# Force-Free Relaxation of Braided Coronal Magnetic Fields

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## Introduction

With its low plasma beta, the dynamics in the solar corona is dominated by magnetic fields. Any static equilibrium can therefore only be obtained if the Lorentz force vanishes, i.e.

$$\mathbf{F}_{\rm L} = \mathbf{J} \times \mathbf{B} = 0 \quad \text{(force-free state)}$$

equivalently:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B}, \quad \mathbf{B} \cdot \nabla \alpha = 0.$$

Magnetic field evolution in the solar corona can be considered to good approximation ideal, i.e. it follows the evolution equations of ideal MHD:

$$\partial \mathbf{B} / \partial t = \nabla \times (\mathbf{U} \times \mathbf{B}).$$

Parker (1972) suggested that under such an evolution sufficiently braided magnetic fields would evolve towards force-free state containing discontinuities corresponding to singular current sheets. His hypothesis has been under debate for 40 years.

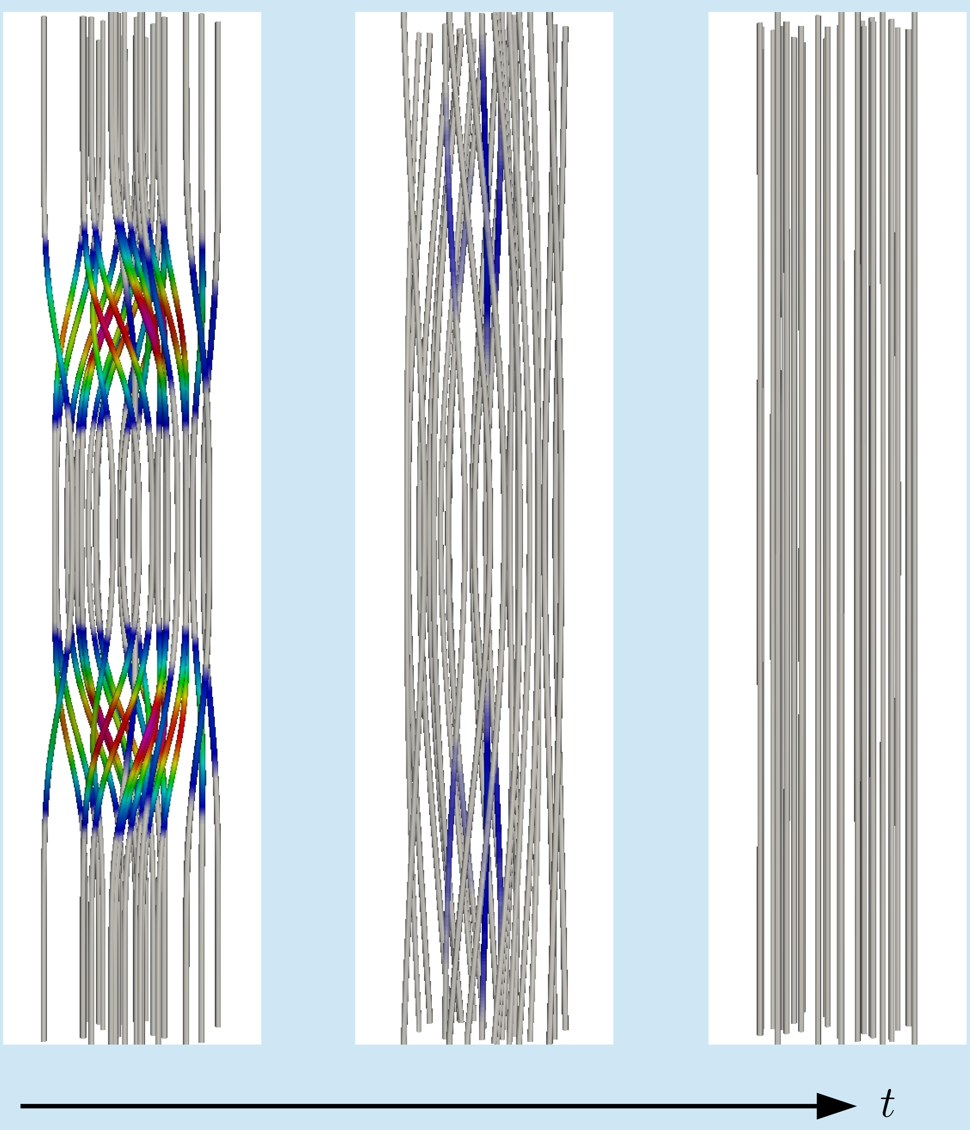
## Problem

Find **force-free** states of initially braided fields under ideal evolution. Test whether or not **current sheets** form and under which conditions. We first need to develop a method to find these.

## Approach

We develop a numerical code for simulating the ideal relaxation of braided magnetic fields. This code uses **Lagrangian** methods for moving grids and recently developed **mimetic** numerical operators which greatly reduce numerical errors. Test configurations are used for benchmarking our code, while highly braided fields are simulated to test Parker's hypothesis. We compare classical numerical operators with the novel mimetic operators.

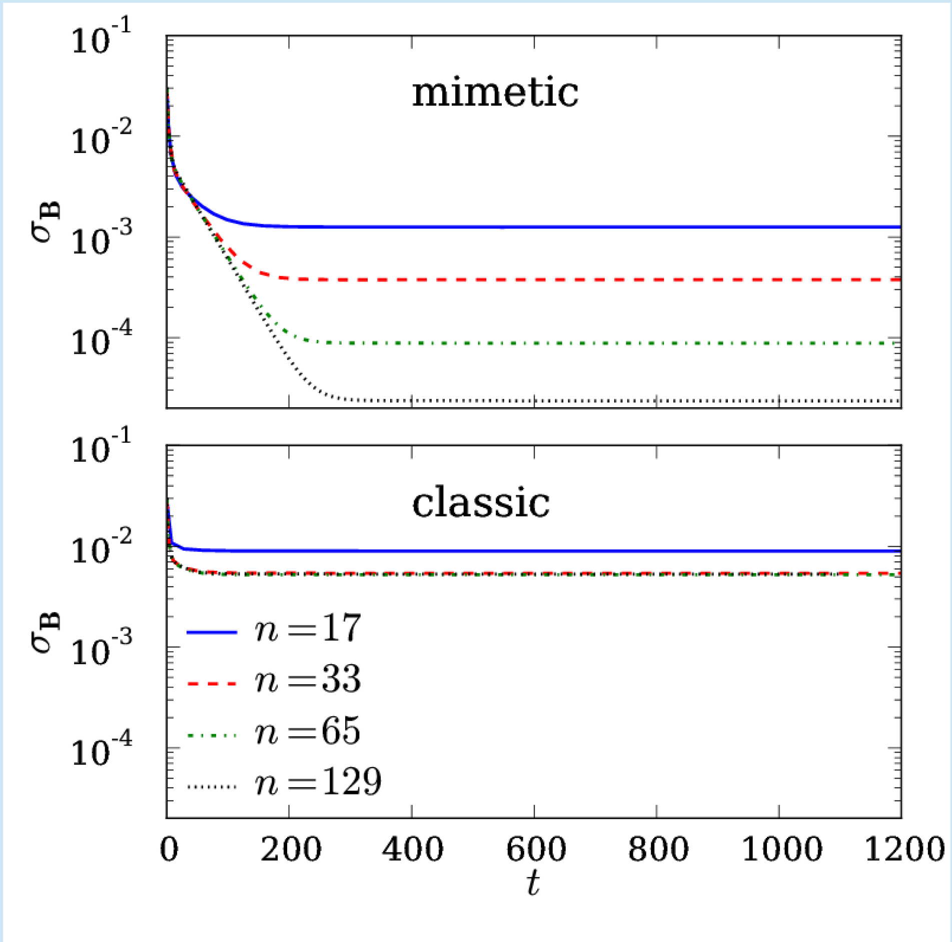
## Field Relaxation



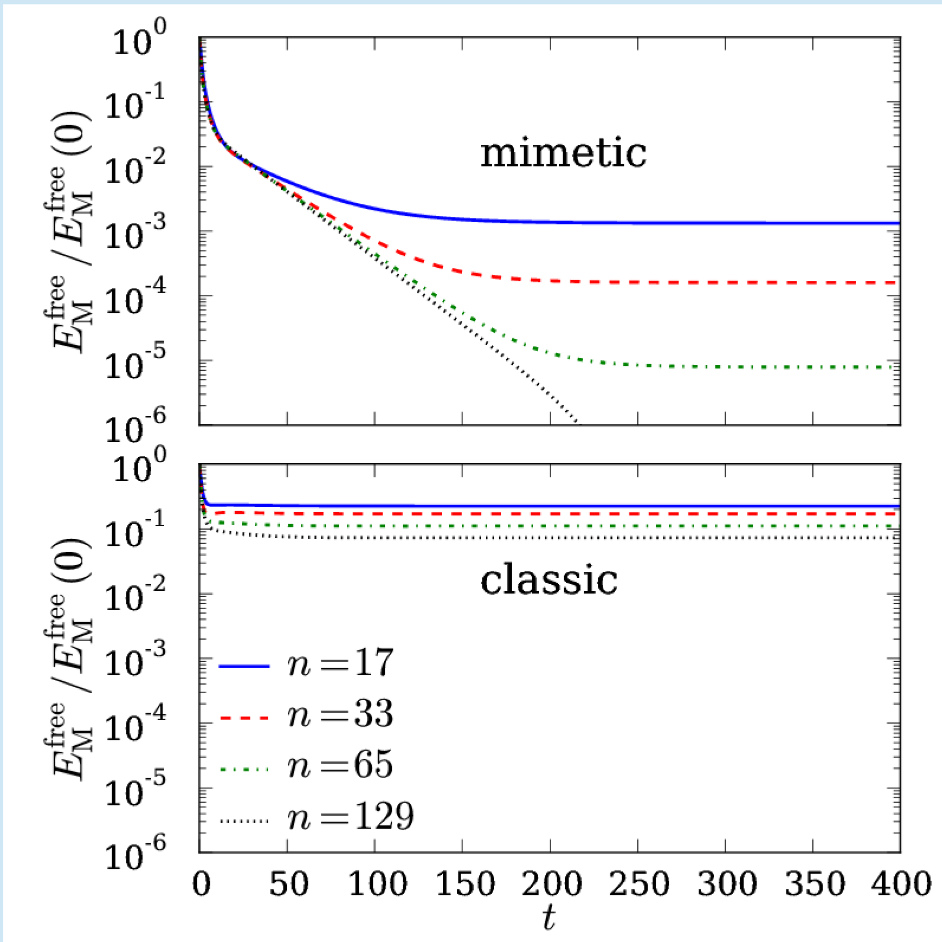

Evolution of the magnetic streamlines for a simply braided magnetic field. As expected, the field relaxes into the potential field with constant z-component.

With our code we are able to simulate the relaxation of magnetic fields into the force-free state, which is a great improvement compared to previous approaches (Craig et al. 1986).

## Quality of the Relaxation




Deviation of the magnetic field from the analytically obtained results as a function of time for different resolutions *n* using mimetic numerical operators (upper panel) and classical numerical operators (lower panel).

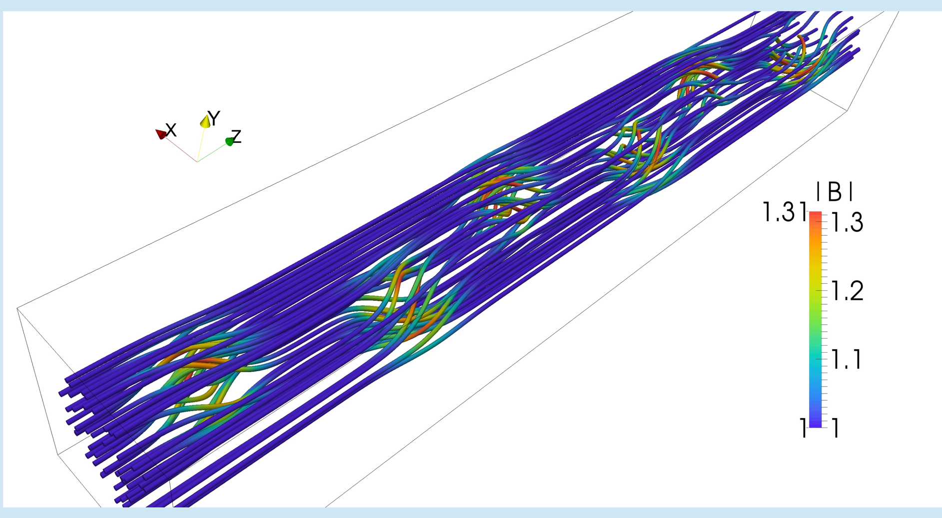



Evolution of the free magnetic energy for different resolutions *n* using mimetic numerical operators (upper panel) and classical numerical operators (lower panel).

By using mimetic numerical operators our code is able to relax the field much closer to the expected minimum energy state, which is the force-free state.

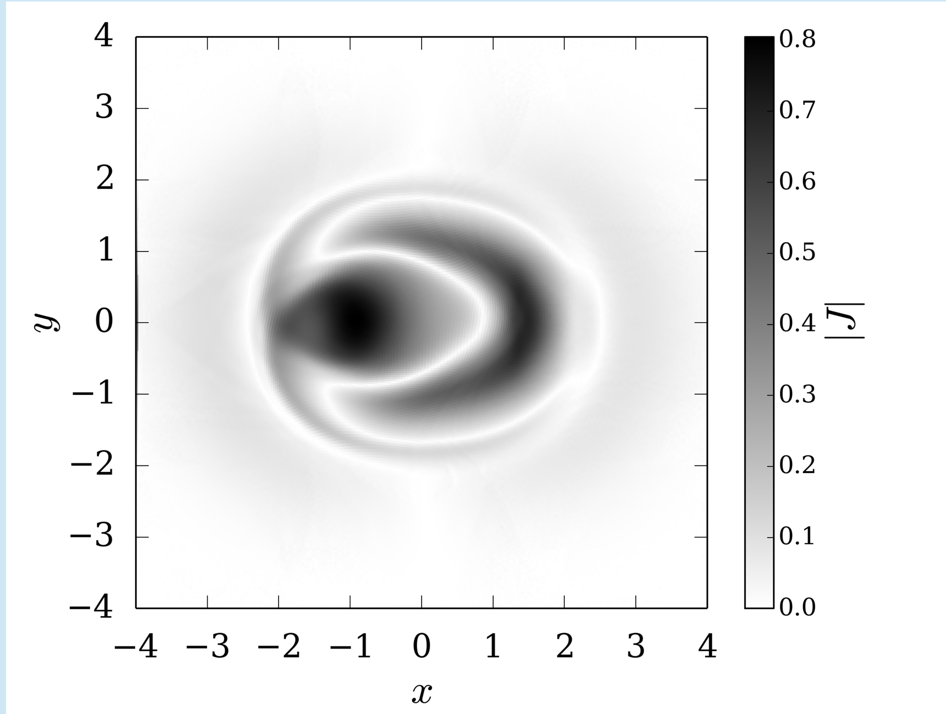
## Strongly Twisted Fields

We use the strongly twisted magnetic field configuration from Wilmot-Smith et al. (2009) and test whether or not singular current sheets develop.




Initial magnetic field configuration used for testing the Parker hypothesis. The colour coding shows the magnetic field strength.

## Strongly Twisted Fields (cont.)

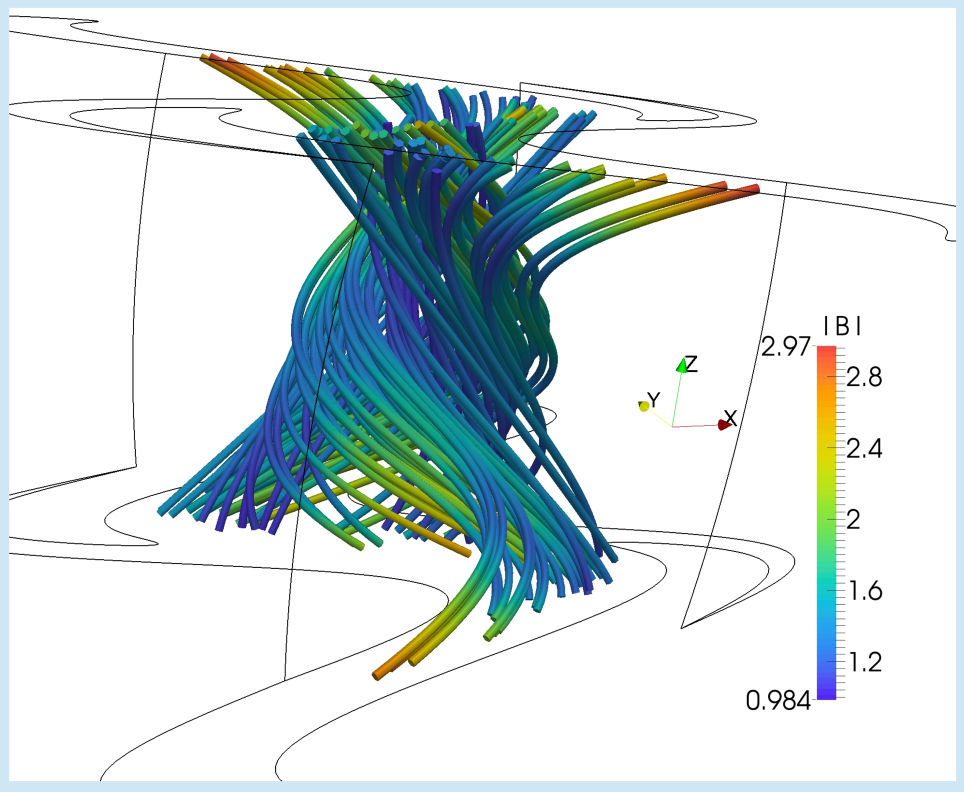



Cut through the xy-plane of the electric current density for the strongly twisted field after relaxation.

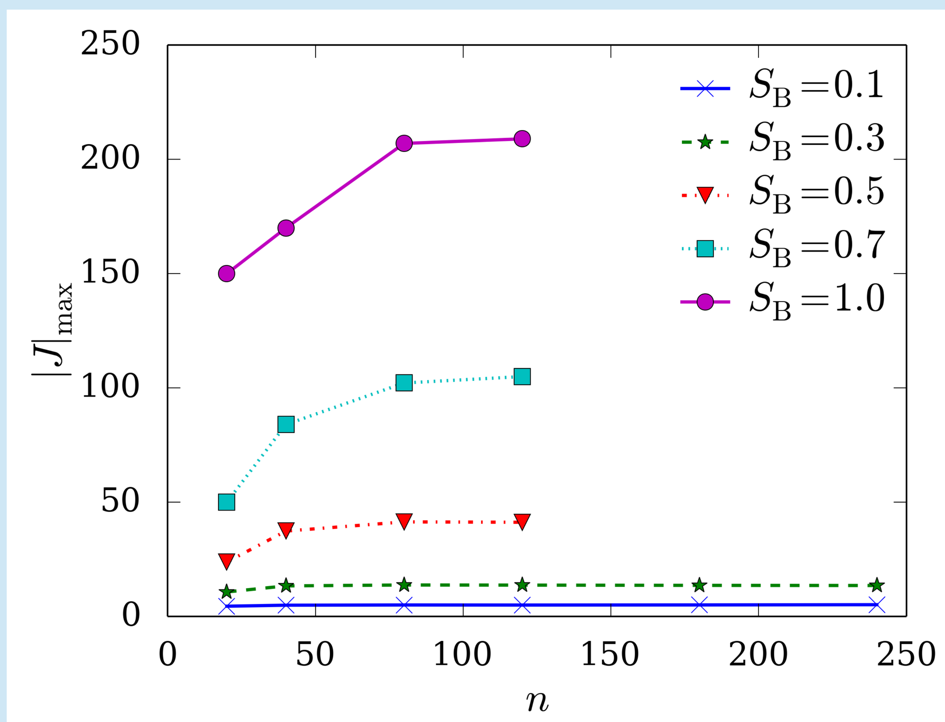
There is no evidence for the formation of singular current sheets. All our simulations show that the spatial variation of the current is well resolved and its strength does not diverge.

## Sheared Field Lines

We make use of the strongly sheared magnetic field configuration introduced by Longbottom et al. (1998) who claimed evidence for singular currents. With our methods we push the maximum resolution to ca. 4 times of theirs.




Initial magnetic field for the sheared field configuration. The colour coding shows the magnetic field strength.




Maximum electric current density after relaxation for the sheared field configuration for different resolutions *n* and shearing strength $S_B$.

In contrast to previous findings we do not observe diverging currents with increasing resolution.

## Contact

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## References


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- Craig, I. J. D. & Sneyd, A. D. 1986, ApJ, **311**, 451
- Wilmot-Smith A. et al. 2009, ApJ, **696**, 1339
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## Conclusions

- We developed a numerical code for the ideal relaxation of magnetic fields using mimetic numerical operators.
- Strong improvements of the relaxation quality are obtained compared to classical methods.
- No evidence for singular current sheets is found for strongly braided or sheared fields.