Magnetic Helicity and its Topological Interpretation

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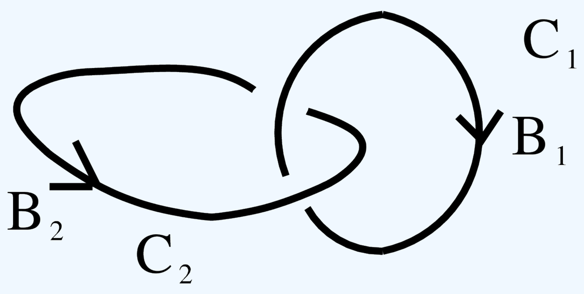
Introduction

Magnetic helicity H is strongly related to the linking of magnetic flux tubes.

Magnetic Helicity: $H = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V$



$$\int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = 2n\Phi_1\Phi_2$$

n = linking number

Φ_i = magnetic fluxes through the rings

Realizability condition: $M(k) \geq k|H(k)|/2\mu_0$

Magnetic energy M(k) has a lower bound in ideal MHD in presence of magnetic helicity (Moffatt 1969).

Approach:

We study the evolution of three flux rings in setups with either zero or finite magnetic helicity. Two configurations have interlocked rings. We perform direct numerical simulations with the Pencil Code.

Model and Setup

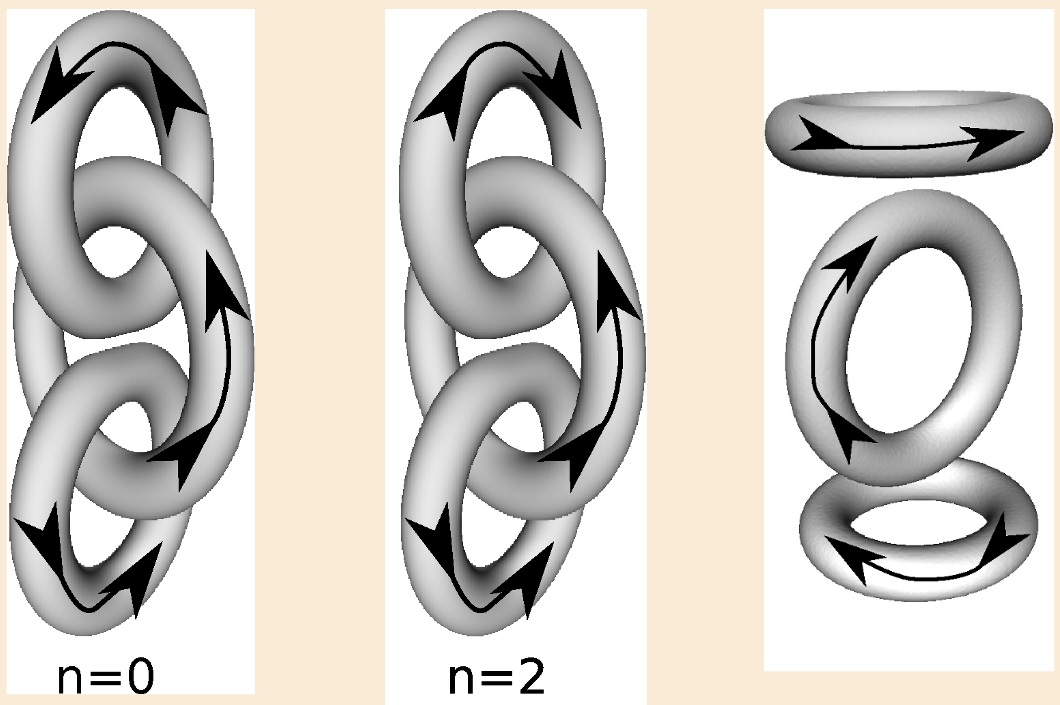


FIG.1. The initial field configuration used in the simulations.
The arrows indicate the direction of the magnetic field.
Left: interlocked configuration with zero helicity.
Center: interlocked configuration with finite helicity.
Right: noninterlocked configuration with zero helicity.

Model equations:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A}$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_s^2 \boldsymbol{\nabla} \ln \rho + \boldsymbol{J} \times \boldsymbol{B}/\rho + \boldsymbol{F}_{\mathrm{visc}}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Results

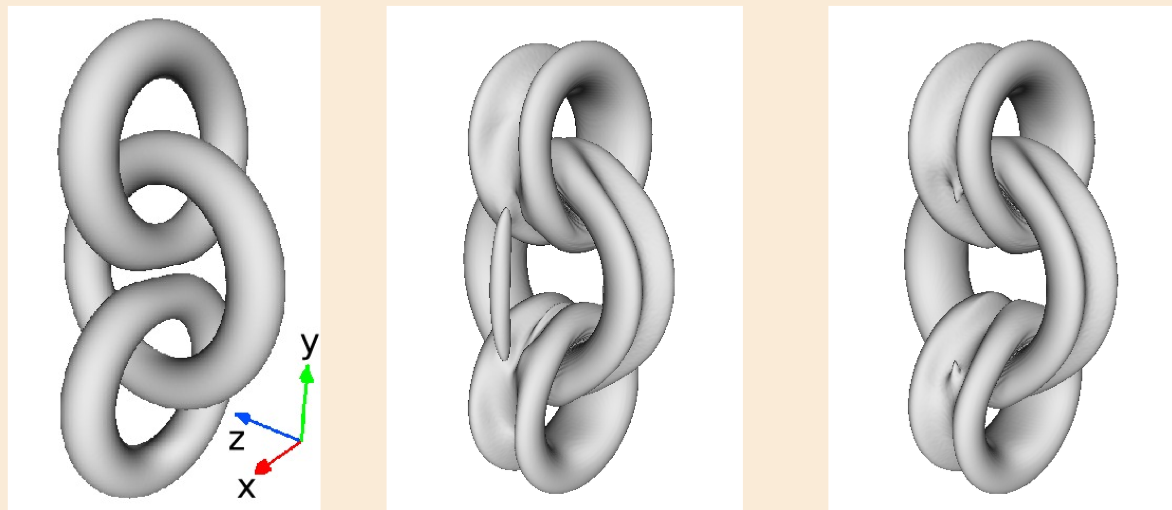


FIG. 2. Triple ring configuration at time τ=0 (left) and τ=0.5 for zero (center) and finite (right) helicity. In the center we can see the emergence of a new flux ring which is absent on the right.

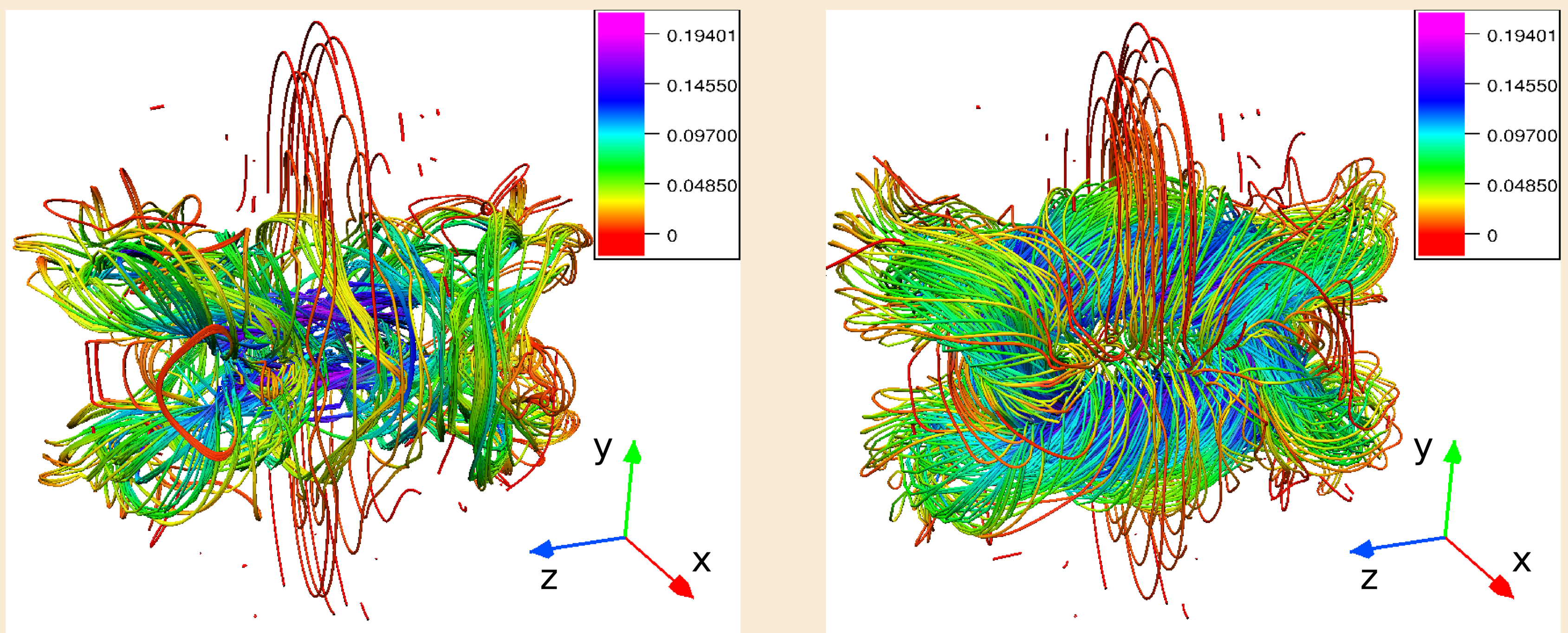


FIG. 3. Magnetic flux tubes at time τ=4 for the zero helicity case (left) and finite helicity case (right). The colours represent the magnitude of the magnetic field.

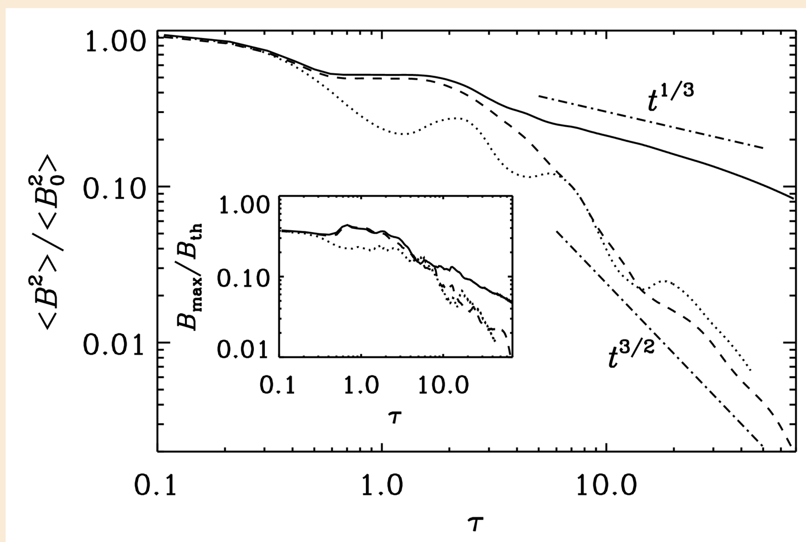


FIG. 4. Magnetic energy decay for the finite helicity case (solid line), zero helicity but interlocked case (dashed line), and non-interlocked case (dotted line).

Conclusions & Outlook

- Decay of magnetic energy is faster with zero helicity.
- Topology of the flux configuration is conserved with helicity.
- Qualitative degree of knottedness unimportant: relative orientation matters.
- Need to consider magnetic helicity in reconnection studies.

References

- F. D. Sordo, S. Candelaresi, and A. Brandenburg, Phys. Rev. E, 81:036401 (2010).
- H. K. Moffatt, J. Fluid Mech. 35, 117 (1969).