The Kinetic Helicity Needed to Drive Large-Scale Dynamos

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Introduction

Helically driven turbulence in a conducting medium can lead to large-scale magnetic fields. Conditions for a positive amplification effect are sufficient helicity of the velocity field and a high enough scale separation ratio between the large scales and the turbulent scales (Brandenburg, 2001). The critical values for the parameters are derived in the mean-field approach and confirmed in 3d direct numerical simulations.

Mean-Field Predictions

Growth rate of the mean-field dynamo:

$$\lambda = \alpha k - \eta_{\rm T} k^2 = (C_\alpha - 1)\eta_{\rm T} k^2$$

$C_\alpha \ = \ \alpha/(\eta_{\rm T} k)$ dynamo number

Large-scale dynamo growth for:

$$C_\alpha^{\rm crit} \ = \ 1$$

Normalized kinetic helicity:

$$\epsilon_{\rm f} = \langle \boldsymbol{\omega} \cdot \boldsymbol{u} \rangle / (k_{\rm f} \langle \boldsymbol{u}^2 \rangle)$$

$$C_\alpha \approx -\frac{\langle \boldsymbol{\omega} \cdot \boldsymbol{u} \rangle}{\iota k_1 u_{\rm rms}^2} = -\frac{\epsilon_{\rm f} k_{\rm f}}{\iota k_1}$$

$$\iota \ = \ 1 + 3/{\rm Re}_{\rm M} \equiv \eta_{\rm T}/\eta_{\rm t}$$

$${\rm Re}_{\rm M} = u_{\rm rms}/(\eta k_{\rm f})$$

→ critical normalized kinetic helicity:

$$\epsilon_{\rm f}^{\rm crit} \ \propto \ (k_{\rm f}/k)^{-1} \quad (1)$$

Connection to 3d turbulence

Saturation:

$$\langle \boldsymbol{J} \cdot \boldsymbol{B} \rangle = \epsilon_{\rm m} k_{\rm m} \langle \boldsymbol{B}^2 \rangle$$

$$B_{\rm sat}^2 / B_{\rm eq}^2 \approx (|C_\alpha|/\epsilon_{\rm m} - 1)\,\iota \quad (2)$$

(Blackman, 2002)

$$B_{\rm eq} = (\mu_0 \overline{\rho})^{1/2} u_{\rm rms}$$

Results by Pietarila Graham

By performing direct numerical simulation in 3 dimensions J. Pietarila Graham et al. (2012) infered a power law dependence with a power of -3, instead of -1:

$$\epsilon_{\rm f}^{\rm crit} \propto (k_{\rm f}/k_1)^{-3}$$

→ conflict with mean-field predictions

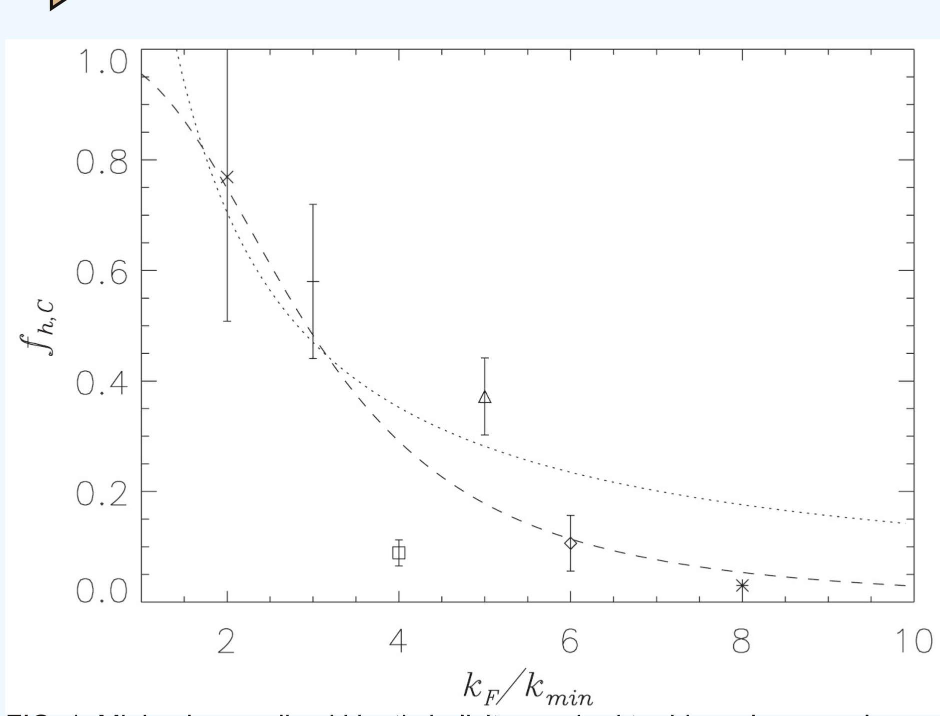

FIG. 1. Minimal normalized kinetic helicity required to drive a large-scale dynamo in dependence of the scale separation ratio (J. Pietarila Graham et al. 2012)

Approach

- Resistive magnetohydrodynamics
- Helical forcing function f
- Triply periodic box (magnetic helicity conservation)

$$\frac{\partial}{\partial t}\boldsymbol{A} = \boldsymbol{U} \times \boldsymbol{B} - \eta \mu_0 \boldsymbol{J}$$

$$\frac{\rm D}{{\rm D}t}\boldsymbol{U} = -c_{\rm s}^2 \boldsymbol{\nabla} \ln \rho + \frac{1}{\rho}\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{F}_{\rm visc} + \boldsymbol{f}$$

$$\frac{\rm D}{{\rm D}t} \ln \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{U}$$

Results

Exponential growth of the mean magnetic field is followed by slow saturation (Fig. 1). At saturation only one average for mean B survives (Fig. 2).

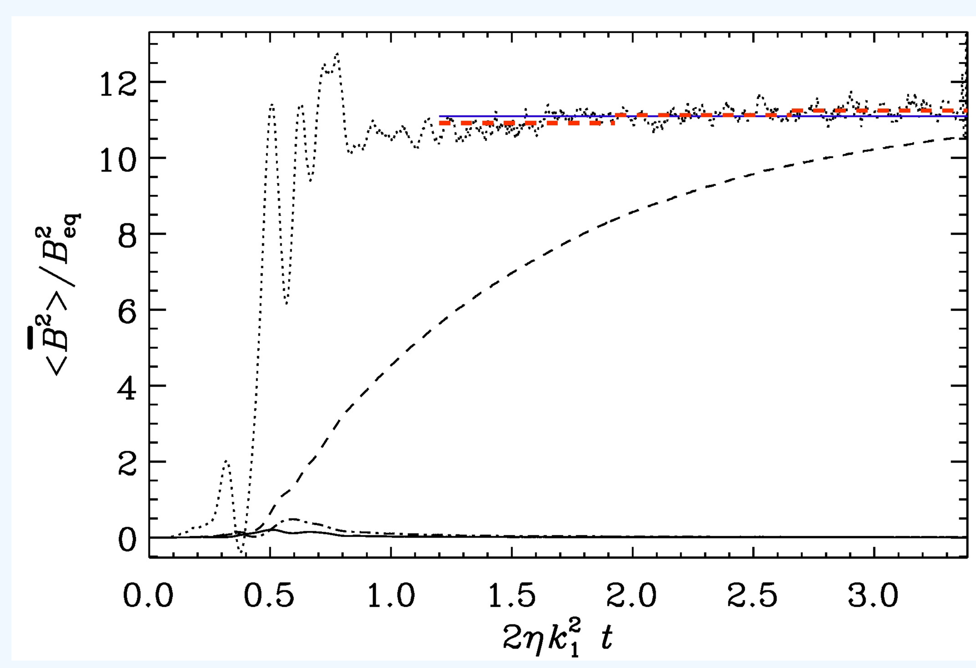

FIG. 1. Mean magnetic energy vs. resistive time calculated as average in the xz direction (dashed line), in xy (solid) and yz (dash-dotted).

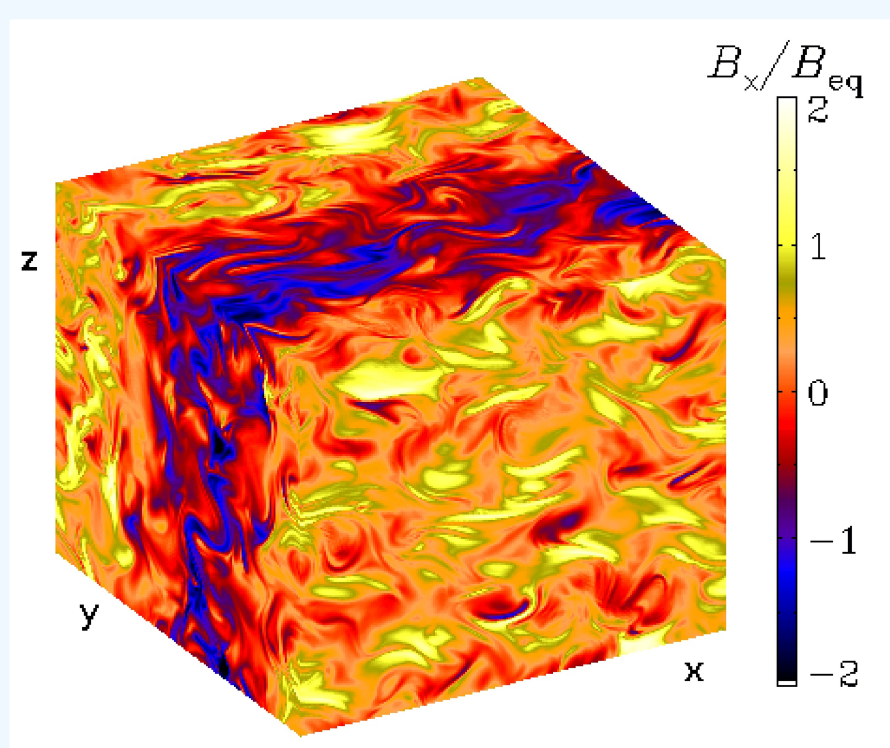

FIG. 2. x-component of the magnetic field on the periphery of the simulation box after resistive saturation.

The linear dependence for the mean magnetic field predicted in eq. (2) is reproduced, apart from a slight offset (Fig. 3).

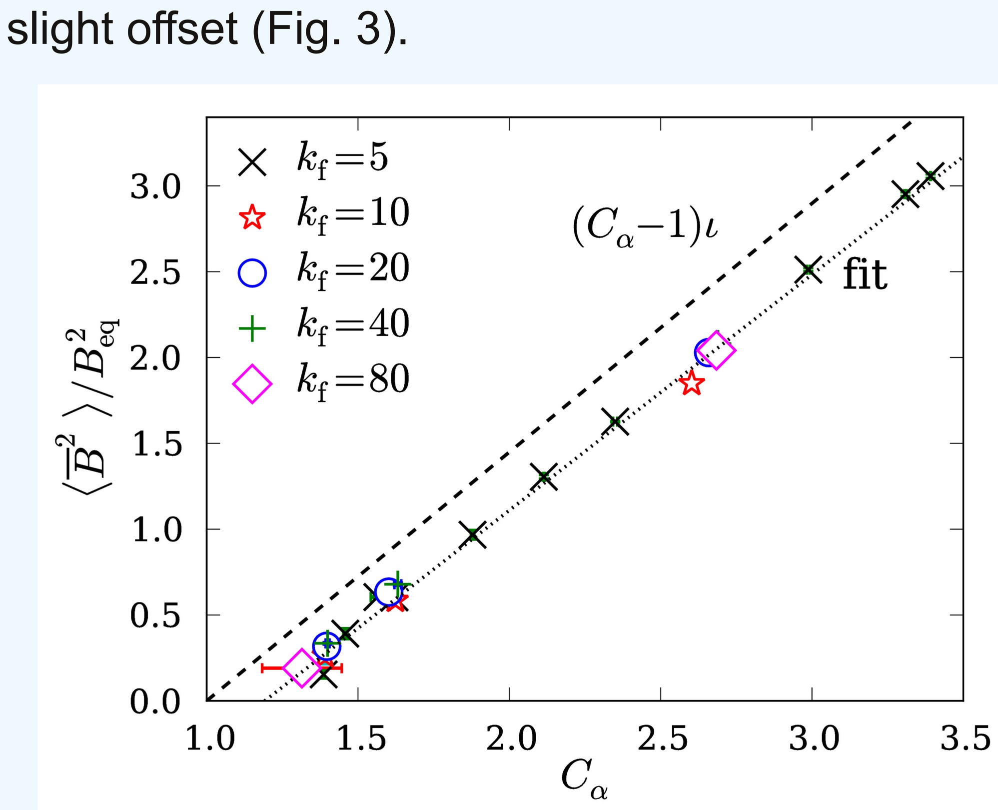

FIG. 3. Steady state values for the normalized mean magnetic enerrgy in dependence of the dynamo number for different scale separation ratios. The dashed line shows the theoretical prediction and the dotted a linear fit.

Results

Saturation values for the mean magnetic energy follow a linear behavior in dependence of the normalized kinetic helicity (Fig. 4).

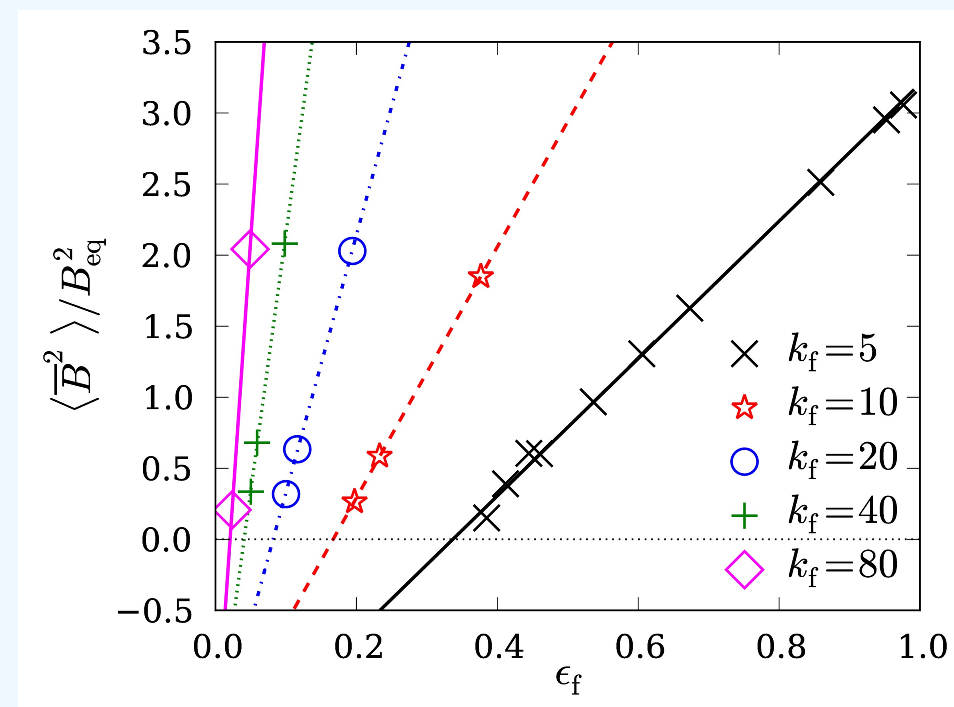

FIG. 4. Steady state values for the normalized mean magnetic energy for various scale separation ratios together with linear fits.

Using the linear fits in Fig. 4 we can extrapolate the critical values for $\epsilon_{\rm f}$ for large-scale dynamo action to occur. The dependence in kf/k1 follows precisely the predictions by equation (1) (Fig. 5), rather than a power -3 (J. Pietarila Graham et al., 2012).

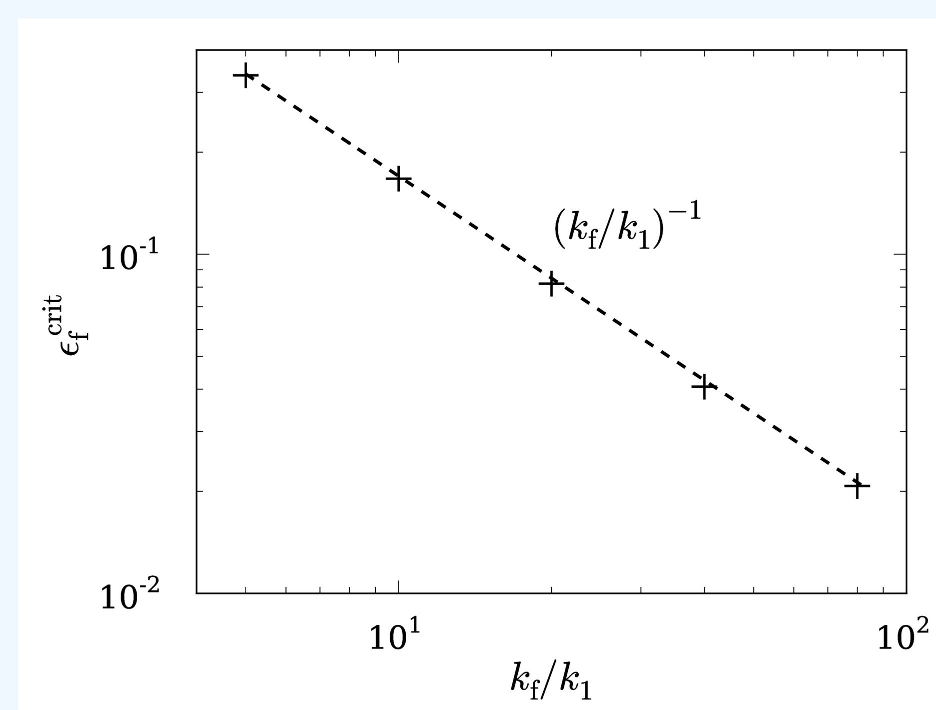

FIG. 5. Steady state values for the normalized mean magnetic energy for various scale separation ratios together with a fit.

Conclusions

- Critical values for large-sale dynamo action confirmed.
- Results by Pietarila Graham contaminated by small-scale dynamo.

References

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