Helicity Fluxes and Gauge Issues in Turbulent Dynamos

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Introduction

Quenching

Large scale magnetic fields in dynamos, like the Sun or Galaxy, are created on dynamical time scales.
In periodic boxes with helicity conservation dynamos grow only on resistive time scales (Brandenburg 2001).

Saturation magnetic energy varies as R_m^{-1} which can be catastrophic for real astrophysical objects (Sun: $R_m = 10^9$, Galaxy: $R_m = 10^{14}$). This catastrophic quenching needs to be alleviated by helicity fluxes, which will also give us fast growing dynamos.

As R_m increases, larger fractions of magnetic energy are stored in smaller scales.

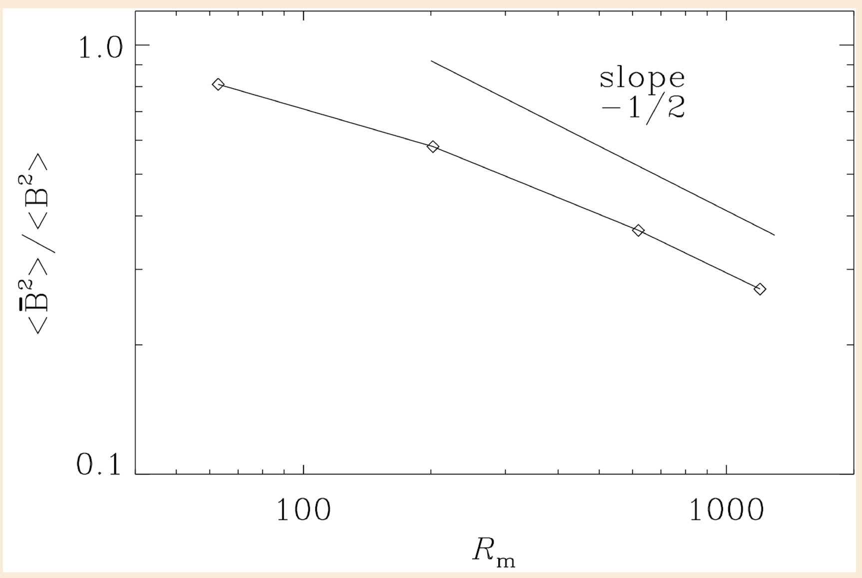

FIG. 1. Mean magnetic energy to the total magnetic energy versus magnetic Reynolds number. (Brandenburg & Dobler 2001)

Gauging the Magnetic Helicity

Magnetic helicity density: $h = \boldsymbol{A}\cdot\boldsymbol{B}$
is gauge dependent: $\boldsymbol{A} \to \boldsymbol{A} + \nabla\Lambda$

- Do simulations show different behaviour for different gauges?
- How do helicity fluxes change?

Approach

Quenching

- 1d mean field simulation
- Linear forcing of kinematic helicity
- Allow for helicity fluxes out of the domain.
- Impose helicity fluxes through the equator.

The equations:

$$\frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left(\frac{\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}}}{B_{eq}^2} + \frac{\alpha_M}{R_m} \right) - \frac{\partial}{\partial z}\mathcal{F}_\alpha$$

$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z\overline{B}_r + \mathcal{E}_\phi\right) + \eta\frac{\partial^2 \overline{B}_r}{\partial z^2},$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z\overline{B}_\phi - \mathcal{E}_r\right) + \eta\frac{\partial^2 \overline{B}_\phi}{\partial z^2} + q\Omega_0\overline{B}_r$$

Approach

Gauging the Magnetic Helicity

Make direct numerical simulations (DNS) of compressible MHD for an isothermal gas with constant sound speed.

Consider the gauges:
- Resistive gauge
- Lorenz gauge
- Weyl gauge

Small-scale and large-scale helicity changes:

$$\partial_t \overline{h}_m^M = 2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}} - 2\eta\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}} - \nabla\cdot\overline{\boldsymbol{\mathcal{F}}}_m^H$$

$$\partial_t \overline{h}_f^M = -2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}} - 2\eta\overline{\boldsymbol{j}\cdot\boldsymbol{b}} - \nabla\cdot\overline{\boldsymbol{\mathcal{F}}}_f^H$$

Helicity fluxes:

$$\overline{\boldsymbol{\mathcal{F}}}_m^H = \overline{\boldsymbol{E}}\times\overline{\boldsymbol{A}} + \overline{\Psi}\,\overline{\boldsymbol{B}},$$
$$\overline{\boldsymbol{\mathcal{F}}}_f^H = \overline{\boldsymbol{e}\times\boldsymbol{a}} + \overline{\psi\boldsymbol{b}},$$

and $\Psi = \overline{\Psi} + \psi$.

Fickian diffusion: $\overline{\boldsymbol{\mathcal{F}}}_f^H = -\kappa_f\nabla\overline{h}_f^M$

The equations for the simulations:

$$D_t\boldsymbol{U} = -c_s^2\nabla\ln\rho + \frac{1}{\rho}\boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{F}_{visc} + \boldsymbol{f}$$
$$D_t\ln\rho = -\nabla\cdot\boldsymbol{U},$$
$$\partial_t\boldsymbol{A} = \boldsymbol{U}\times\boldsymbol{B} - \eta\mu_0\boldsymbol{J} - \nabla\Psi,$$

Results

Quenching

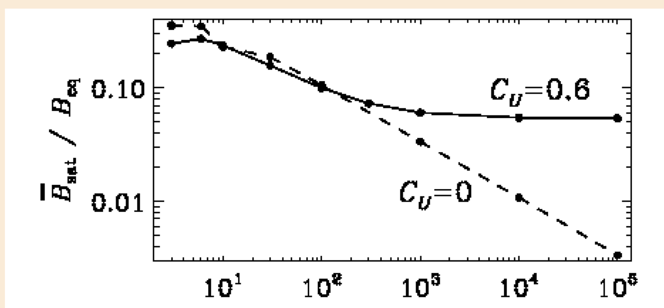

FIG. 2. Saturation magnetic energy versus magnetic Reynolds number R_m with advective helicity flux (solid line). Compare the case without the flux (dashed line). The catastrophic quenching gets alleviated by helicity fluxes.

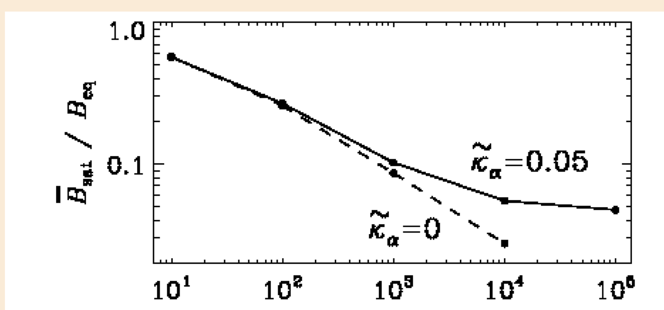

FIG. 3. Saturation magnetic energy versus magnetic Reynolds number R_m with diffusive helicity fluxes through the equator (solid line). Compare the case without this flux (dashed line). Also here the quenching gets alleviated.

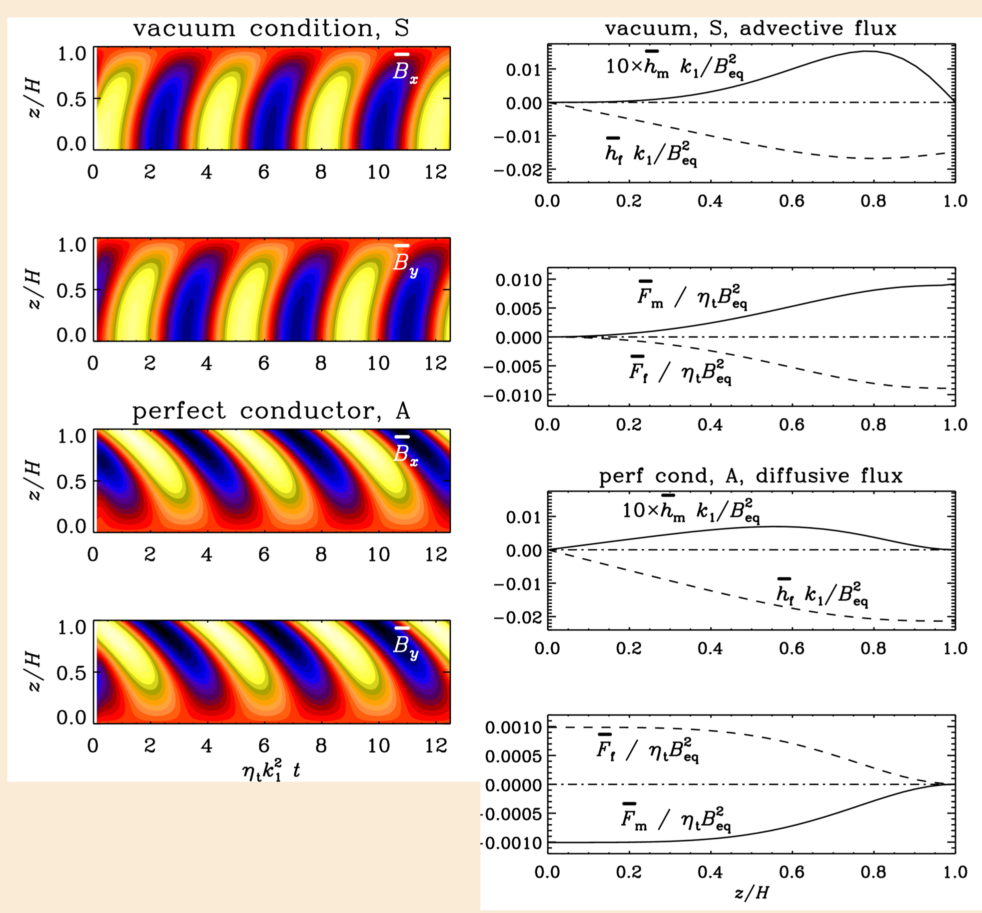

Fig. 4. Left: Space-time diagrams for the mean values of B_x and B_y.
Right (upper): z-profile of the small and large scale helicities and their fluxes in a simulation allowing for advective fluxes out of the domain.
Right (lower): z-profile of the small and large scale helicities and their fluxes in a simulation including Fickian diffusion through the equator.

Results

Gauging the Magnetic Helicity

Equator ward migration

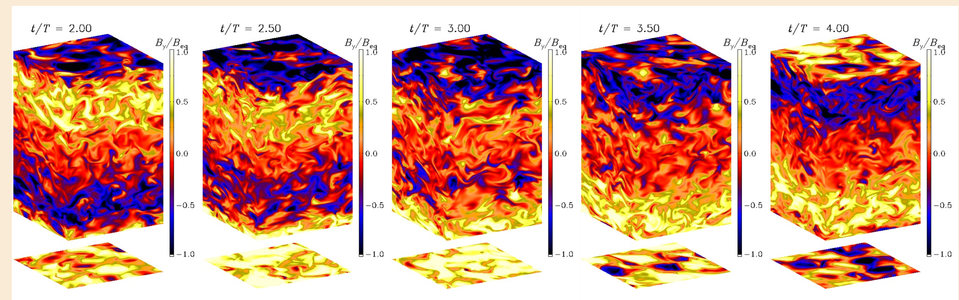

FIG. 5. B_y component of the magnetic field at the perifery of the domain at different times. Note the equator ward migration of the large scale field.

Fickian diffusion can model small scale helicity fluxes through the equator.

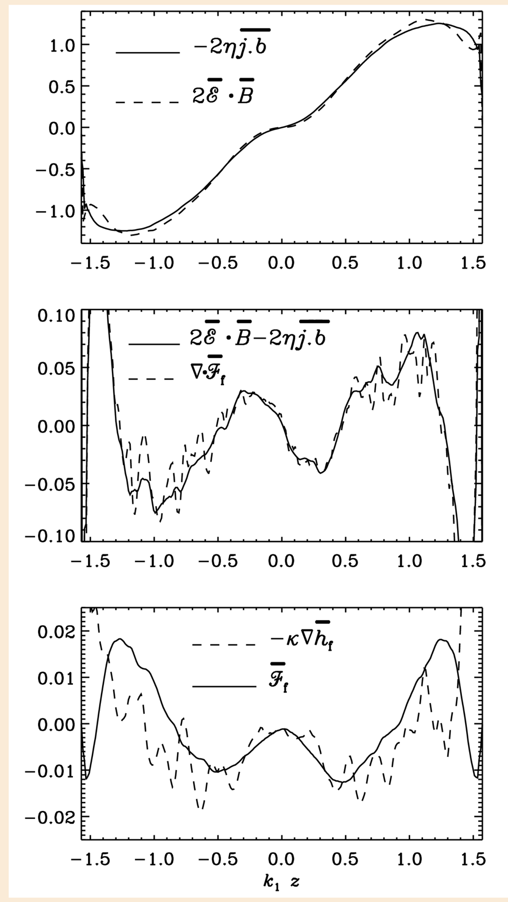

FIG. 6. z-dependence of the contributions to the helicity fluxes (upper two panels) and diffusive helicity flux (lower panel) .

Conclusions

- Magnetic helicity fluxes out of the domain can alleviate catastrophic quenching for high magnetic Reynolds numbers.
- Magnetic helicity fluxes through the equator can also alleviate the quenching.

- Magnetic helicity fluxes in DNS are independent of the gauge.
- Magnetic helicity fluxes follow a Fickian diffusion law. $\kappa_f \approx 0.3\eta_t$

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