effect and alpha-quenching.
- Magnetic helicity fluxes.

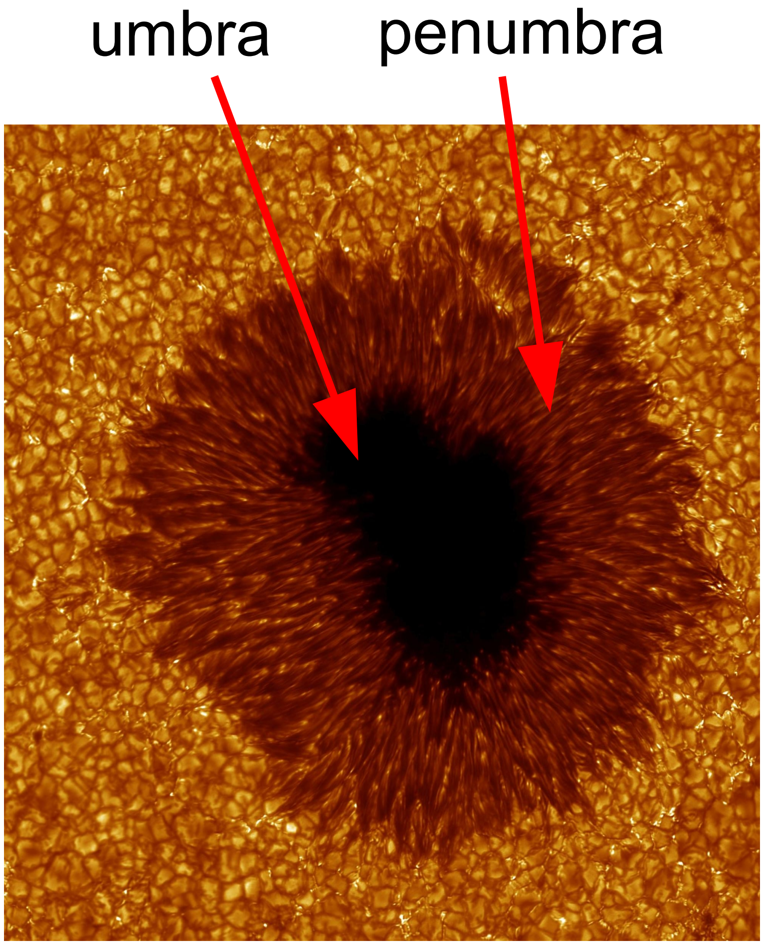
Solar Dynamics Observatory (SDO)



2nd July 2012, Intensity

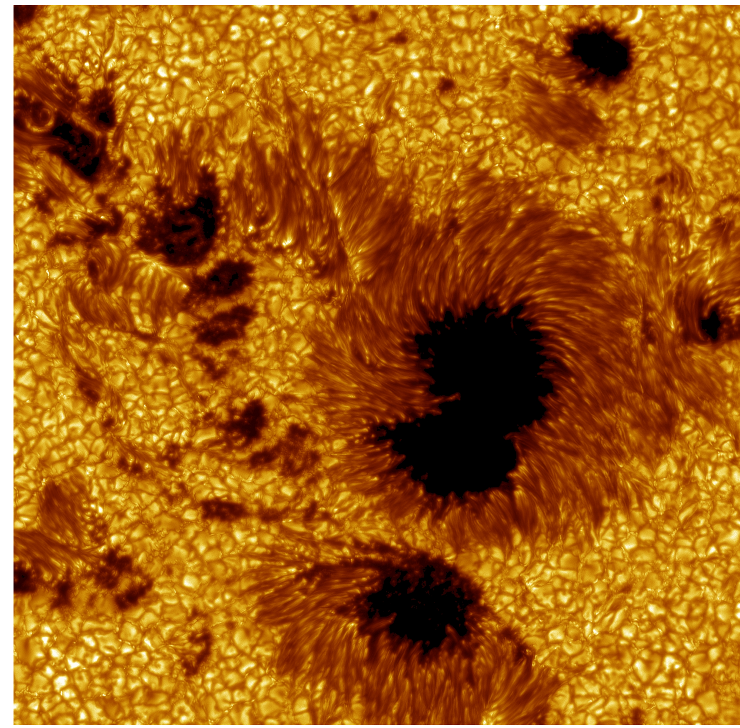
2nd July 2012, Magnetogram

Swedish Solar Telescope (SST)

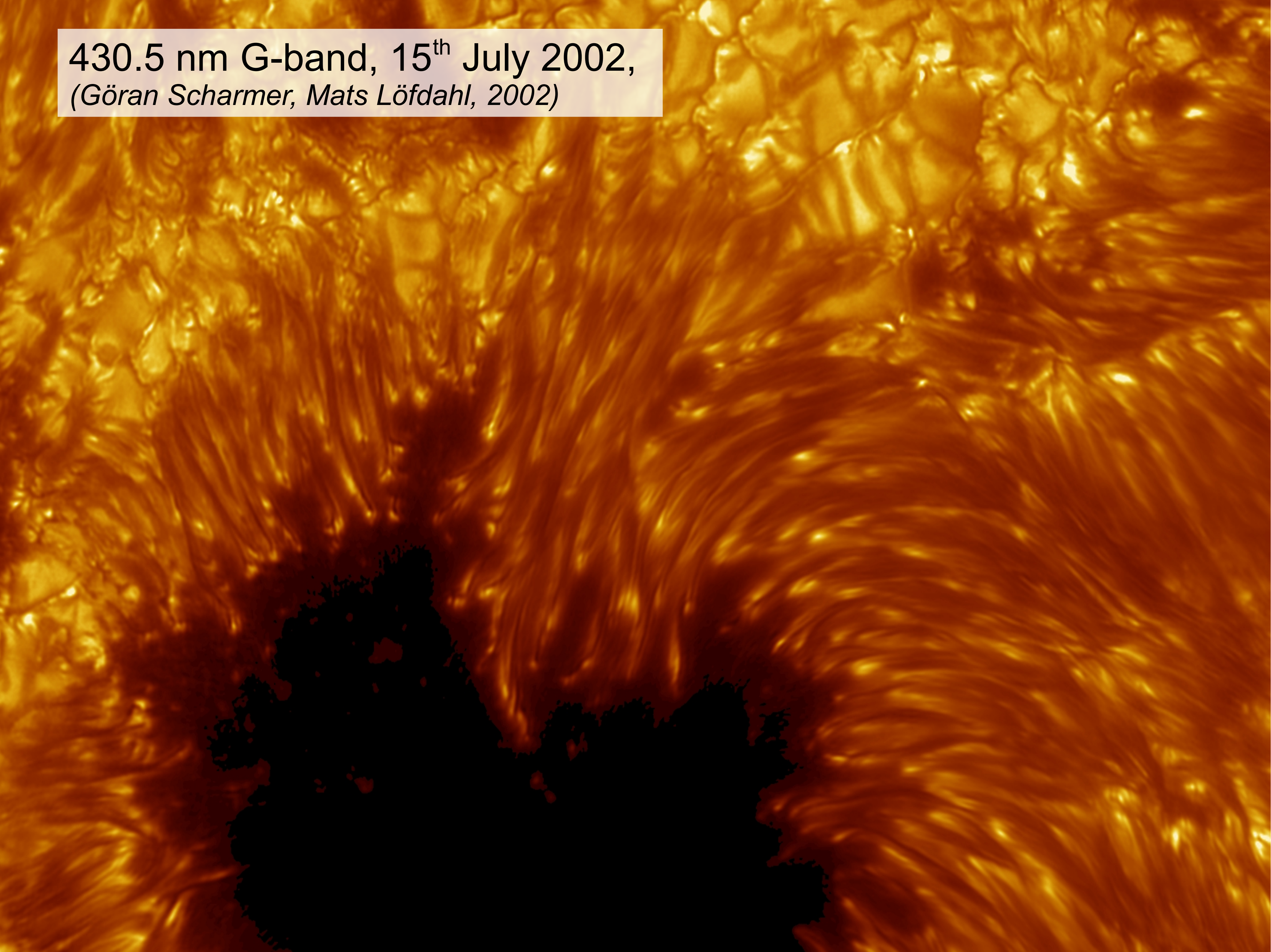


430.5 nm (G-band), 3rd July 2003,
(Dan Kiselman, Mats Löfdahl, 2003)

La Palma
(Göran Scharmer)

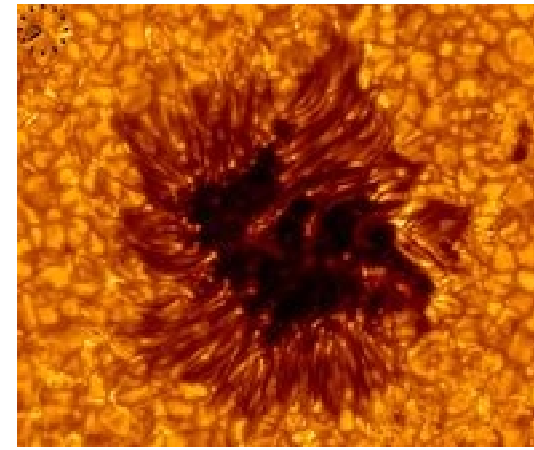


487.7 nm, 15th July 2002,
(Göran Scharmer, Mats Löfdahl, 2002)

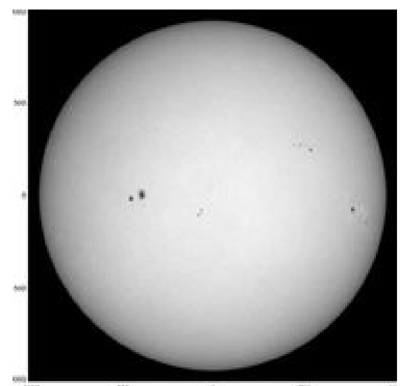

430.5 nm G-band, 15th July 2002,
(Göran Scharmer, Mats Löfdahl, 2002)

Swedish Solar Telescope (SST)

1h quiet Sun, 656.3 nm,
18th June 2006,
(Luc Rouppe van der Voort, Oslo, 2006)

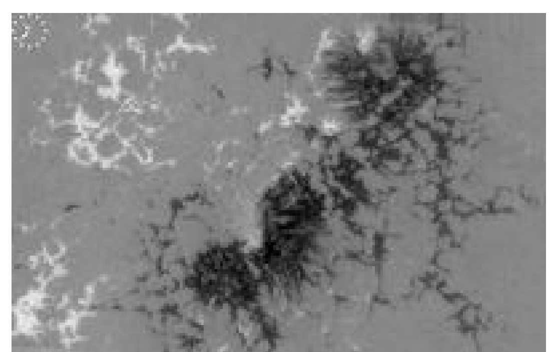


Zoom from SOHO/MDI field of view to SST resolution, August 2004,
(Michiel van Noort, Luc Rouppe van der Voort, Mats Carlsson, Oslo, 2004)

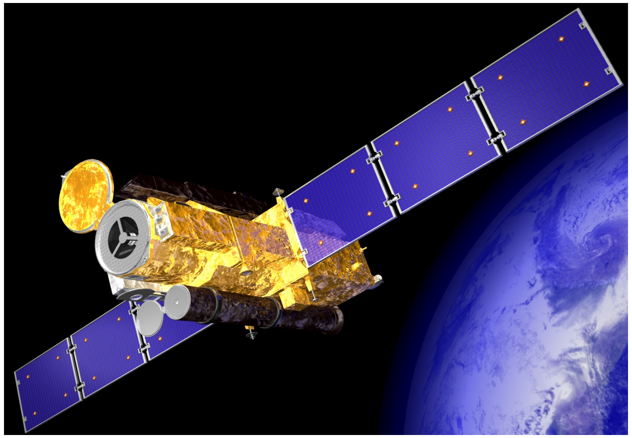


Sunspot 41 min, 430.5 nm G-band, 20th August 2004,
(Michiel van Noort and Luc Rouppe van der Voort, Oslo, 2004)

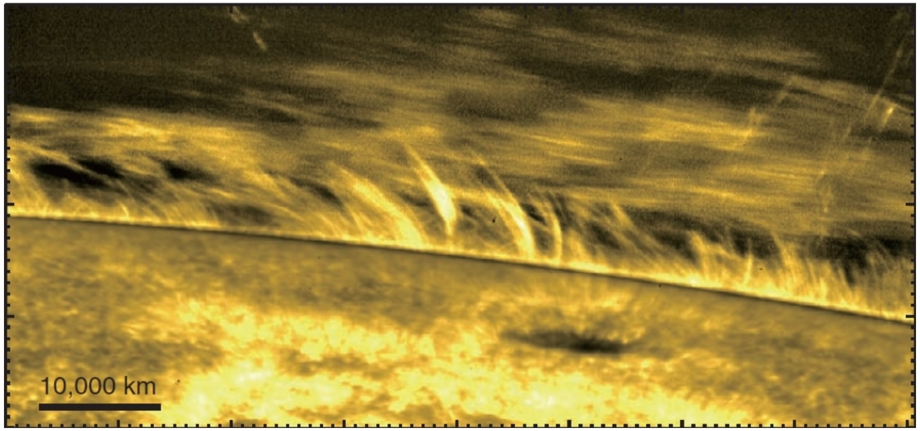
Sunspot group magnetogram, 21st August 2004,
(Michiel van Noort and Luc Rouppe van der Voort, Oslo, 2004)



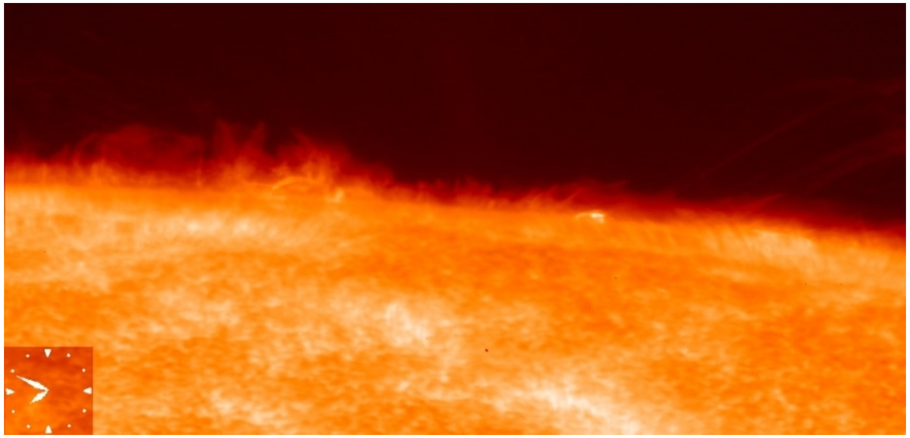
Hinode ひので (Solar-B)



(JAXA)



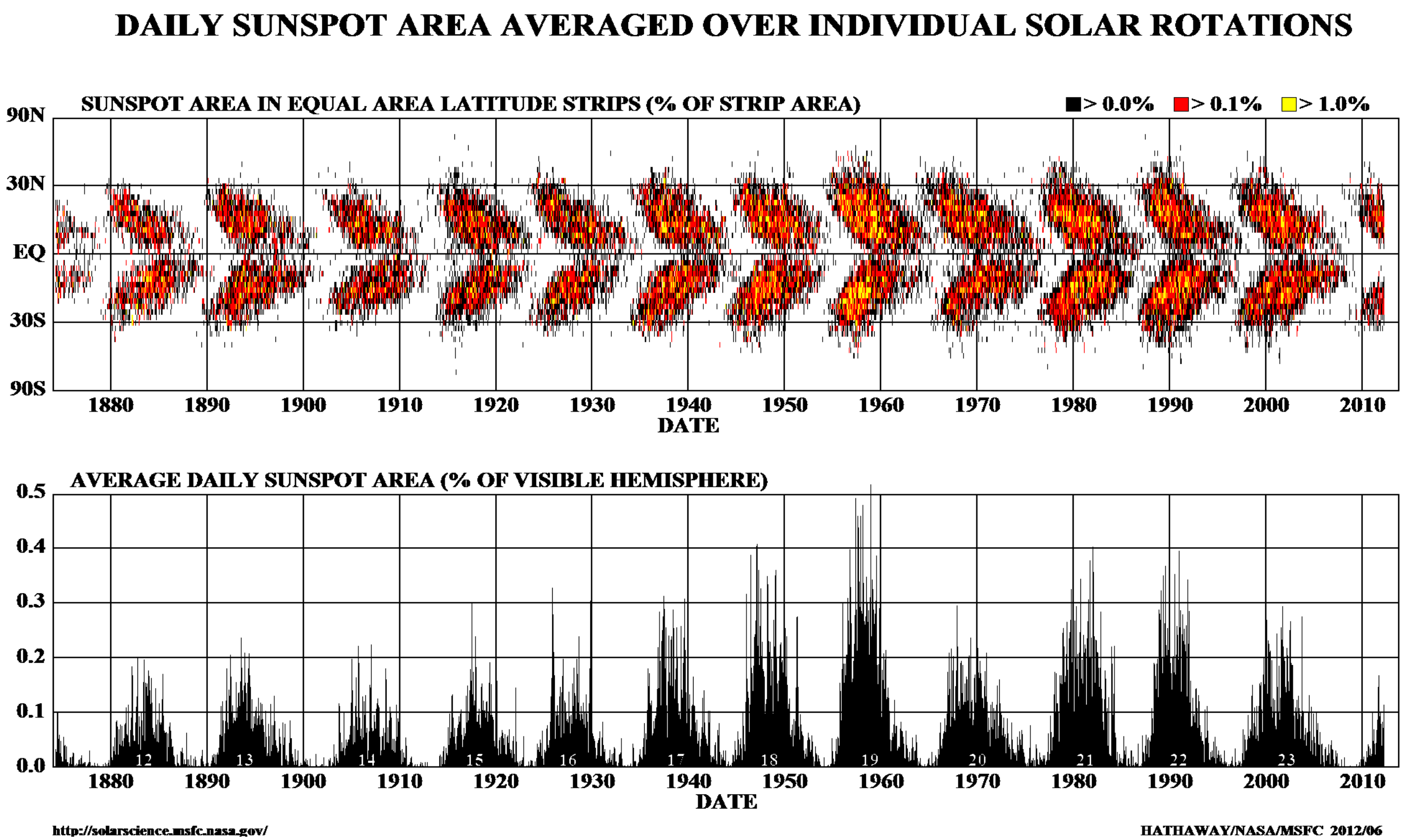
Solar prominence,
9th November 2006,
(Okamoto, T.J. et al., 2007)



Eruption observed in Ca II H (397nm) above a Sun spot,
http://solarb.msfc.nasa.gov/news/movies.html

Solar Magnetic Field

11 year cycle

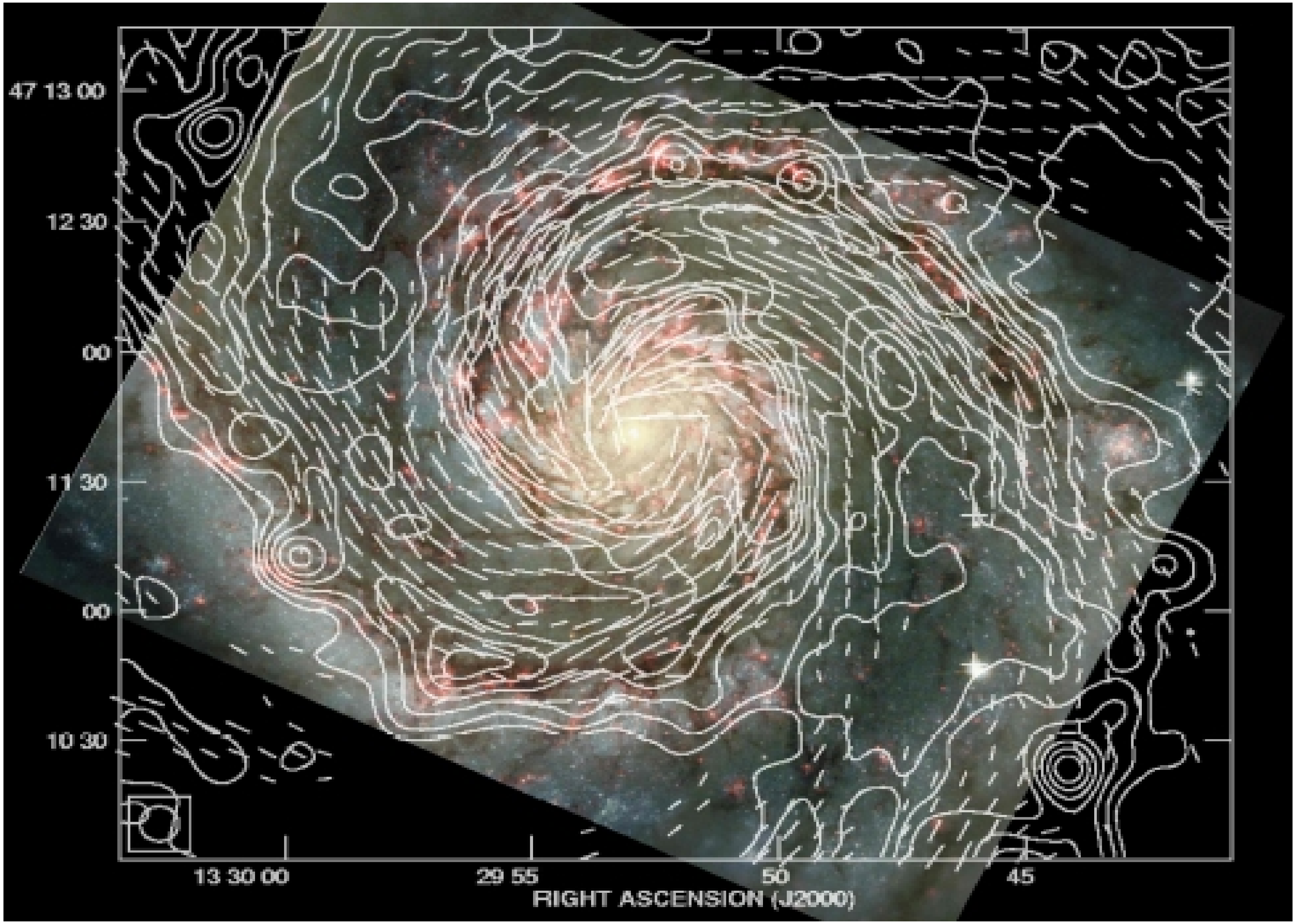


蝶形図

→ dynamo working

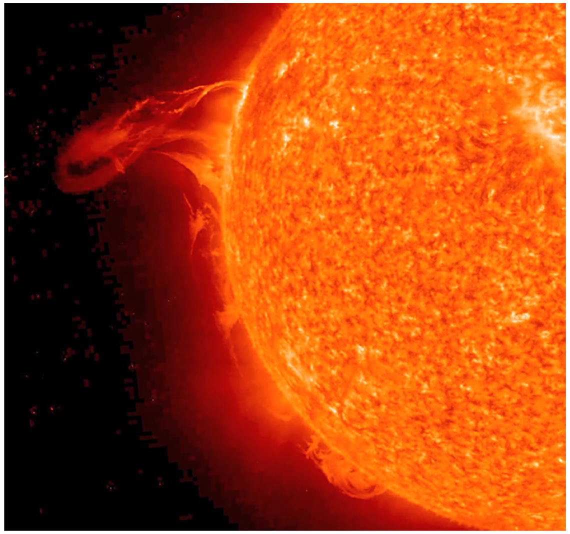
(Hathaway/NASA)

Galactic Magnetic Fields

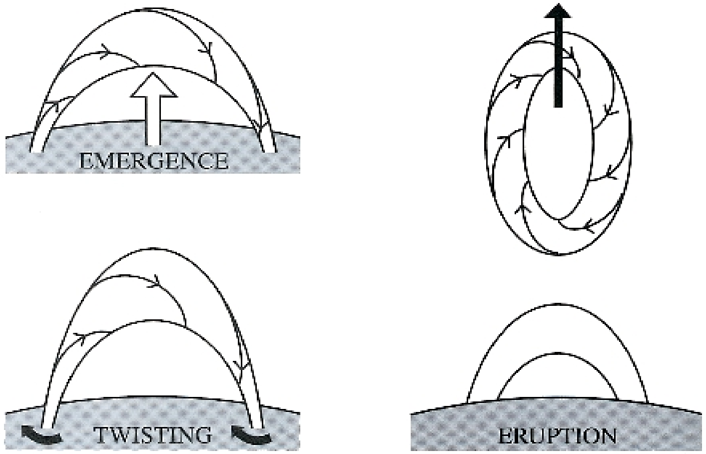


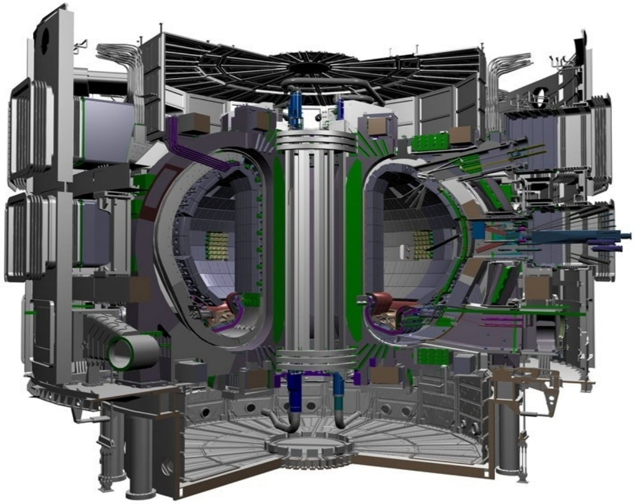
Galaxy M51, radio + optical
(Fletcher et al. 2011)

Twisted Magnetic Fields



SOHO, 7^{th} May 2010



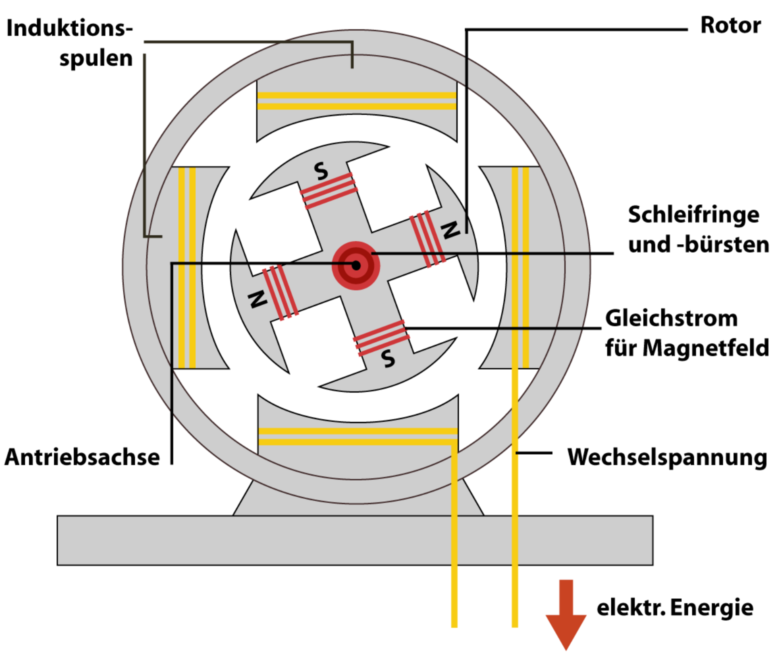


ITER

- Twisted fields are more likely to erupt, *(Canfield et al. 1999)*
- Twist increases the stability of magnetic fields in tokamaks.

Dynamo Effect

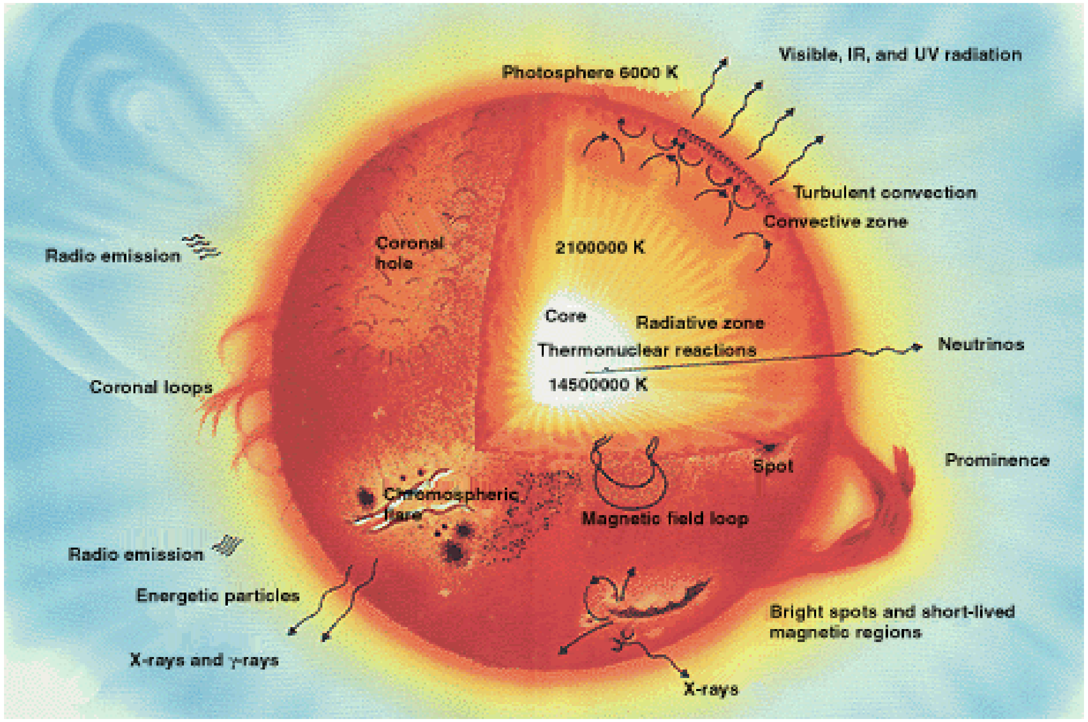
kinetic motion → induction → electric energy



electric power generator
(Wikipedia, user: Kuntoff, 2005)

$$\partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}$$

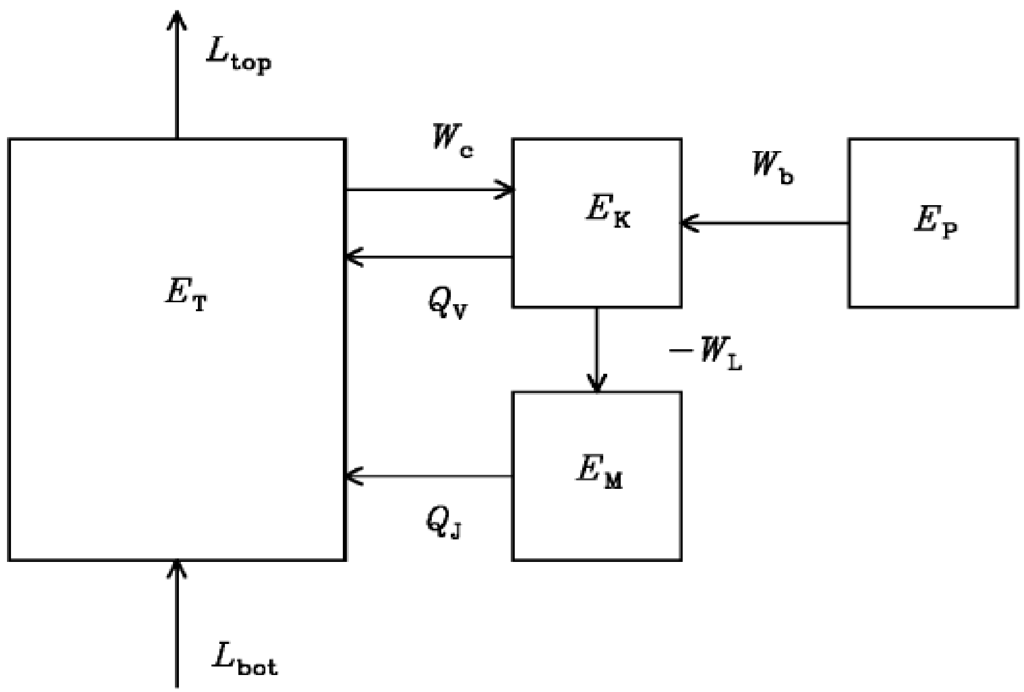
turbulent motion → induction → magnetic energy



Solar model
(NASA)

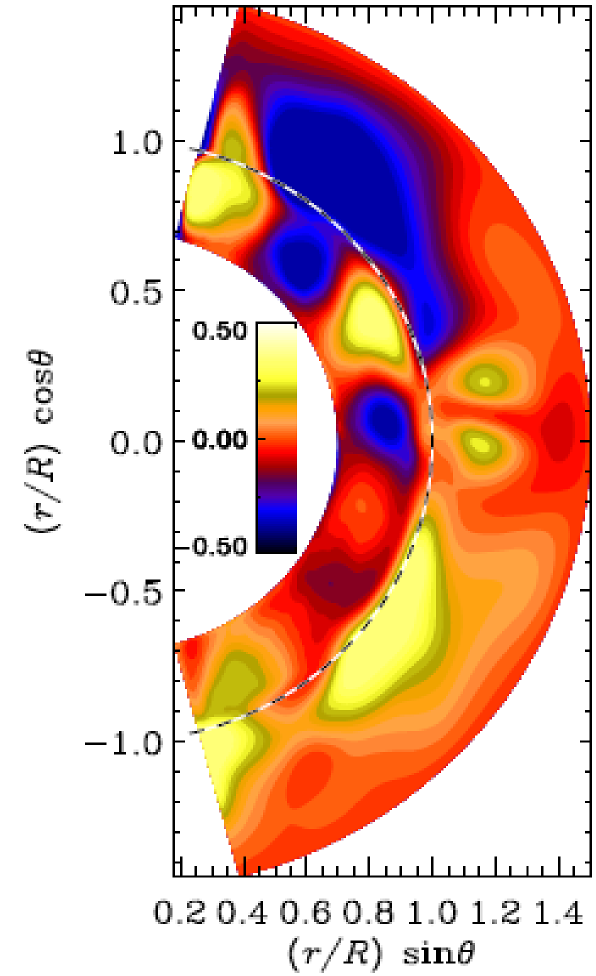
$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B}$$

Turbulent Dynamo Schematics



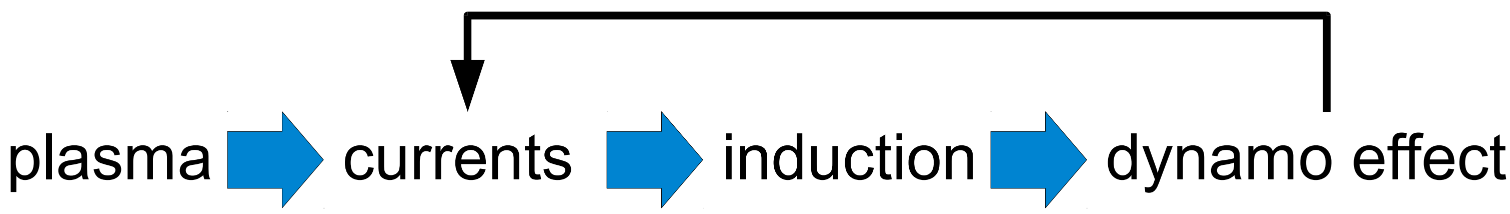
Energy budget for a dynamo.
(Brandenburg et al., 1996)

$E_T, E_K, E_M, E_P =$
thermal, kinetic, magnetic and potential energy



$\langle \overline{B}_\phi \rangle_t$ for a convection driven dynamo.
(Warnecke et al., 2012)

Dynamo Mechanism



Equations of **magnetohydrodynamics** (MHD):

Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Momentum equation: $$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

Continuity equation: $$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Advective derivative: $$\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U} \cdot \boldsymbol{\nabla}$$

Mean-Field Formalism

Mean-field decomposition: $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$

Reynolds rules: $\overline{\boldsymbol{B}_1 + \boldsymbol{B}_2} = \overline{\boldsymbol{B}}_1 + \overline{\boldsymbol{B}}_2, \quad \overline{\overline{\boldsymbol{B}}} = \overline{\boldsymbol{B}}, \quad \overline{\boldsymbol{b}} = 0$

$$\overline{\partial_\mu \boldsymbol{B}} = \partial_\mu \overline{\boldsymbol{B}}, \quad \mu = 0, 1, 2, 3$$

Mean-field induction equations:

$$\partial_t \overline{\boldsymbol{B}} = \eta \nabla^2 \overline{\boldsymbol{B}} + \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}})$$

$$\partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{G}) + \boldsymbol{\nabla} \times (\boldsymbol{u} \times \overline{\boldsymbol{B}}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{b}$$

Electromotive force (emf): $\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$

$$\boldsymbol{G} = \boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}$$

Mofatt, H. K., 1978, Krause and Raedler, 1980

Electromotive Force

The EMF is assumed to be linear and homogeneous in $\overline{\boldsymbol{B}}$.

$$\mathcal{E}_i(x,t) = \mathcal{E}_i^{(0)}(x,t) + \int\int_\alpha K_{ij}(x,x',t,t')\overline{B}_j(x-x',t-t')\ \mathrm{d}^3x'\ \mathrm{d}t'$$

Taylor expansion:

$$\overline{B}_j(x',t) = \overline{B}_j(x,t) + (x'_k - x_k)\frac{\partial \overline{B}_j(x,t)}{\partial x_k} + \ldots$$

Assume local and instantaneous dependence of $\overline{\boldsymbol{\mathcal{E}}}$ on $\overline{\boldsymbol{B}}$.

$$\overline{\mathcal{E}}_i = \alpha_{ij}\overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k} + \ldots$$

For a turbulent system without preferred direction, i.e. $\boldsymbol{U} = 0$:

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_{\mathrm{t}}\nabla\times\overline{\boldsymbol{B}}$$

$$\partial_t\overline{\boldsymbol{B}} = \alpha\boldsymbol{\nabla}\times\overline{\boldsymbol{B}} + \eta_{\mathrm{T}}\boldsymbol{\nabla}^2\overline{\boldsymbol{B}}$$

Nonlinear Alpha-Effect

Helical forcing: $\alpha = \alpha_\mathrm{K} = -\tau \overline{\boldsymbol{\omega} \cdot \boldsymbol{u}}/3$

Back reaction of $\overline{\boldsymbol{B}}$ on α

➡ Correction: α should depend on $\overline{\boldsymbol{B}}$

$$\alpha = \alpha_\mathrm{K}(1 - \overline{\boldsymbol{B}}^2/B_\mathrm{eq}^2) \quad (\overline{\boldsymbol{B}}^2 \ll B_\mathrm{eq}^2)$$ *(Roberts 1975)*

Algebraic (conventional) quenching:

$$\alpha = \frac{\alpha_\mathrm{K}}{1 + \overline{\boldsymbol{B}}^2/B_\mathrm{eq}^2}$$ *(Ivanova 1977)*

Catastrophic quenching (fit):

$$\alpha = \frac{\alpha_\mathrm{K}}{1 + R_\mathrm{m}\overline{\boldsymbol{B}}^2/B_\mathrm{eq}^2}$$

(Vainshtein 1992)

Sun:	$R_\mathrm{m} = 10^9$
Galaxies:	$R_\mathrm{m} = 10^{18}$

Alpha-Effect

α effect: $\alpha = \alpha_{\rm K} + \alpha_{\rm M}$ (magnetic helicity conservation)

$$\alpha_{\rm K} = -\tau\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}/3$$

$$\alpha_{\rm M} = \tau\overline{\boldsymbol{j}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \tau k^2\overline{\boldsymbol{a}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \overline{h}_{\rm m}$$

(Pouquet et al. 1976)

helically driven dynamo $\overline{h}_{\rm K,f} = \overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}$

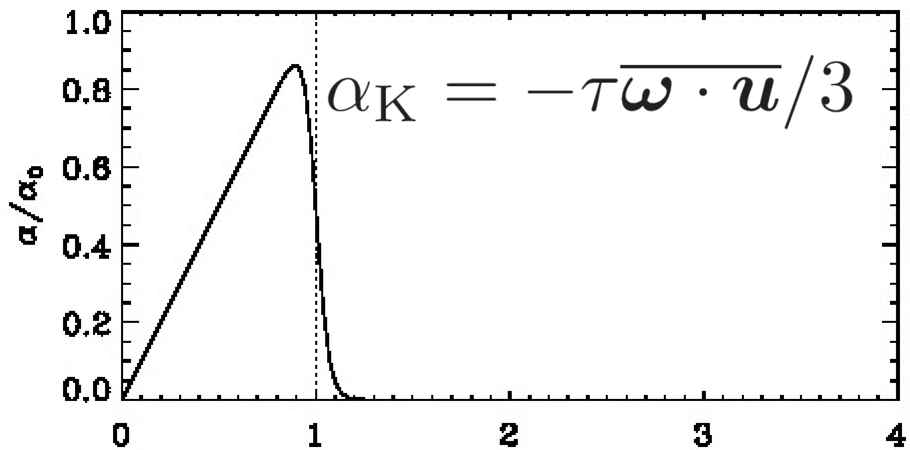
- production of magnetic helicity $\overline{h}_{\rm M,f} = \overline{\boldsymbol{a}\cdot\boldsymbol{b}}$
- total magnetic helicity conservation $\overline{h}_{\rm M,m} = \overline{\boldsymbol{A}}\cdot\overline{\boldsymbol{B}}$

$\overline{\boldsymbol{a}\cdot\boldsymbol{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^9$ galaxies: ${\rm Re}_{\rm M} = 10^{18}$

Magnetic Helicity Fluxes

$$\frac{\partial \alpha_{\rm M}}{\partial t} = -2\eta_{\rm t} k_{\rm f}^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm M}}{R_{\rm m}} \right) - \frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha$$

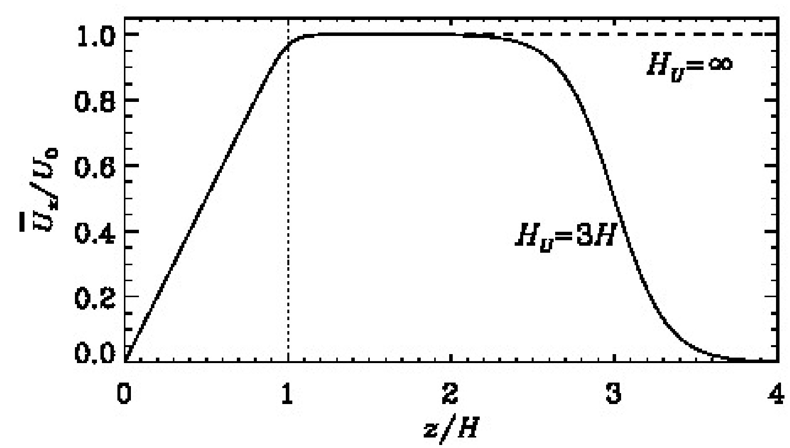


advective:
$\alpha_{\rm M} \overline{\boldsymbol{U}}$

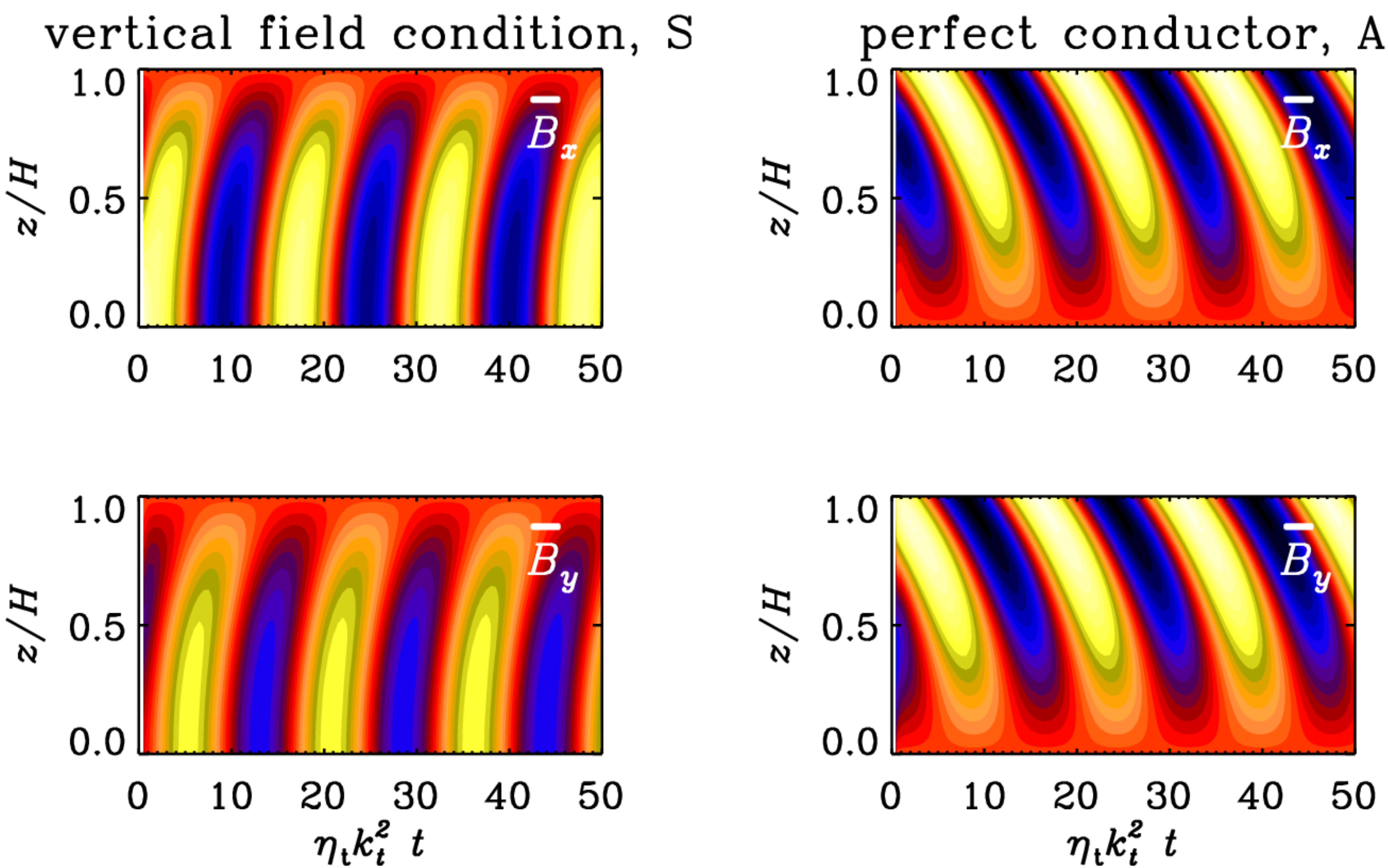
α diffusion
$\kappa_\alpha \frac{\partial \alpha_{\rm M}}{\partial z}$

$$\frac{\partial \overline{h}_{\rm m}}{\partial t} = 2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_{\rm m}$$

$$\frac{\partial \overline{h}_{\rm f}}{\partial t} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{j} \cdot \boldsymbol{b}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_{\rm f}$$



Dynamo Waves

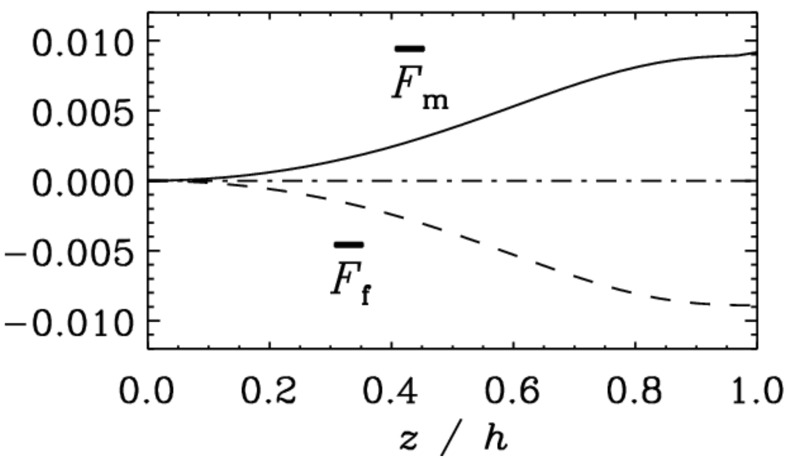


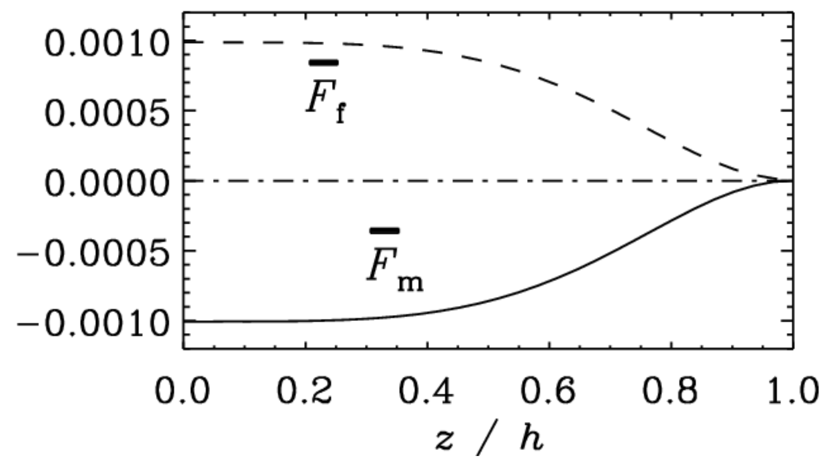
Magnetic Felicity Fluxes

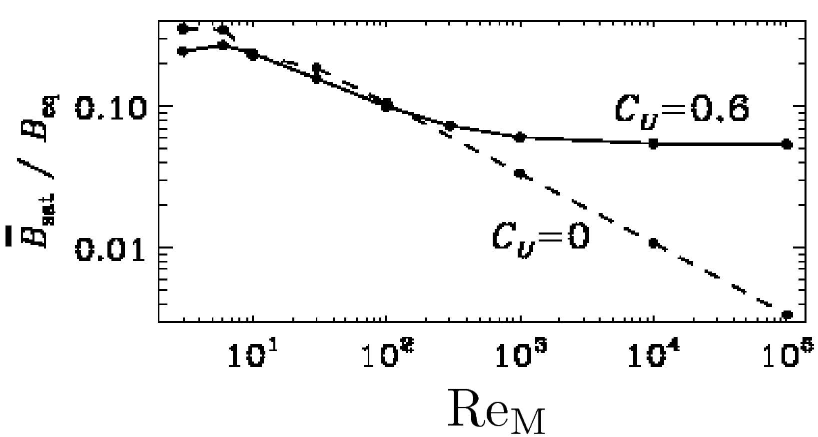
open boundary
symmetric
wind

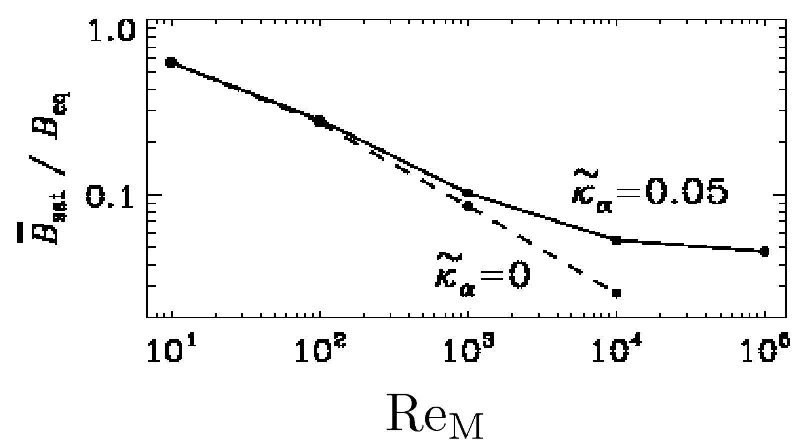
vs.

closed boundary
antisymmetric
κ_α









Conclusions

- Helical turbulence can drive large-scale dynamo action.
- Convective motions in plasma drive dynamos.
- Dynamical alpha-quenching as more self-consistent model.

- Advective magnetic helicity fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

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Appendix

Viscous force: $\boldsymbol{F}_{\text{visc}} = \rho^{-1} \boldsymbol{\nabla} \cdot 2\nu\rho\mathbf{S}$

Strain tensor: $\mathsf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla} \cdot \boldsymbol{U}$

Sound speed: $c_{\text{S}} = \sqrt{\gamma \frac{p}{\rho}}$