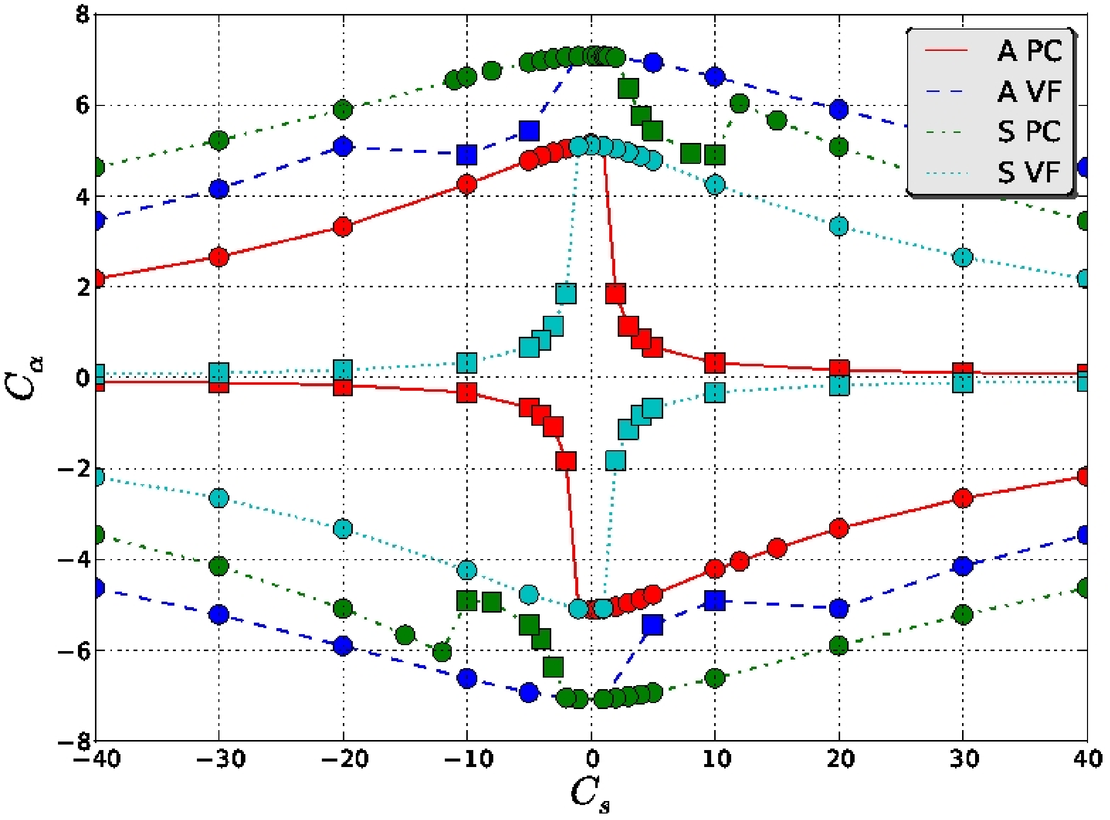
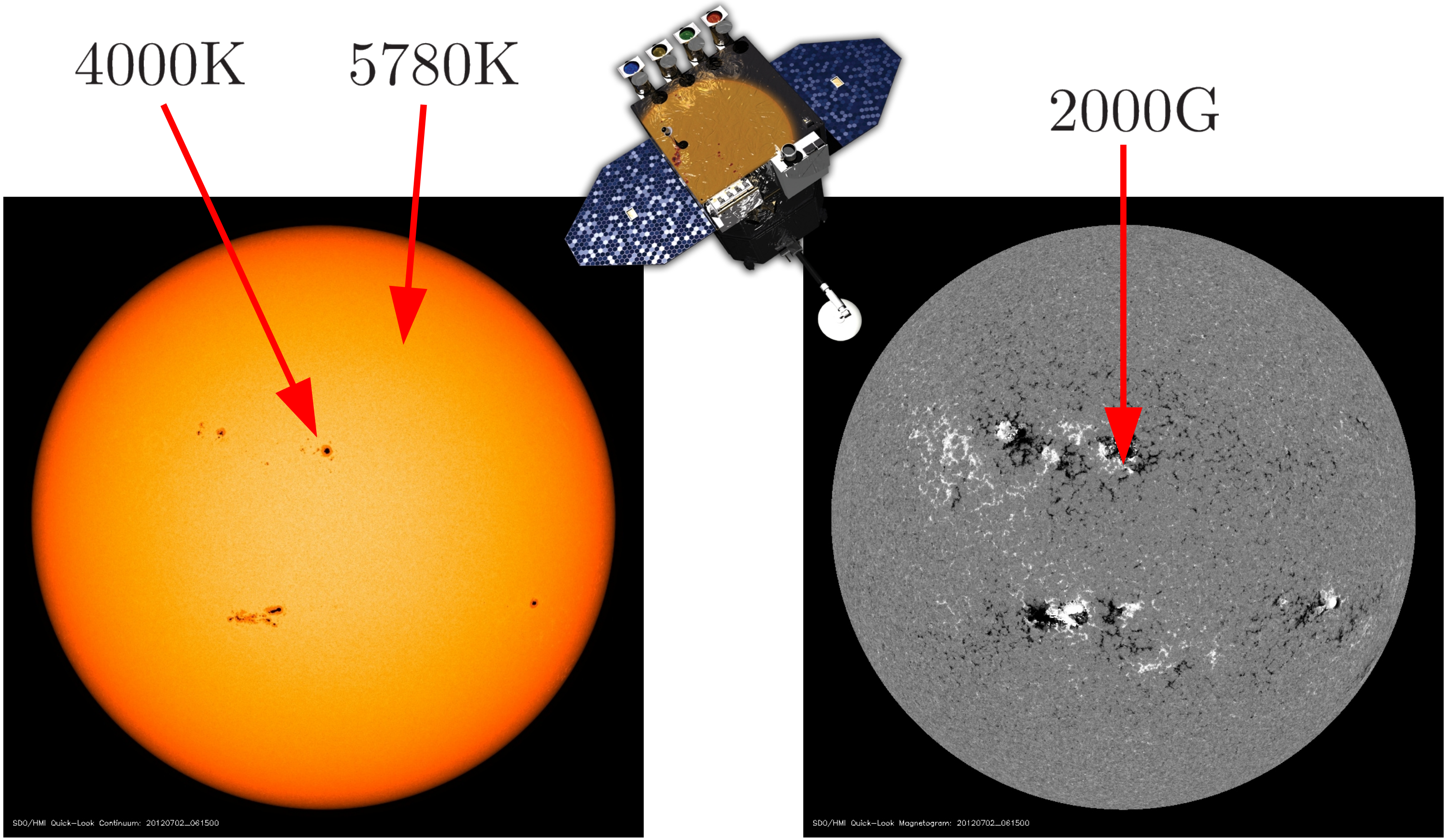
Magnetic helicity fluxes in dynamically quenched dynamos

Simon Candelaresi



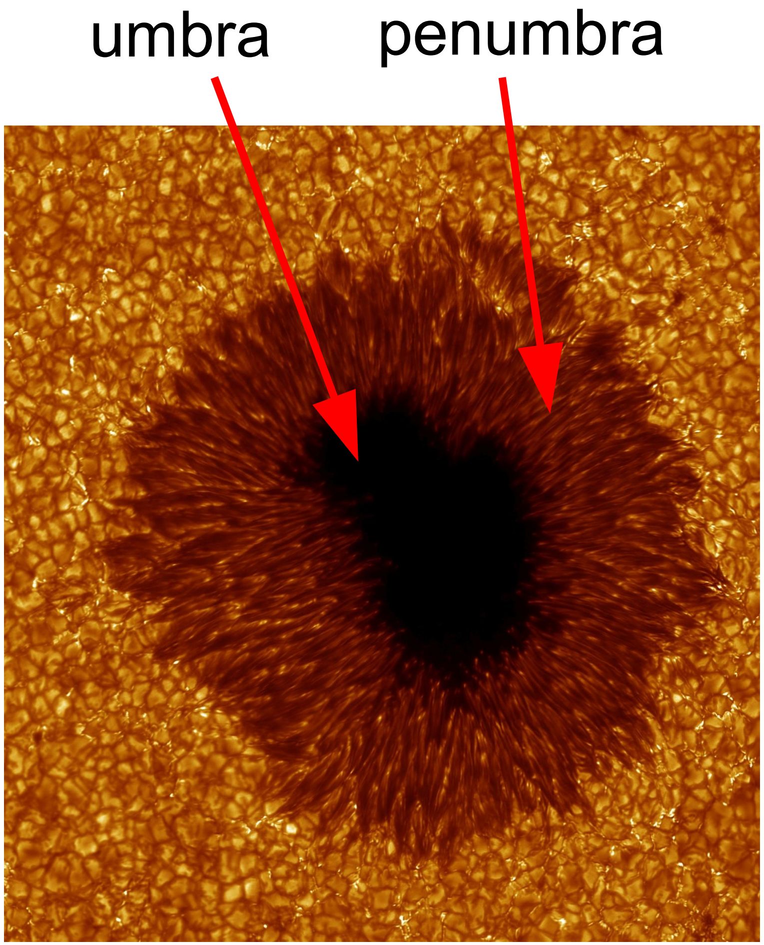
Solar Dynamics Observatory (SDO)



2nd July 2012, Intensity

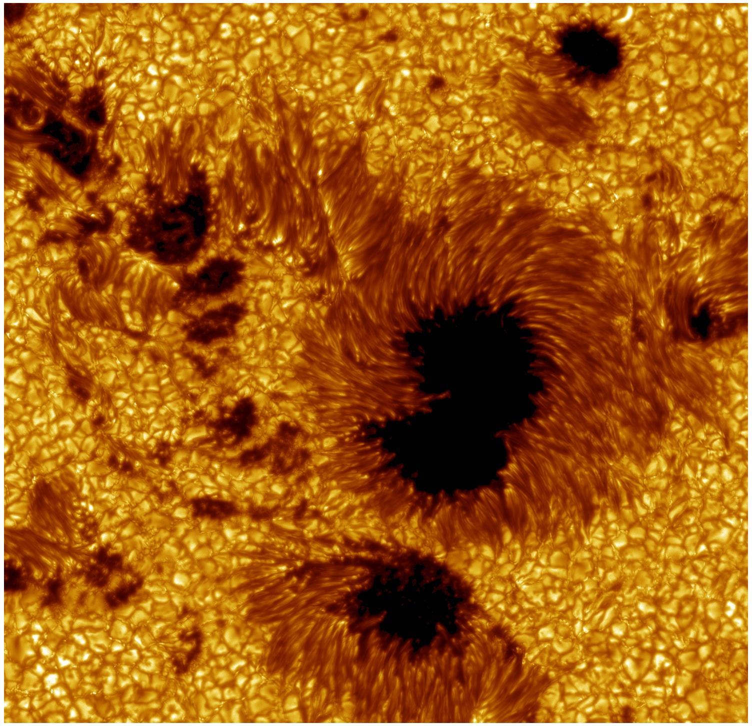
2nd July 2012, Magnetogram

Swedish Solar Telescope (SST)

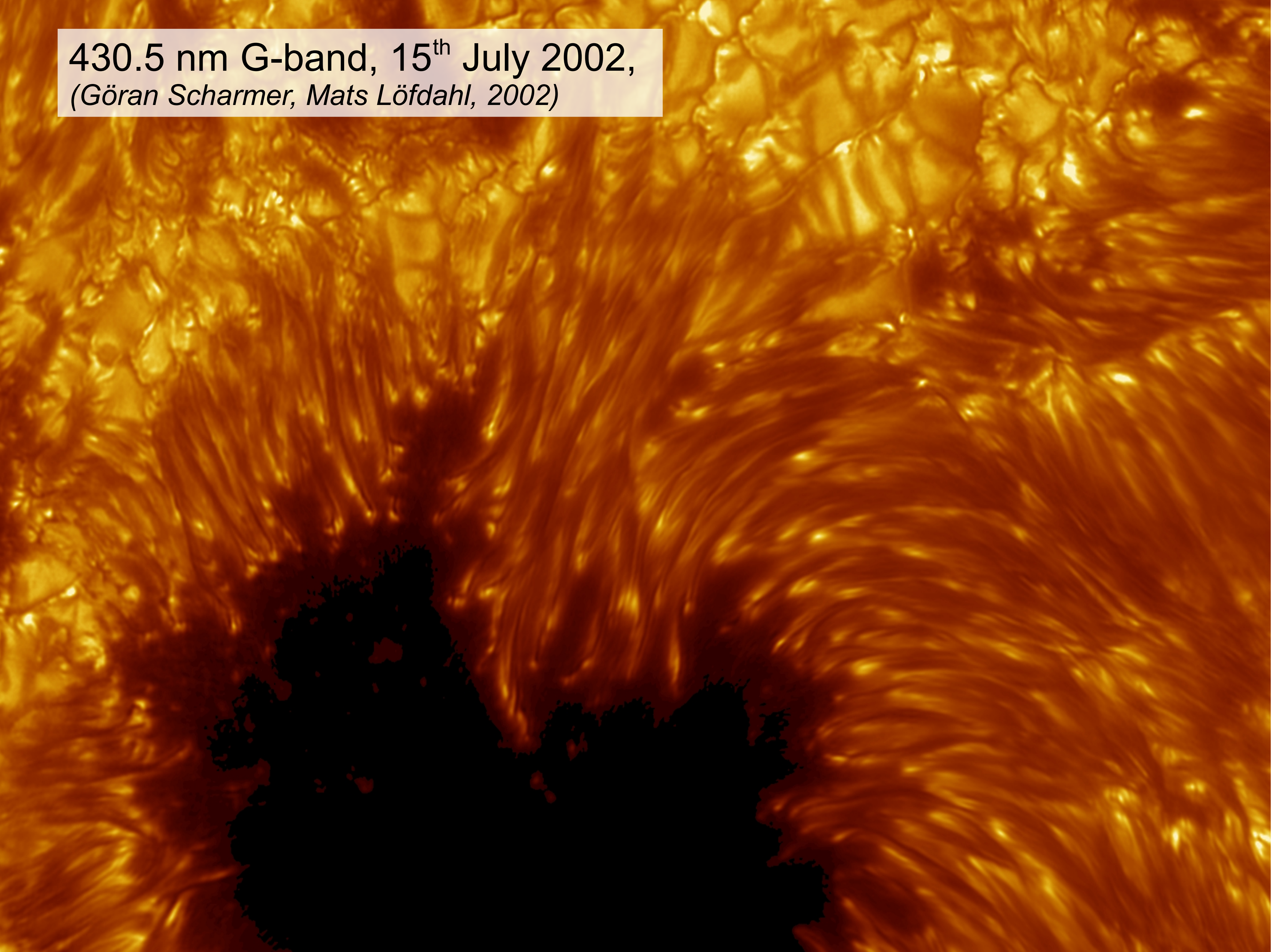


430.5 nm (G-band), 3rd July 2003,
(Dan Kiselman, Mats Löfdahl, 2003)

La Palma
(Göran Scharmer)



487.7 nm, 15th July 2002,
(Göran Scharmer, Mats Löfdahl, 2002)

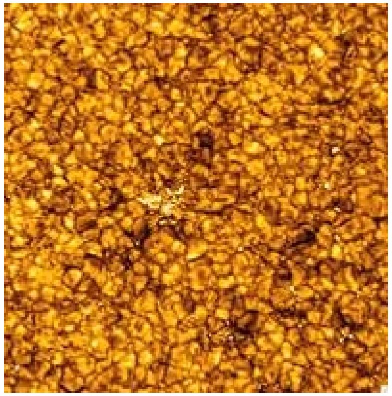


430.5 nm G-band, 15th July 2002,

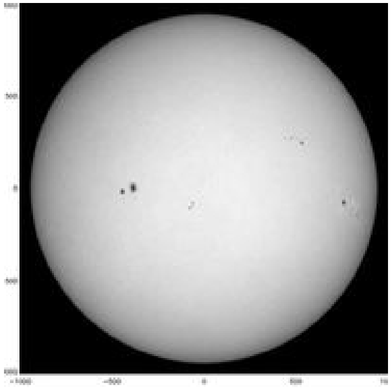
(Göran Scharmer, Mats Löfdahl, 2002)

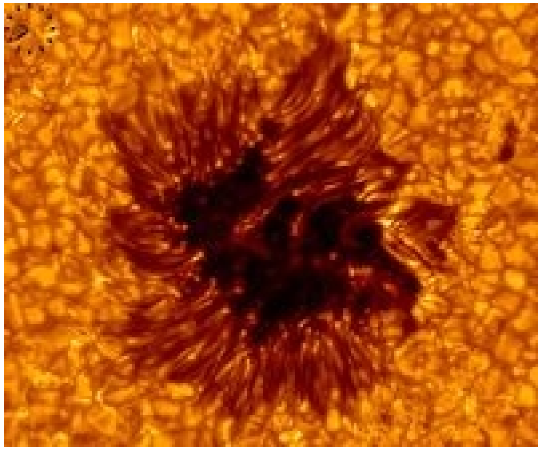
Swedish Solar Telescope (SST)

1h quiet Sun, 656.3 nm,
18th June 2006,
(Luc Rouppe van der Voort, Oslo, 2006)



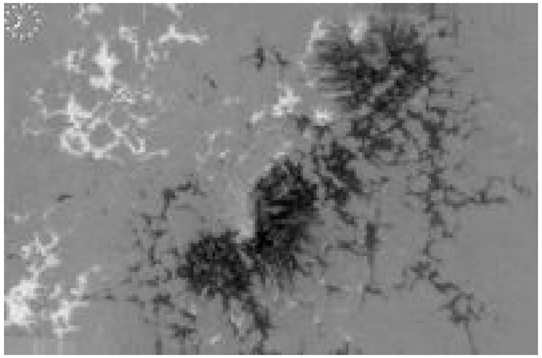
Zoom from SOHO/MDI field of view to SST resolution, August 2004,
(Michiel van Noort, Luc Rouppe van der Voort, Mats Carlsson, Oslo, 2004)



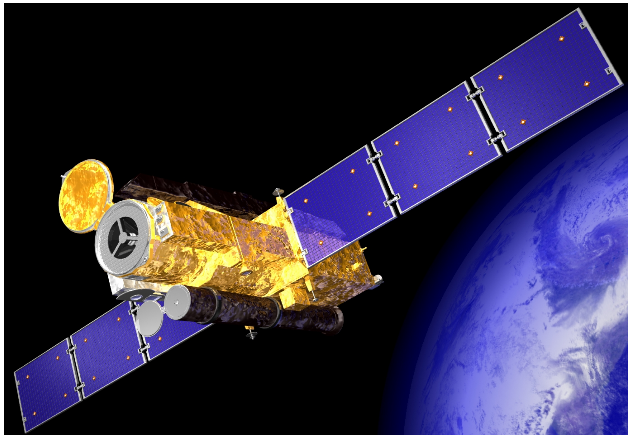


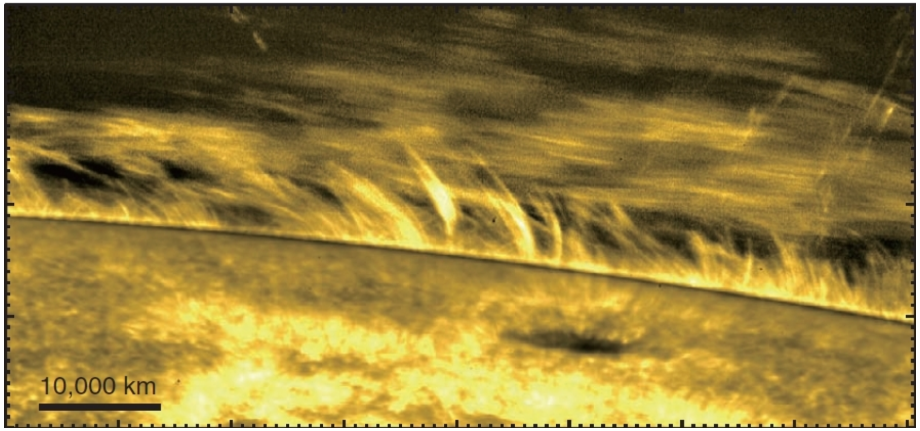
Sunspot 41 min, 430.5 nm G-band, 20th August 2004,
(Michiel van Noort and Luc Rouppe van der Voort, Oslo, 2004)

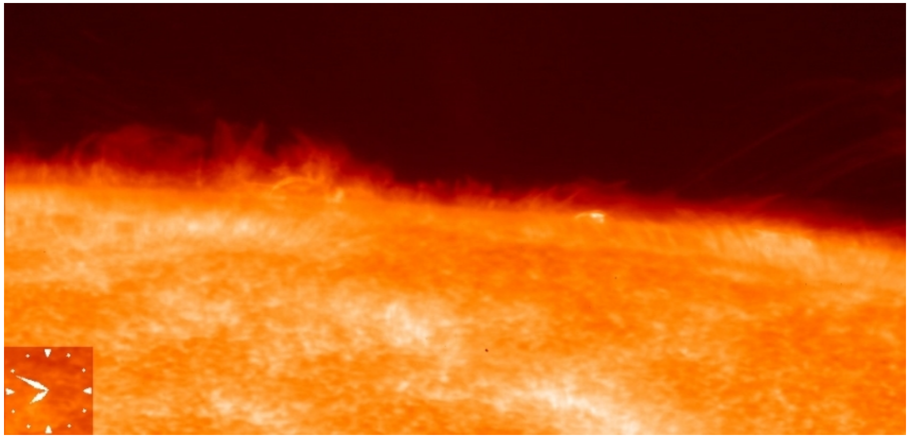
Sunspot group magnetogram,
21st August 2004,
(Michiel van Noort and Luc Rouppe van der Voort, Oslo, 2004)



Hinode ひので (Solar-B)


(JAXA)



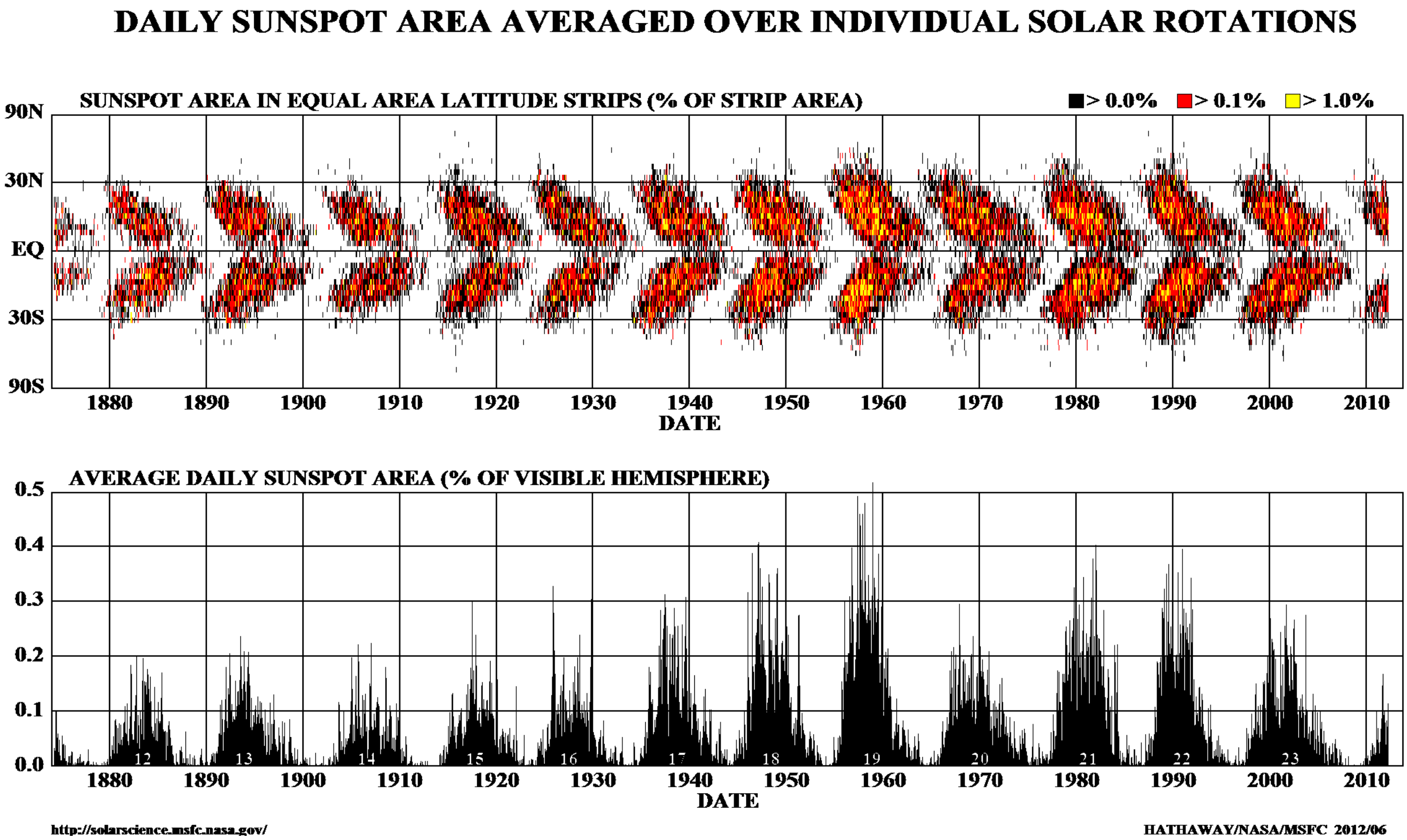


Solar prominence,
9th November 2006,
(Okamoto, T.J. et al., 2007)

Eruption observed in Ca II H (397nm) above a Sun spot,
http://solarb.msfc.nasa.gov/news/movies.html

Solar Magnetic Field

11 year cycle

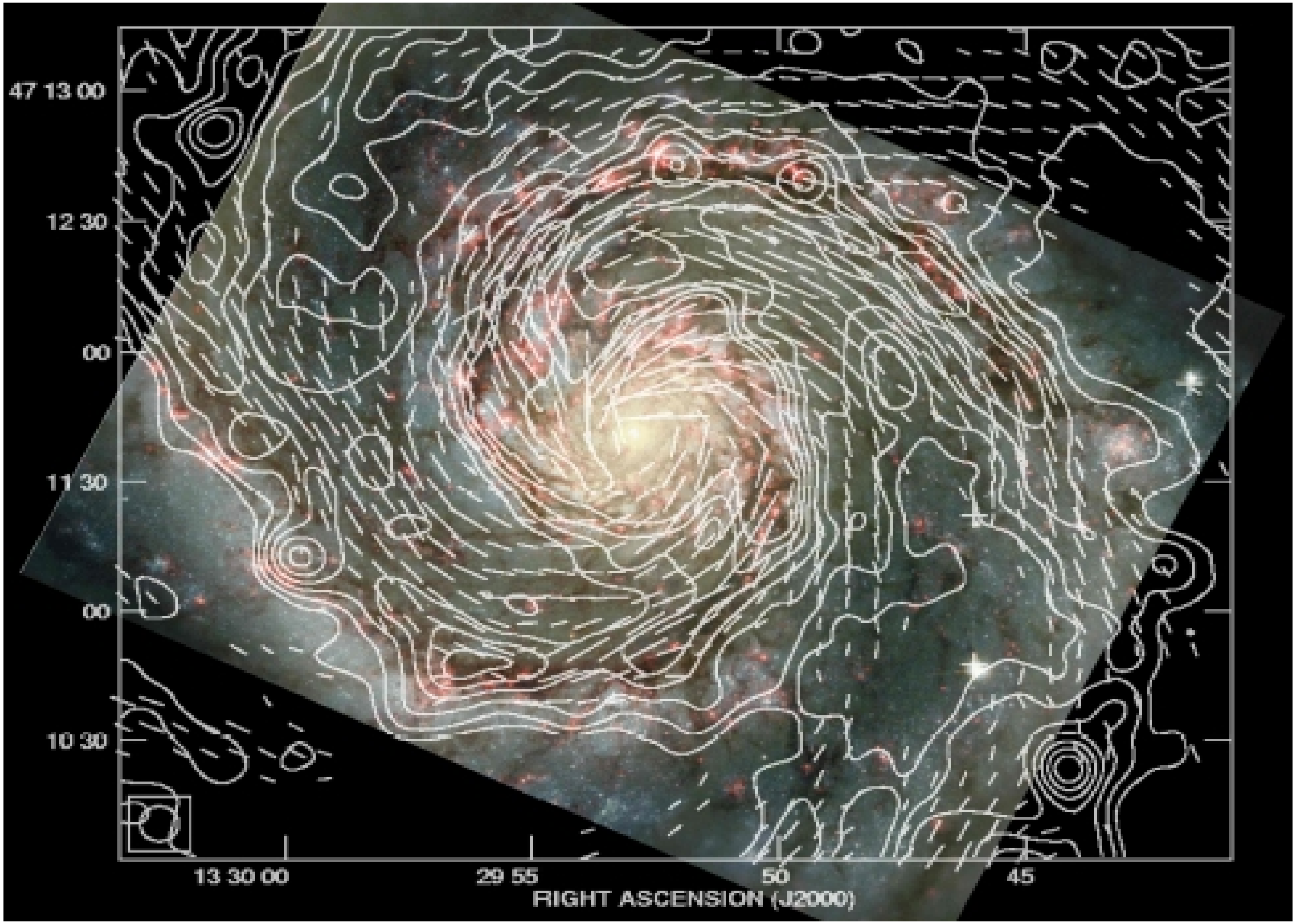


蝶形図

➡ dynamo working

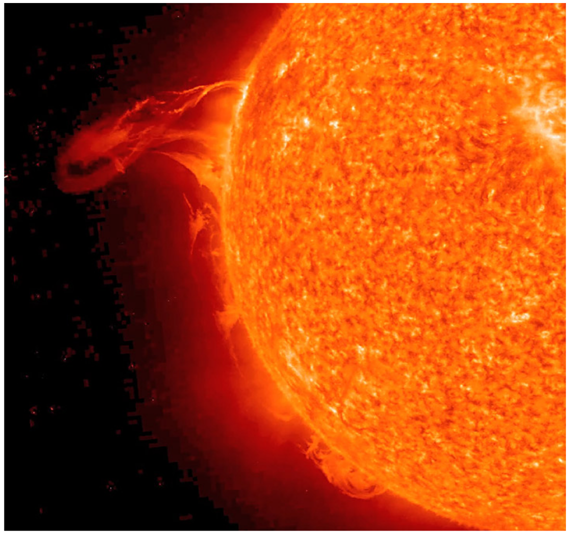
(Hathaway/NASA)

Galactic Magnetic Fields

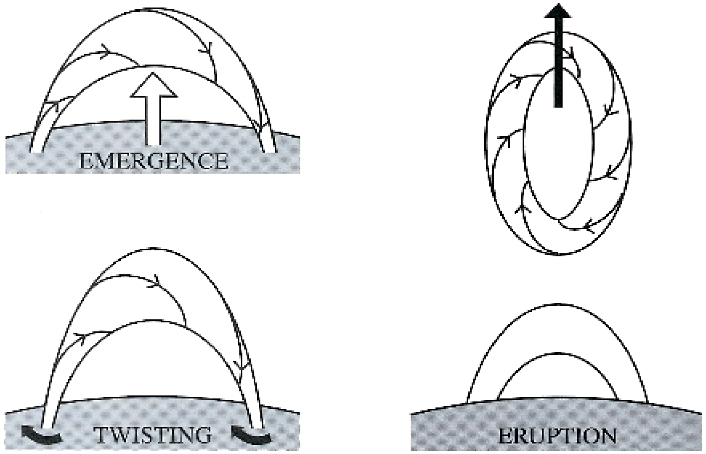


Galaxy M51, radio + optical
(Fletcher et al. 2011)

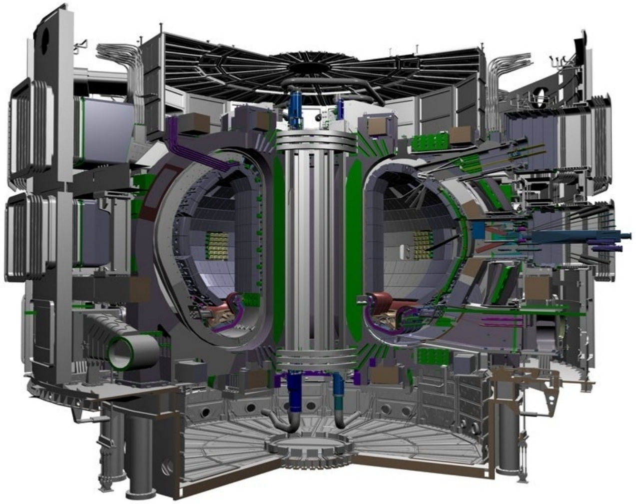
Twisted Magnetic Fields



SOHO, 7th May 2010



- ☀ Twisted fields are more likely to erupt, *(Canfield et al. 1999)*



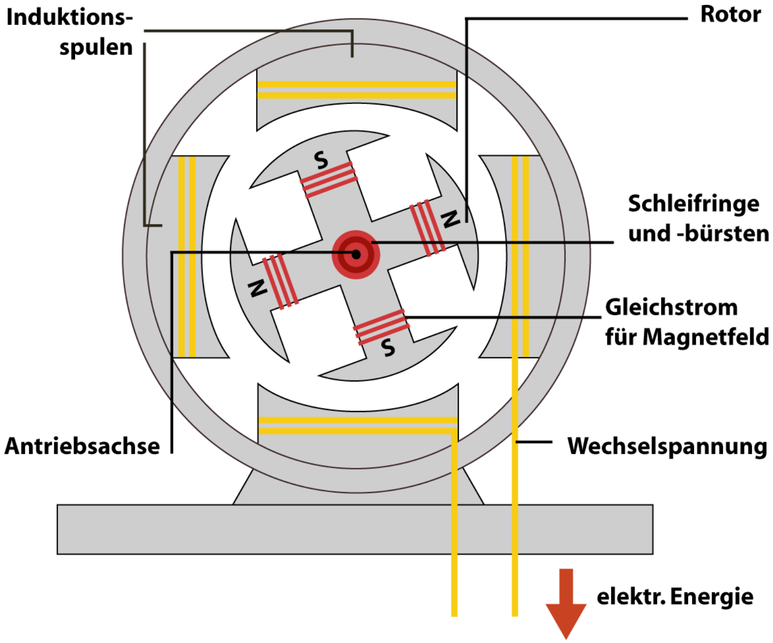
ITER

- ✿ Twist increases the stability of magnetic fields in tokamaks.

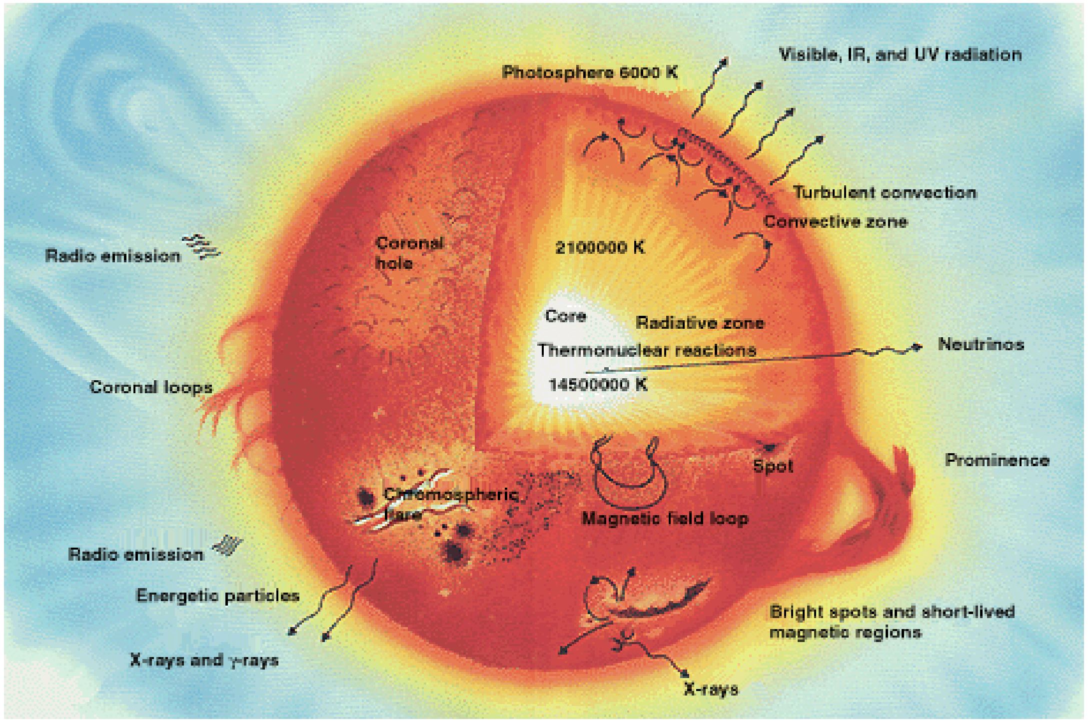
Dynamo Effect

kinetic motion → induction
→ electric energy

turbulent motion → induction
→ magnetic energy



electric power generator
(Wikipedia, user: Kuntoff, 2005)

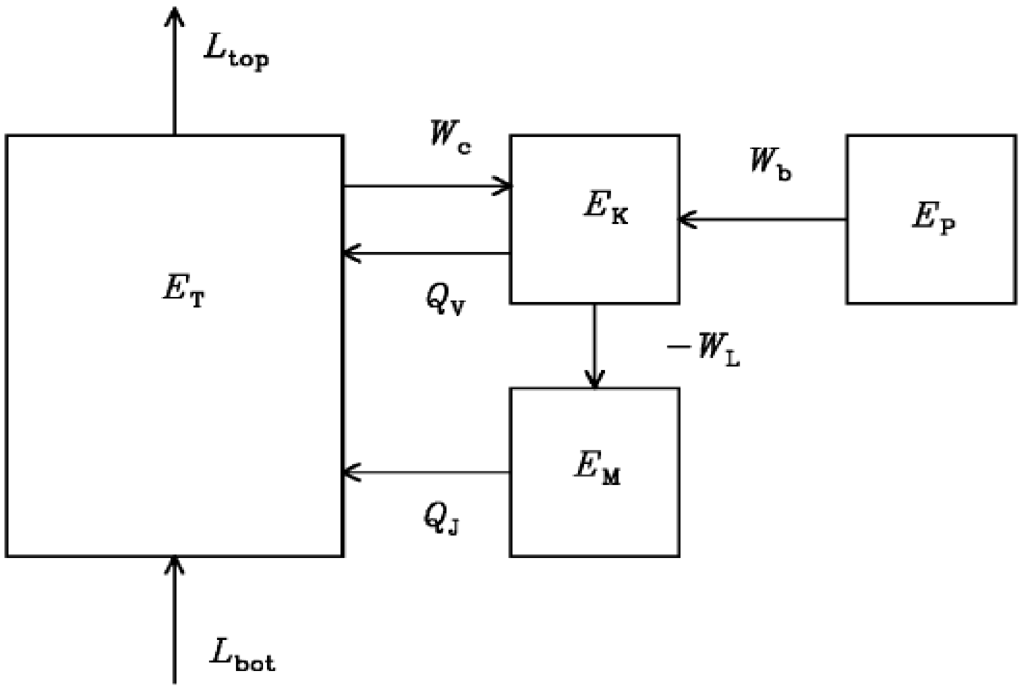


Solar model
(NASA)

$$\partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}$$

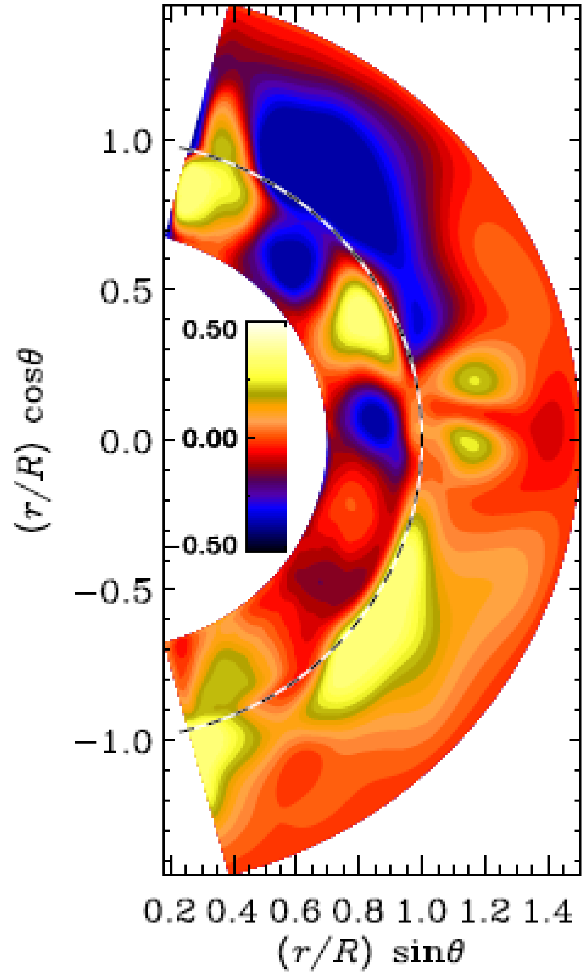
$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B}$$

Turbulent Dynamo Schematics



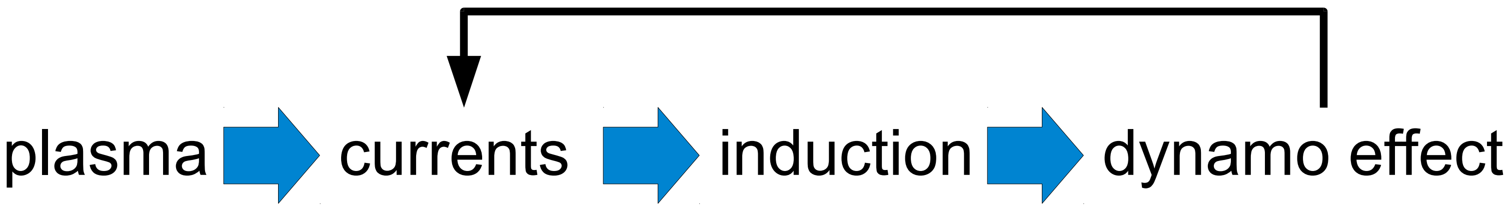
Energy budget for a dynamo.
(Brandenburg et al., 1996)

$E_{\mathrm{T}}, E_{\mathrm{K}}, E_{\mathrm{M}}, E_{\mathrm{P}} =$
thermal, kinetic, magnetic and potential energy



$\langle \overline{\boldsymbol{B}}_{\phi} \rangle_t$ for a convection driven dynamo.
(Warnecke et al., 2012)

Dynamo Mechanism



plasma → currents → induction → dynamo effect

Equations of **magnetohydrodynamics** (MHD):

Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

Momentum equation: $$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_{\mathrm{S}}^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

Continuity equation: $$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Advective derivative: $$\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{U} \cdot \boldsymbol{\nabla}$$

Mean-Field Formalism

Mean-field decomposition: $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$

Reynolds rules: $\overline{\boldsymbol{B}_1 + \boldsymbol{B}_2} = \overline{\boldsymbol{B}}_1 + \overline{\boldsymbol{B}}_2, \quad \overline{\overline{\boldsymbol{B}}} = \overline{\boldsymbol{B}}, \quad \overline{\boldsymbol{b}} = 0$

$$\overline{\partial_\mu \boldsymbol{B}} = \partial_\mu \overline{\boldsymbol{B}}, \quad \mu = 0, 1, 2, 3$$

Mean-field induction equations:

$$\partial_t \overline{\boldsymbol{B}} = \eta \nabla^2 \overline{\boldsymbol{B}} + \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}})$$

$$\partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{G}) + \boldsymbol{\nabla} \times (\boldsymbol{u} \times \overline{\boldsymbol{B}}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{b}$$

Electromotive force (emf): $\boxed{\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \boldsymbol{b}}}$

$$\boldsymbol{G} = \boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}$$

Mofatt, H. K., 1978, Krause and Raedler, 1980

Electromotive Force

The EMF is assumed to be linear and homogeneous in $\overline{\boldsymbol{B}}$.

➡ $$\mathcal{E}_i(x,t) = \mathcal{E}_i^{(0)}(x,t) + \int\int_{\alpha} K_{ij}(x,x',t,t')\overline{B}_j(x-x',t-t')\;\mathrm{d}^3x'\;\mathrm{d}t'$$

Taylor expansion:

$$\overline{B}_j(x',t) = \overline{B}_j(x,t) + (x'_k - x_k)\frac{\partial \overline{B}_j(x,t)}{\partial x_k} + \ldots$$

Assume local and instantaneous dependence of $\overline{\boldsymbol{\mathcal{E}}}$ on $\overline{\boldsymbol{B}}$.

➡ $$\overline{\mathcal{E}}_i = \alpha_{ij}\overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k} + \ldots$$

For a turbulent system without preferred direction, i.e. $\boldsymbol{U} = 0$:

$$\boxed{\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_{\mathrm{t}}\nabla\times\overline{\boldsymbol{B}}}$$

$$\boxed{\partial_t\overline{\boldsymbol{B}} = \alpha\boldsymbol{\nabla}\times\overline{\boldsymbol{B}} + \eta_{\mathrm{T}}\boldsymbol{\nabla}^2\overline{\boldsymbol{B}}}$$

Alpha-Effect

α effect: $\alpha = \alpha_{\rm K} + \alpha_{\rm M}$

$$\alpha_{\rm K} = -\tau\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}/3$$

$$\alpha_{\rm M} = \tau\overline{\boldsymbol{j}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \tau k^2\overline{\boldsymbol{a}\cdot\boldsymbol{b}}/(3\overline{\rho}) = \overline{h}_{\rm m}$$

helically driven dynamo $\overline{h}_{\rm K,f} = \overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}$

➡ production of magnetic helicity $\overline{h}_{\rm M,f} = \overline{\boldsymbol{a}\cdot\boldsymbol{b}}$

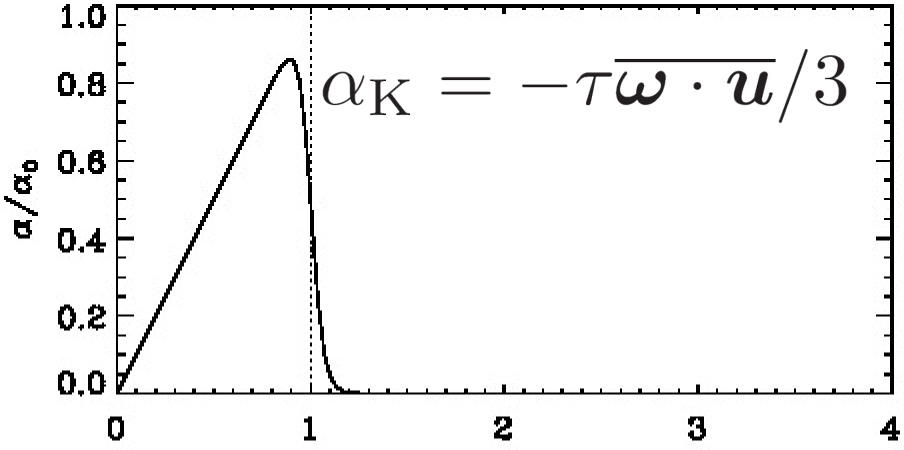
➡ total magnetic helicity conservation $\overline{h}_{\rm M,m} = \overline{\boldsymbol{A}}\cdot\overline{\boldsymbol{B}}$

$\overline{\boldsymbol{a}\cdot\boldsymbol{b}}$ works against dynamo: $E_{\rm M} \propto 1/{\rm Re}_{\rm M}$ $\quad {\rm Re}_{\rm M} = \dfrac{UL}{\eta}$

Sun: ${\rm Re}_{\rm M} = 10^9$ galaxies: ${\rm Re}_{\rm M} = 10^{18}$

Magnetic Helicity Fluxes

$$\frac{\partial \alpha_{\rm M}}{\partial t} = -2\eta_{\rm t} k_{\rm f}^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm M}}{R_{\rm m}} \right) - \underline{\frac{\partial}{\partial z} \overline{\mathcal{F}}_\alpha}$$

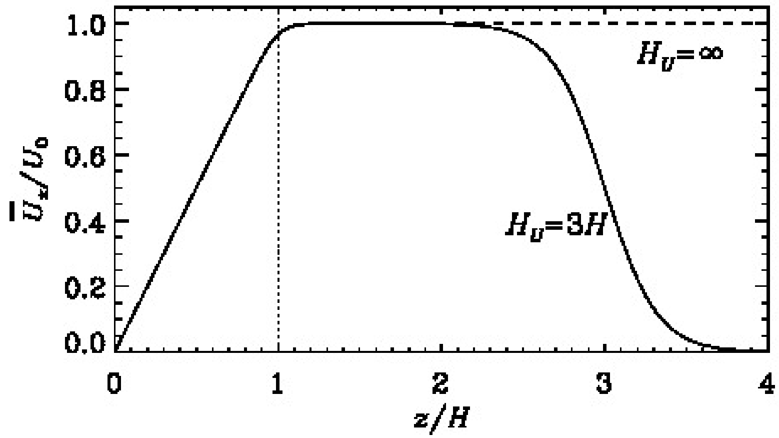


advective:
$\alpha_{\rm M} \overline{\boldsymbol{U}}$

α diffusion
$\kappa_\alpha \dfrac{\partial \alpha_{\rm M}}{\partial z}$

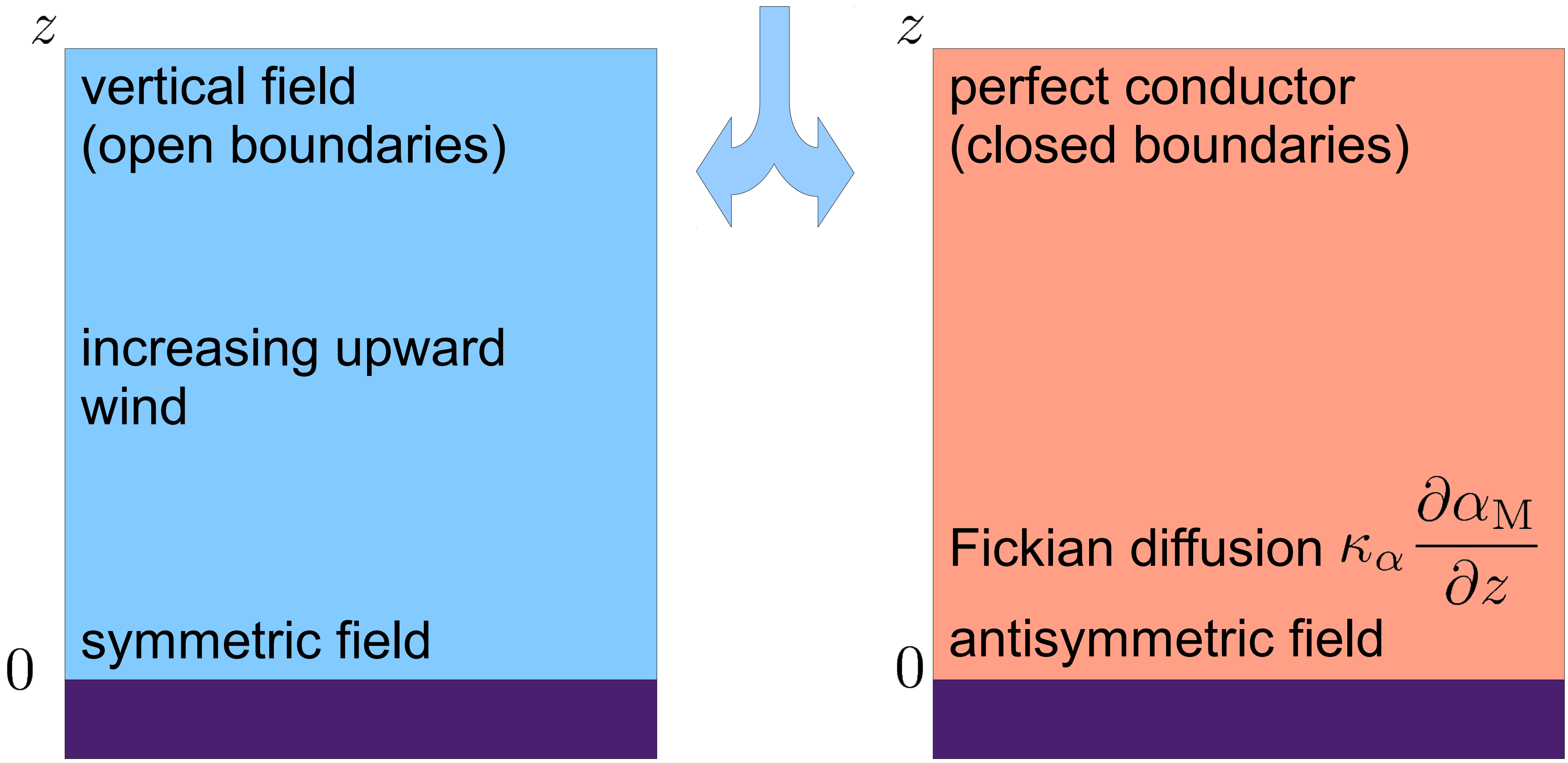
$$\frac{\partial \overline{h}_{\rm m}}{\partial t} = 2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_{\rm m}$$

$$\frac{\partial \overline{h}_{\rm f}}{\partial t} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta\mu_0 \overline{\boldsymbol{j}} \cdot \overline{\boldsymbol{b}} - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{F}}_{\rm f}$$



Magnetic Helicity Fluxes

Solve equations for one hemisphere.
Impose (anti)symmetric field at the equator.



vertical field
(open boundaries)

increasing upward
wind

symmetric field

perfect conductor
(closed boundaries)

Fickian diffusion $\kappa_\alpha \frac{\partial \alpha_{\mathrm{M}}}{\partial z}$

antisymmetric field

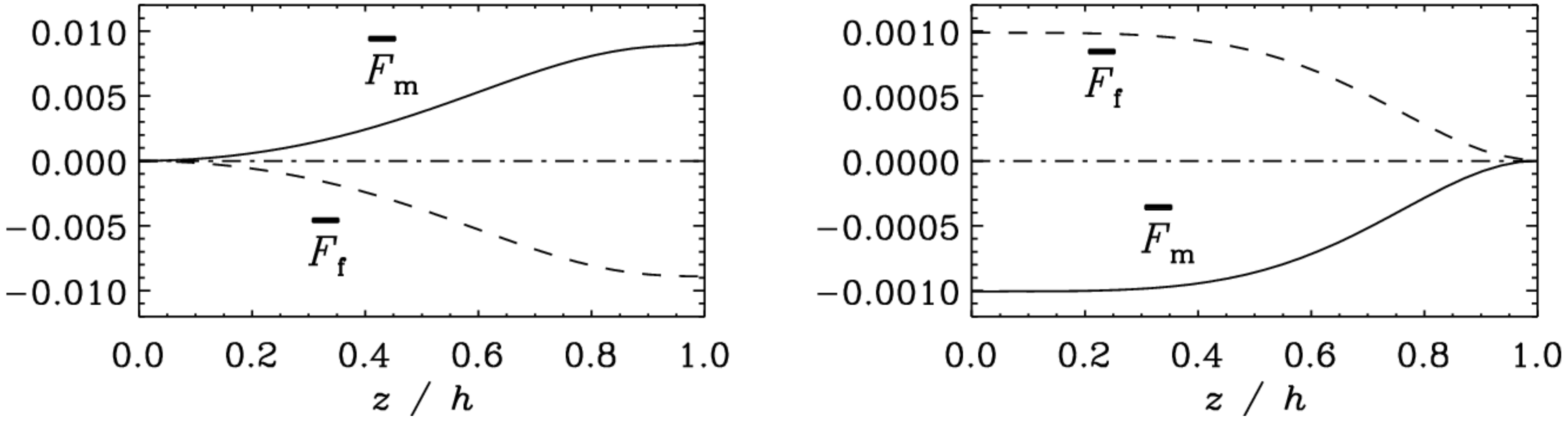
$$\mathrm{Re}_{\mathrm{M}} = \frac{U_{\mathrm{rms}} L}{\eta}$$

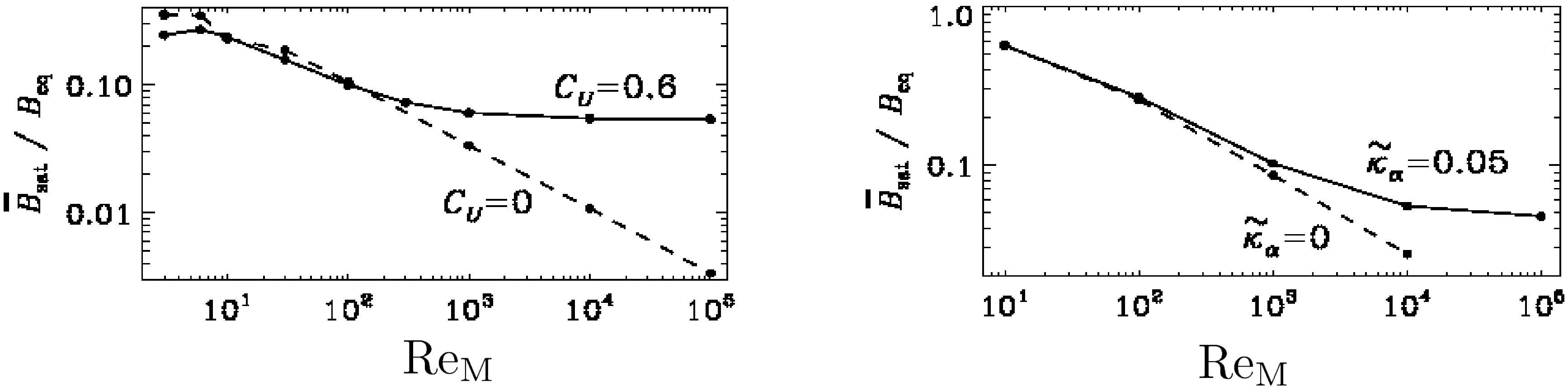
Magnetic Felicity Fluxes

open boundary
symmetric
wind

vs.

closed boundary
antisymmetric
κ_α



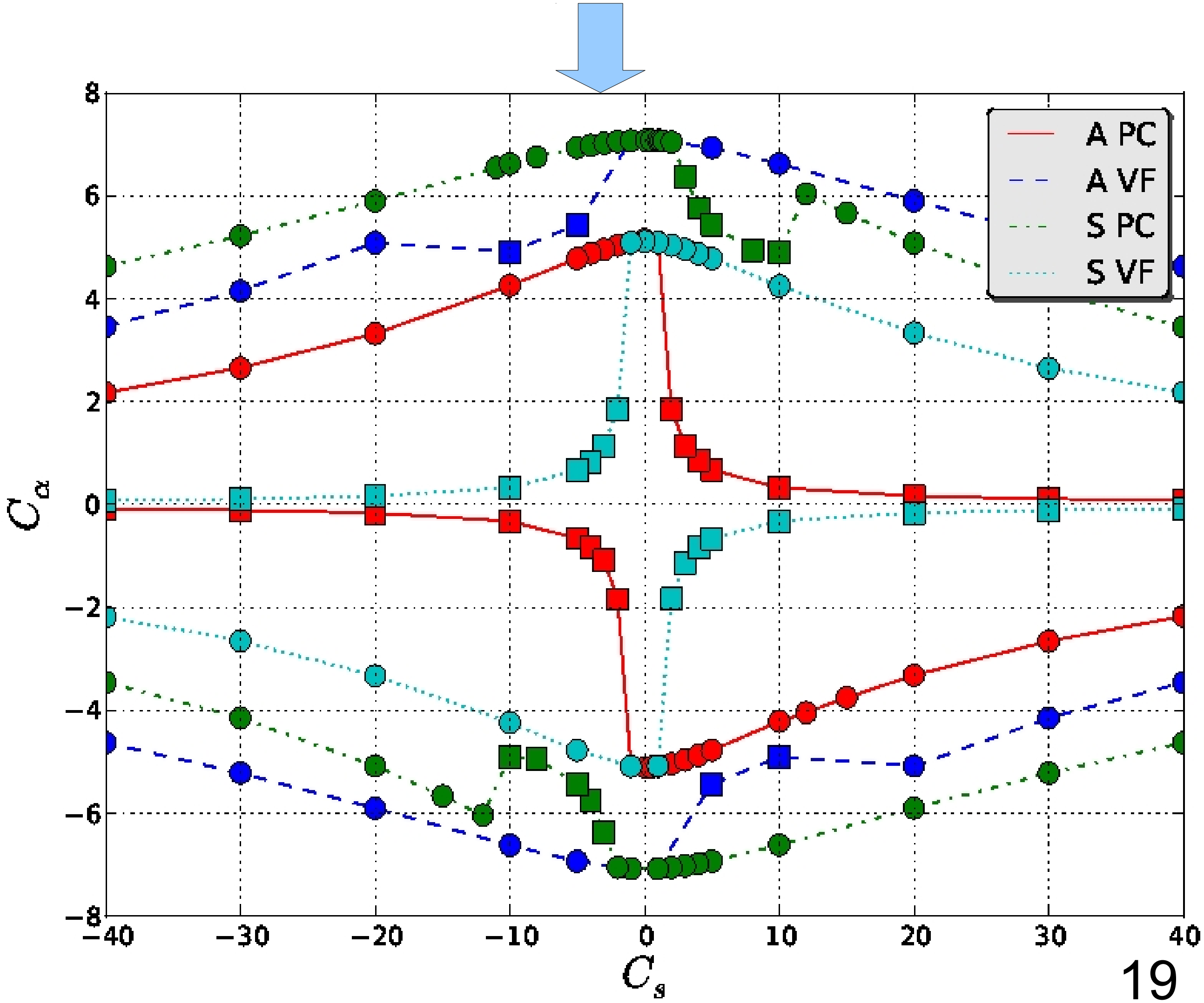


Adding Shear

Critical values for the forcing and the shearing amplitude

Shearing velocity field:

$$\overline{\boldsymbol{U}} = \begin{pmatrix} 0 \\ Sz \\ 0 \end{pmatrix}$$

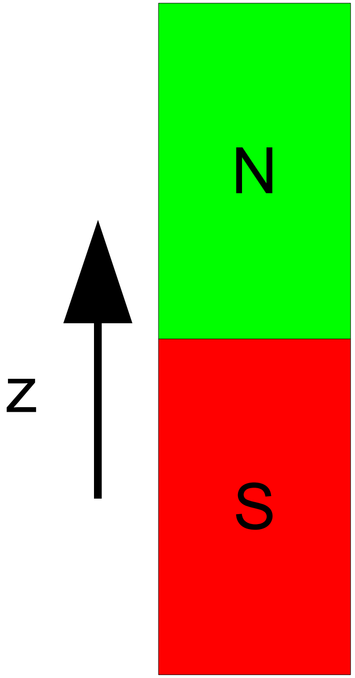


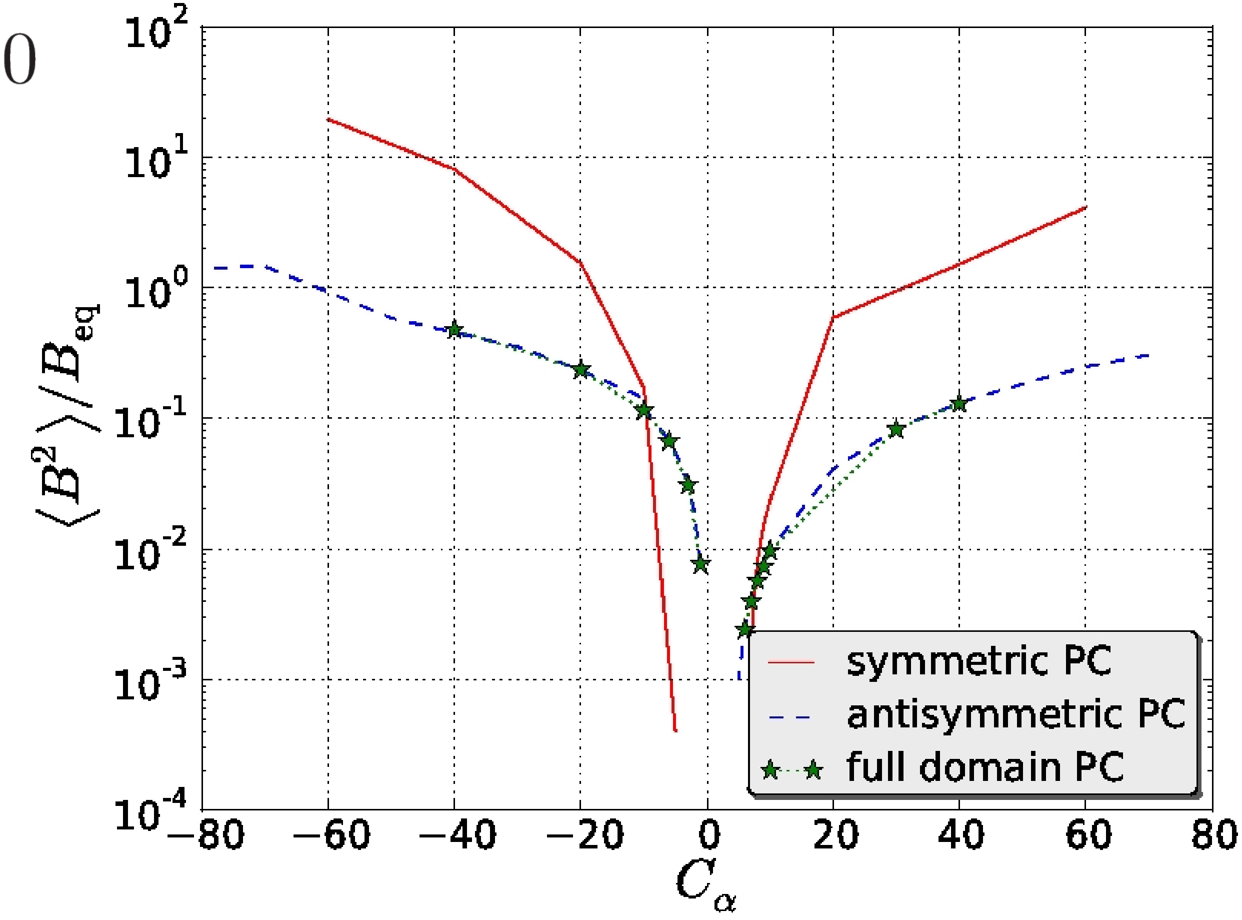
Full Domain

Imposed parity in the hemispheric model is artificial.

➡ Include both hemispheres.

$$C_{\mathrm{S}} = -10$$



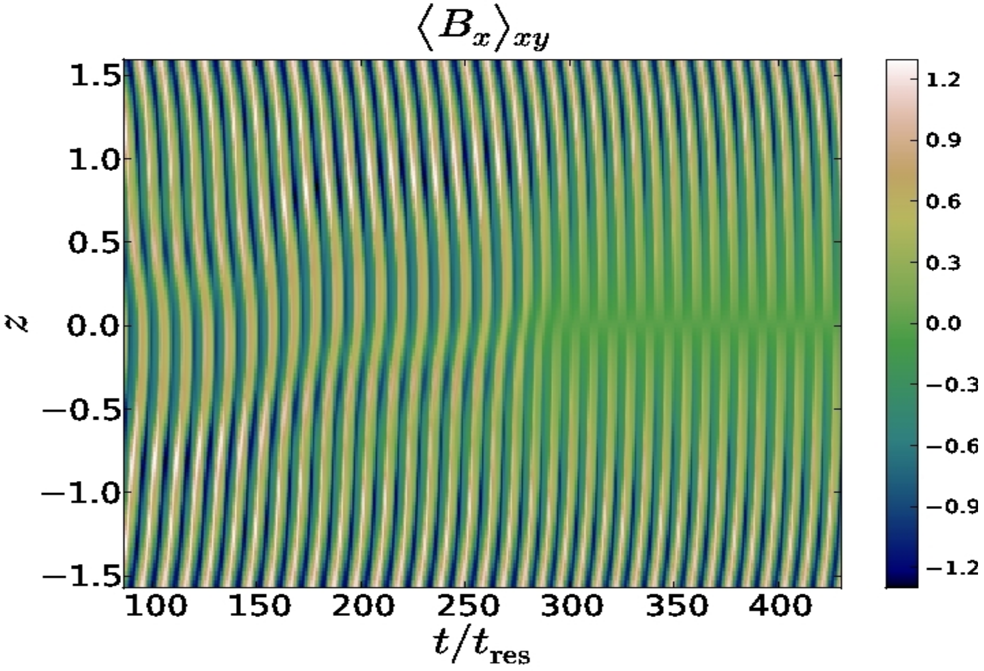


Preferred antisymmetric mode?

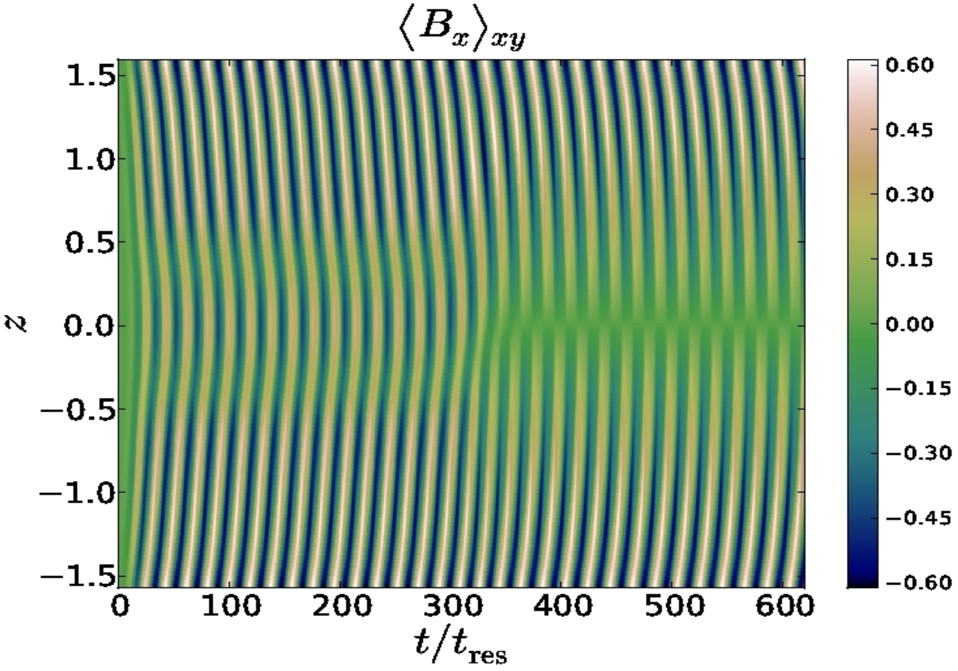
Parity Change

Look at the parity of the magnetic field $\overline{B}_y$

Random initial field



Symmetric initial field



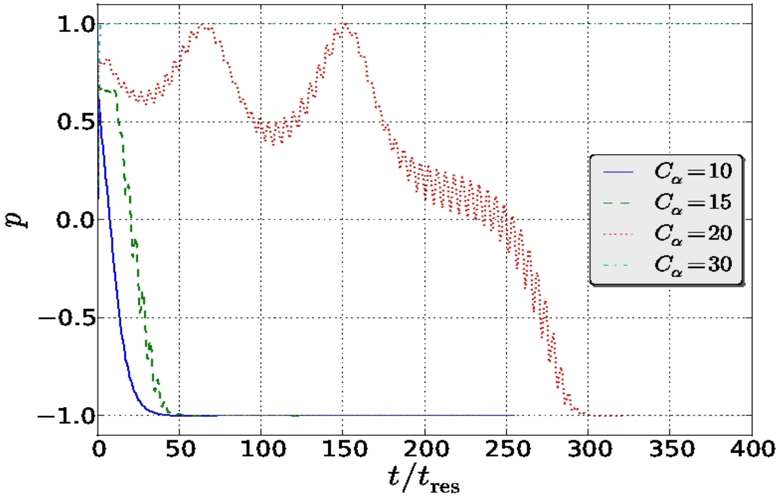
➡ The antisymmetric solution seems to be the preferred one.

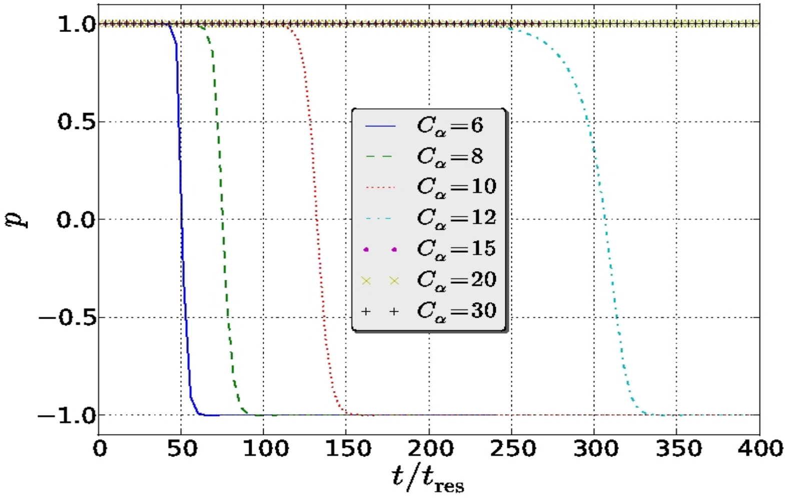
Parity Change

Random initial field

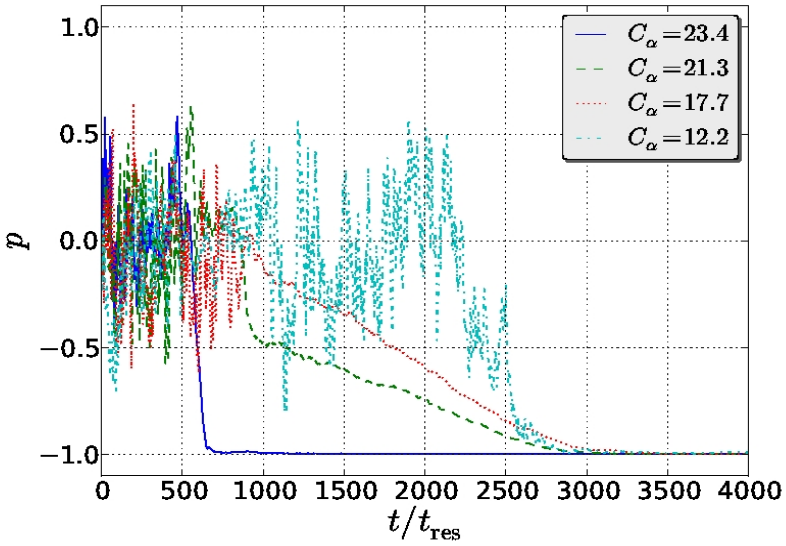
Symmetric initial field

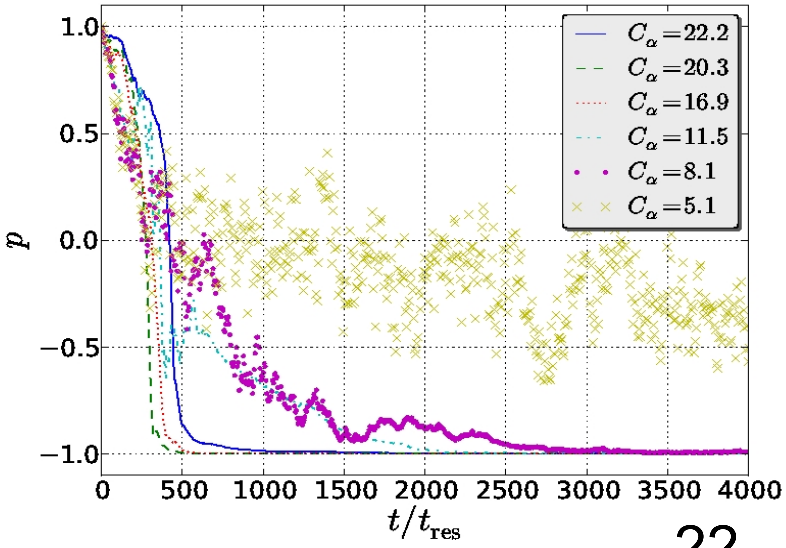
MF





DNS





Conclusions

- Helical turbulence can drive large-scale dynamo action.
- Convective motions in plasma drive dynamos.

- Advective magnetic helicity $\alpha_{\rm f}$ fluxes can alleviate catastrophic quenching.
- Diffusive magnetic helicity fluxes can alleviate catastrophic quenching.

- Symmetric mode is unstable.
- The antisymmetric mode seems to be the preferred one.

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`www.nordita.org/~iomsn`

Appendix

Viscous force: $\boldsymbol{F}_{\rm visc} = \rho^{-1}\boldsymbol{\nabla}\cdot 2\nu\rho\mathsf{\mathbf{S}}$

Strain tensor: $\mathsf{S}_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{U}$

Sound speed: $c_{\rm S} = \sqrt{\gamma\frac{p}{\rho}}$